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ABSTRACT

It is suggested that the energy difference between the Gamow-Teller and Fermi resonances is given, in units of MeV, by $E_{GT} - E_F = 26A^{-1/3} - 18.5\ (N-Z)/A$. The consequences of this result on the strength of the spin and isospin dependent residual interaction, as well as on the effective axial-vector coupling constant, are discussed.

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The Fermi (F) resonance has been known since the discovery of the well-defined multiplet structure by Anderson and Wong[1], but the Gamow-Teller (GT) one, although foreseen in the early 1960's[2-6], was observed experimentally for the first time only in 1975, by Doering et al.[7] (through the $^{90}Zr(p,n)^{90}Nb$ reaction at incident proton energies of 35 and 45 MeV). This GT resonance was later on confirmed by a new study at E_p = 120 MeV[8]. The gathering of the GT strength in ^{208}Pb was detected even more recently, by Horen et al.[9], in the neutron spectra resulting from bombardment with 120 and 160 MeV protons. Almost simultaneously, Horen et al.[10] have reported a sistematic study of the energetics of the GT strength for a number of targets with A > 90, with incident proton energies of 120, 160 and 200 MeV. They have also suggested that the energy difference between the GT and F resonances can be represented approximately by (in units of MeV)

$$E_{GT} - E_F = 6.7 - 60T_oA^{-1} \quad , \qquad (1)$$

where T_o = (N-Z)/2 is the isospin of the target nucleus.

Quite recently, Sterrenburg et al[11] have performed a (p,n) reaction study with 45 MeV protons at small angles on 17 targets ranging from ^{90}Zr to ^{208}Pb. Common features of the spectra were: i) a sharp peak representing the F resonance and ii) a broad peak, interpreted as a GT state, at a slightly higher but target dependent excitation energy.

The above mentioned systematic behaviour of the energy difference $E_{GT}-E_F$, throughout the periodic table, calls for a discussion in the framework of schematic models, in order to elucidate upon the details of the underlying structure.

As the ground state correlations are, in general, negligible for F and GT excitations (in nuclei with an appreciable neutron excess), it is enough to work within the Tamm-Dancoff approximation. We will use a schematic residual force of the form[12-15]

$$H = \frac{\kappa}{2} \sum_{\mu_\tau} \mathcal{M}^\dagger(\tau=1,\mu_\tau).\mathcal{M}(\tau=1,\mu_\tau), \tag{2}$$

where κ is the coupling constant and

$$\mathcal{M}(\tau=1,\mu_\tau) = \begin{cases} \sum_{i=1}^{A} \tau_{\mu_\tau}(i) & \text{for the F mode,} \\ \sum_{i=1}^{A} \vec{\sigma}(i)\tau_{\mu_\tau}(i) & \text{for the GT mode.} \end{cases} \tag{3}$$

Furthermore, a degenerate model for the single- particle energies is assumed,

$$E^{(o)} = \varepsilon - \frac{V_1}{A} T_o + \Delta E_{Coul}, \tag{4}$$

where V_1 is the symmetry potential, ΔE_{Coul} is the Coulomb energy displacement and ε represents the average single particle energy. For the F resonance $\varepsilon \simeq 0$, while for the GT mode this quantity should be of the same order of magnitude as the spin-orbit splitting $\Delta_{\ell s}$, i.e.,

$$\varepsilon_{GT} \simeq \Delta_{\ell s} \simeq 20\ A^{-\frac{1}{3}}\ \text{MeV}, \tag{5}$$

where we have employed the estimate $\mathcal{E} \simeq A^{\frac{1}{3}}$ [12).]

As the interaction (2) shifts the collective states, both F and GT, by $4\kappa T_o$, the perturbed energies are

$$E = \varepsilon - T_o(V_1/A - 4\kappa) + \Delta E_{Coul}, \tag{6}$$

In the case of the F resonance (ε_F=0) this expression just reflects the self-consistency between the Hartree-Fock field and the collective motion[16]; namely as the excitation energy E_F should equal ΔE_{Coul}, one has κ_F=$V_1/4A$. (Care must be taken in attaching a physical meaning to the effective mass when this self-consistency is not fullfiled). Consequently,

$$E_{GT} - E_F = \varepsilon_{GT} + 4T_o(\kappa_{GT} - \kappa_F). \tag{7}$$

If, consistent with later discussion, it is assumed that both κ_F and κ_{GT} are proportional to A^{-1}, one sees that the second term of this expression has the same mass and charge dependence as the second term in eq. (1). However, from the estimate (5) one would expect an $A^{-\frac{1}{3}}$ dependence for the first term in eq. (1) instead of it being a constant. This fact induced us to try to adjust the difference E_{GT}-E_F by a function of the form $C_1 A^{-1/3} + C_2\, T_o A^{-1}$, C_1 and C_2 being free parameters. Inclusion of 25 experimental data[8-11], as shown in fig. 1, yields (in MeV)

$$E_{GT} - E_F = 26\ A^{-\frac{1}{3}} - 37\ T_o A^{-1}, \tag{8}$$

with a χ^2=2.33. The same procedure was repeated by omitting the mass dependence $A^{-1/3}$ and the result was (in MeV)

$$E_{GT} - E_F = 7.0 - 57.8\ T_o A^{-1}, \tag{9}$$

with χ^2=5.08. The small difference between eqs. (1) and (9) arises from the fact that we have considered more experimental data than Horen et al.[10].

First of all it should be noted that the coefficient of the T_o/A term is considerably reduced when the $A^{-\frac{1}{3}}$ dependence is introduced. Furthermore, as the first term in eq. (8) has a very simple physical meaning, and as eq. (8) reproduces the experimental data much better than eq. (9), in the following discussion we omit further references to the latter. That is, it will be assumed that the correct A and T_o dependence for the energy difference E_{GT}-E_F is the one given by eq. (8). Consequntely, from eq. (7) we have

$$(\kappa_F - \kappa_{GT})A = 9.25\ \text{MeV}. \tag{10}$$

Adopting now for the symmetry potential the values suggested by Bohr and Mottelson[12], and remembering that $A\kappa_F = V_1/4$, we obtain

$$A\kappa_F = 25 \text{ MeV}, \quad A\kappa_{GT} = 15.75 \text{ MeV}, \quad \text{for } V_1 = 100 \text{ MeV}, \tag{11a}$$

and

$$A\kappa_F = 32.5 \text{ MeV}, \quad A\kappa_{GT} = 23.25 \text{ MeV}, \quad \text{for } V_1 = 130 \text{ MeV}. \tag{11b}$$

The above κ_{GT} should be used when only nucleonic degrees of freedom are used. When the effect of the coupling between the nucleon and the isobar $\Delta(1230)$ is included, the GT strength is [17,18]

$$\kappa_{GT}^{\Delta} = \frac{\kappa_{GT}}{1 + \kappa_{GT}\, \chi_{\Delta}^{(o)}}, \tag{12a}$$

where $\chi_{\Delta}^{(o)}$ is the unperturbed isobar-hole response function. The inverse relation reads

$$\kappa_{GT} = \frac{\kappa_{GT}^{\Delta}}{1 - \kappa_{GT}^{\Delta}\, \chi_{\Delta}^{(o)}}. \tag{12b}$$

By employing the constituent quark-model value of 72/25 for the ratio of the $\pi N\Delta$ and πNN coupling constants (i.e., $g^2_{\pi N\Delta}/g^2_{\pi NN} \equiv r_{\pi N\Delta}$), Bohr and Mottelson[17] have shown that

$$\chi_{\Delta}^{(o)} = -\frac{64}{25}\frac{A}{E_{\Delta}}, \tag{13a}$$

where $E_{\Delta} \simeq 300$ MeV if one ignores nuclear excitation energies in comparison with the $\Delta - N$ mass difference. Consequentely, from the results (11) we see that $A\kappa_{GT}^{\Delta} = 18.2$ and 29.0 MeV for $V_1 = 100$ and 130 MeV, respectively. The second of these values for κ_{GT}^{Δ} is the same as that obtained by Bohr and Mottelson[17] from the analysis of quenching phenomena in the magnetic dipole moments in the region of ^{208}Pb.

When the experimental value $r_{\pi N\Delta} = 4$, is used rather than that given by the quark model, as done by Brown and Rho[18], one has

$$\chi_{\Delta}^{(o)} = -\frac{32}{9}\frac{A}{E_{\Delta}}. \tag{13b}$$

With this value for the response function the above mentioned results for κ_{GT}^{Δ} are sightly increased. (It is worth noting that Vento et al.[19] have indicated quite recently that the experimental values for the axial-vector coupling constant g_A and for the ratio $r_{\pi N\Delta}$ provide consistent indications of substantial bag deformation.)

We can easily relate the coupling constants κ_F and κ_{GT} of the separable force given by eq. (2), which has an infinite range, with the parameters f_o' and g_o' of the zero-range Migdal force[20-22]

$$F^{ph}(\vec{r}_1,\vec{r}_2) = C(f_o'\vec{\tau}_1\cdot\vec{\tau}_2 + g_o'\vec{\tau}_1\cdot\vec{\tau}_2\ \vec{\sigma}_1\cdot\vec{\sigma}_2)\ \ \delta(\vec{r}_1-\vec{r}_2) \tag{14}$$

where $C = 380$ MeV fm^3.[≠1] The corresponding relations are

$$\kappa_F = \frac{Cf_o'}{\Omega}, \quad \kappa_{GT} = \frac{Cg_o'}{\Omega}, \tag{15}$$

where Ω stands for the nuclear volume. When the nucleus is treated as a Fermi gas $\Omega = (4\pi/3)r_o^3 A = 0.58\ (4\pi A\,\text{fm}^3)$ with $r_o \simeq 1.2$ fm. A more reliable estimate can be obtained from the shell model, where Ω^{-1} is the average value of the radial integrals

≠1 It should be noted that the Migdal force is the direct term of a δ- function interaction.

$$I(\ell_1\ell_2\ell_1'\ell_2') = \frac{1}{4\pi}\int R_{n_1\ell_1}(r)\, R_{n_2\ell_2}(r)\, R_{n_1'\ell_1'}(r)\, R_{n_2'\ell_2'}(r)\, r^2 dr \ . \qquad (16)$$

Using wave functions of an harmonic oscillator potential, one has[4,5)]

$$\Omega = \langle I(\ell_1\ell_2\ell_1'\ell_2')\rangle_{av}^{-1} \simeq 4\pi A \text{ fm}^3 \ . \qquad (17)$$

Consequently, from eqs.(10),(11),(15)and (17),we obtain that

$$f_o' - g_o' \simeq 0.31, \qquad (18)$$

and

$$f_o' \simeq 0.83\ , \quad g_o' \simeq 0.52, \qquad (19a)$$

and

$$f_o' \simeq 1.07\ , \quad g_o' \simeq 0.77, \qquad (19b)$$

for V_1 = 100 and 130 MeV, respectively.

Quite recently, Bertsch et al.[23)] have pointed out that the systematics of the GT states, when studied with a zero range interaction, require an interaction strength g_o' of about 0.53 to 0.63 ($V_{\sigma\tau} = Cg_o' \simeq$ 200-240 MeV fm^{-3}). With a similar value for g_o'(=0.65) Krewald et al.[24)] and Osterfeld et al.[25)] have succeeded in reproducing both the allowed and first forbidden charge-exchange resonances in ^{208}Pb. On the other hand, from the investigation of magnetic states in ^{12}C and ^{16}O a value of g_o'=0.75 was obtained[26)]. In addition, Brown and Rho[18)] have shown that, when one uses a $\pi + \rho$ exchange force in the nucleon space and includes the screening due to isobar-hole excitations, there results an effective coupling constant g_o' equal to 0.9.

For a δ-interaction of the form[5,6)]

$$V(\vec{r}_1,\vec{r}_2) = -2\pi(V_s P_s + V_t P_t)\ \delta(\vec{r}_1-\vec{r}_2), \qquad (20)$$

where P_s and P_t are singlet-even and triplet-even projection operators one has

$$\kappa_F = \frac{\pi(3V_t - V_s)}{4\Omega}\ ,\quad \kappa_{GT} = \frac{\pi(V_s + V_t)}{4\Omega}\ . \qquad (21)$$

With the values derived here for κ_F and κ_{GT} we get

$$V_t = 163 \text{ MeV fm}^3,\ V_s = 89 \text{ MeV fm}^3,\ (V_1 = 100 \text{ MeV}), \qquad (22a)$$

and

$$V_t = 223 \text{ MeV fm}^3,\ V_s = 149 \text{ MeV fm}^3, (V_1 = 130 \text{ MeV}), \qquad (22b)$$

when Ω=4πA fm^3. Ikeda[5)] has employed, in his study of the F and GT resonances in ^{208}Pb, a value of 50 MeV fm^3 for V_s,and he varied V_t from 83 to 200 MeV fm^3.

Let us now estimate the effective axial-vector coupling constant for allowed β decay processes. In the context of the approximation used here it reads

$$\left(\frac{g_A^{eff}}{g_A}\right)_{GT} = \frac{1 + \kappa_{GT}\chi_\Delta^{(o)}}{1 - \kappa_{GT}\chi_N^{(o)}} = \frac{1}{1 - \kappa_{GT}^{\Delta}(\chi_N^{(o)} + \chi_\Delta^{(o)})}\ , \qquad (23)$$

where

$$\chi_N^{(o)} = -\frac{4T_o}{E_{GT}^{(o)}} \qquad (24)$$

is the unpertubed particle-hole response function and $E_{GT}^{(o)} = \Delta_{\ell s} - 4T_o\kappa_F + \Delta E_{Coul}$. For example, with V_1=130 MeV one has that $(g_A^{eff}/g_A)_{GT} \simeq$ 0.5, 0.5 and 0.4 for ^{90}Zr, ^{116}Sn and ^{208}Pb, respectively. A similar result for the quenching effect on the axial-vector coupling constant can be found in the literature[28)].

We can summarize our considerations by saying that the energy difference between the Gamow Teller and Fermi resonances obeys the relation (8), which has a very simple physical meaning. This result was used to fix the coupling strength of the spin-isospin dependent residual interaction. It is worth noticing that the values presented for this coupling constant are based upon the underlying assumption of the self-consistency between the Hartree-Fock field and the collective motion. We also estimate the effective axial-vector coupling constant for allowed β decay.

After finishing the present work we have learned about a similar study performed by Suzuki[26)] in ^{48}Ca, ^{90}Zr and ^{208}Pb. His value for $(\kappa_F - \kappa_{GT})$ is smaller than ours by a factor of two.

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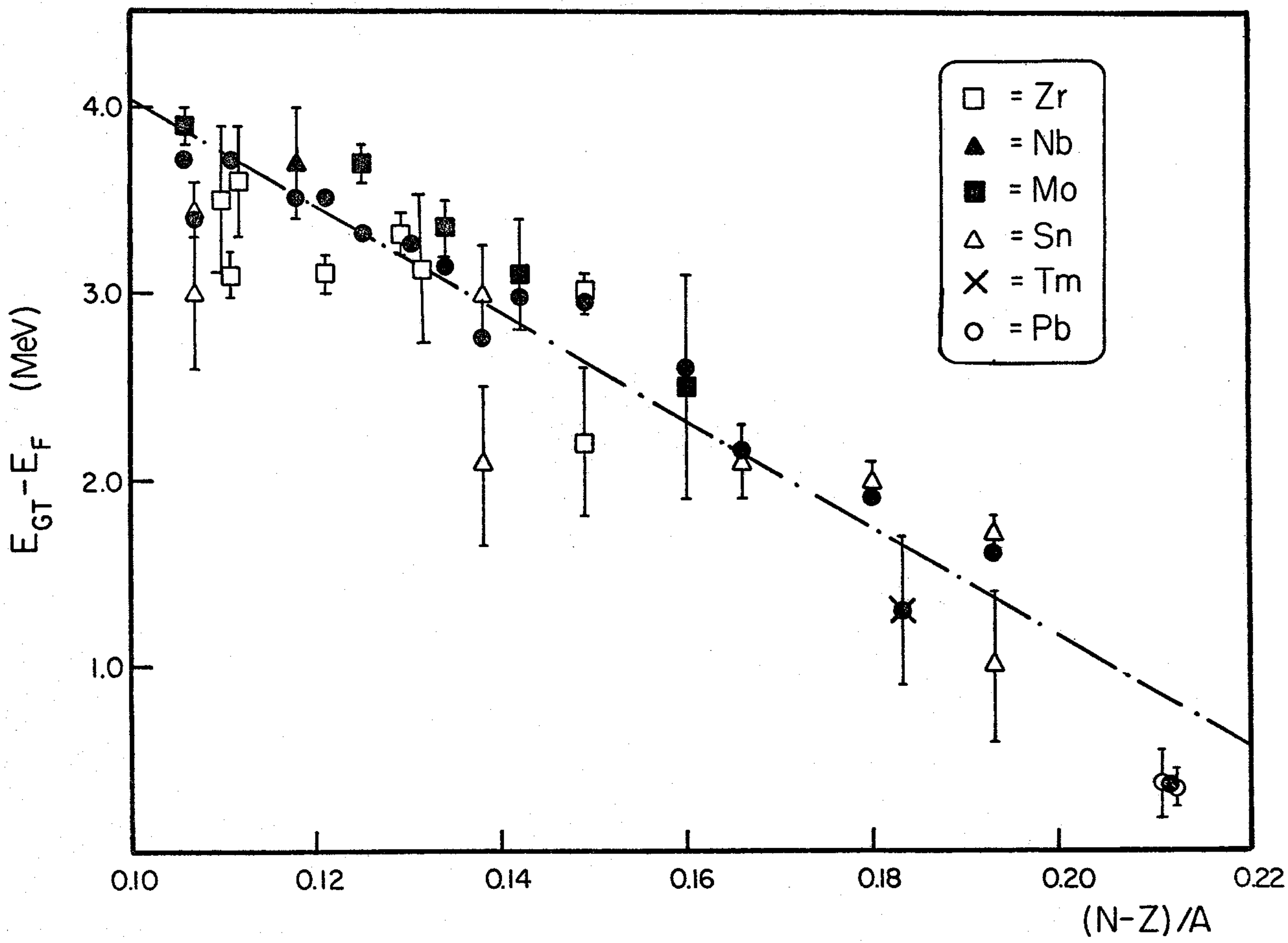
Figure caption

Fig. 1 – Plot of $E_{GT} - E_F$ versus $(N-Z)/A$. The experimental data were taken from ref. 8) for ^{90}Zr, from ref. 10) for ^{90}Zr, ^{92}Zr, ^{94}Zr, ^{112}Sn, ^{116}Sn, ^{124}Sn, ^{169}Tm and ^{208}Pb and from ref. 11) for ^{90}Zr, ^{91}Zr, ^{92}Zr, ^{94}Zr, ^{96}Zr, ^{93}Nb, ^{94}Mo, ^{96}Mo, ^{97}Mo, ^{98}Mo, ^{100}Mo, ^{112}Sn, ^{116}Sn, ^{120}Sn, ^{122}Sn, ^{124}Sn and ^{208}Pb. When the experimental results overlap (in the case of $^{90,92}Zr$ and ^{208}Pb) we displace them slightly with respect to the correct value of $(N-Z)/A$ for the sake of clarity. The values calculated by means of eq. (8) are indicated by full circles, while the dash-dotted line corresponds to eq. (9).



$E_{GT} - E_F$ (MeV)
4.0
3.0
2.0
1.0
0.10
0.12
0.14
0.16
0.18
0.20
0.22
(N-Z)/A
□ = Zr
▲ = Nb
■ = Mo
△ = Sn
× = Tm
○ = Pb