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NON-LINEAR σ MODEL WITH SUPERGRAVITY

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ABSTRACT

We study a locally supersymmetric version of the O(N) non-linear σ model within 1/N expansion scheme, implying interesting features due to the mass generation.

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I. INTRODUCTION

Two dimensional models have been extensively used as a laboratory for higher dimensions, and we are confident to say that in many cases this laboratory was indeed very useful, and current ideas such as confinement and θ vacua (1) appeared first in two dimensions. Many methods were applied succesfully in toy models also in this simplified cases as 1/N expansion (2), instanton gas contribution (3), solitons (4), role of higher conservation laws (5), bound-state problems (6), examples of non-trivial S matrices (7), tests of general hypotesis of quantum field theory (1), renormalization program (8), etc.

We are particularly interested in studying quantum supergravity in two dimensions, so that we are able to exploit all conceptual richness emerging in this context. In two dimensions there does not exist a kinetic term for the Graviton either for the Gravitino fields. Any quadratic non-trivial function for these field is originated through matter field (spinless bosons and spin 1/2 fermions) quantum fluctuations. This can be called as induced supergravity. Ordinary quantum gravity is non-renormalizable (9,10) and quantum supergravity leads, in particular cases, to a renormalizable theory (11,12,13). Indeed, we show that, up to one-loop order, the Graviton two point function exibits only logaritmic ultravioled divergences which can be shown to be zero in two dimensions. This means that quadratic and linear ultraviolet divergences were cancelled out due to supersymmetry.

Some years ago, Deser and Zumino (14) constructed a classical locally supersymmetric lagrangian for the spinning string. We use this approach with the global supersymmetric non-linear O(N) σ model (15). This furnishes us with a 1/N expandable supergravity model, so that we obtain to leading order, the Graviton and Gravitino (induced) propagators. The Gravitino propagator has a zero mass pole, which in two dimensions means confinement[1,16]. But here the question arises, about what is confined. We guess that only states of zero supersymmetric charge survive in this theory. Classicaly it is also possible to define a conserved non-local charge, which if survives quantization, would provide a factorizable S-matrix[9]. Because of confinement we think that there is an anomaly, as in the case of the CP^{n-1} model[17].

In section II we perform the functional integration on the bosonic and fermionic fields obtaining an effective action. Next, section III, we make use of 1/N expansion and calculate the Gravitino and Graviton quadratic functions, up to leading order. The physical interpretation of the results are given in section IV, and finally in section V we draw conclusions.

II. EFFECTIVE ACTION

By imposing local supersymmetry[15] on the non-linear O(N) σ model, it can be shown that the lagrangian density

$$\mathcal{L} = \frac{N}{2f} \sqrt{-g} \Big[\frac{1}{2} g^{\mu\nu}\partial_\mu n_i \partial_\nu n_i + \frac{i}{2} \bar{\psi}^\alpha_i (\partial\!\!\!/)_{\alpha\beta}\psi^\beta_i + + \frac{1}{8} (\bar{\psi}^\alpha_i\psi^\alpha_i)^2 + \bar{\psi}^\alpha_i(\gamma^\mu\gamma^\nu)_{\alpha\beta}\, \partial_\nu n_i G^\beta_\mu - \frac{1}{4}(\bar{\psi}^\alpha_i\psi^\alpha_i)\; \bar{G}^\alpha_\mu(\gamma^\nu\gamma^\mu)_{\alpha\beta}G^\beta_\nu\Big] \qquad (1)$$

is invariant under the following locally supersymmetric transformations for the fields(*)

(*) See Appendix B for invarianceproof of (1).

$$\delta n_i(x) = \bar{\varepsilon}(x)_\alpha \psi^\alpha_i(x) \qquad (2a)$$

$$\delta\psi^\alpha_i(x) = -i(\partial_\mu n_i(x) + \bar{G}^\beta_\mu(x)\psi^\beta_i(x))(\gamma^\mu(x)\varepsilon(x))^\alpha + + \frac{1}{2} n_i(x)(\bar{\psi}^\beta_j(x)\psi^\beta_j(x))\varepsilon^\alpha(x) \qquad (2b)$$

$$\delta e^\mu_a(x) = 2i\, \bar{G}^{\mu\alpha}(x)(\gamma_a)_{\alpha\beta}\varepsilon(x)^\beta \qquad (2c)$$

$$\delta G^\alpha_\mu(x) = -(D_\mu\varepsilon(x))^\alpha \qquad (2d)$$

where:

i) $g^{\mu\nu}(x) = e^\mu_a(x)\, e^\nu_b(x)\eta^{ab}$ (η^{ab} is the - flat space - Minkowskian metric(*))

ii) $\gamma^\nu_{\alpha\beta}(x) = \gamma^a_{\alpha\beta}e^\mu_a(x)$ ($e^\mu_a(x)$ is the "tetrad" Gauge field assoaciated to local general coordinate transformation)

iii) $G^\alpha_\mu(x)$ is the real Gravitino field-associated to local supersymmetry.

iv) $\psi^\alpha_i(x)$ and $n^\alpha_i(x)$ are N component (i=1,...,N) real fermion and boson fields (α is the fermionic index - α=1,2), that obey the constraints

$$n^T_i n_i = 1 \quad \text{and} \quad \bar{\psi}^i n_i = 0$$

Also,

(*) See Appendix A for conventions.

$$D_\mu(x) = \partial_\mu + \frac{1}{2}\gamma_5\omega_\mu(x)$$

where $\omega_\mu(x)$ is the spin connection.

Rescaling* η and ψ

$$\eta \to \eta' = (\frac{N}{2f})^{-1/2}\eta \quad \text{and} \quad \psi' = (\frac{N}{2f})^{-1/2}\psi$$

and writting the Green function functional generator associated to (1), gives:

$$Z(\bar{J},\ldots) = \int [d\psi] \ldots [de^\mu_a]\,\delta_x(\bar{\psi}\eta)\,\delta_x(\eta^2 - \frac{N}{2f})\,\times$$

$$\exp i\int d\hat{x}^2\sqrt{-g}\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta + \frac{i}{2}\bar{\psi}\not{D}\psi +$$

$$+\frac{f}{4N}(\bar{\psi}\psi)^2 + \bar{\psi}\gamma^\mu\gamma^\nu\partial_\nu\eta G_\mu - \frac{1}{4}(\bar{\psi}\psi)(\bar{G}_\mu\gamma^\nu\gamma^\mu G_\nu) +$$

$$+\frac{\bar{J}\eta}{2} + \frac{\bar{\xi}\psi}{2} + \ldots\frac{\bar{j}\mu}{2}G\mu\Big] \qquad (3)$$

Since,

$$\delta_x(\eta^2 - \frac{N}{2f})\,\delta_x(\bar{\psi}\eta)\,\exp\frac{if}{4N}\int d\hat{x}^2\sqrt{-g}(\bar{\psi}\psi)^2 =$$

$$= \int[dc][d\phi][d\alpha]\exp\int d\hat{x}^2\sqrt{-g}\Big[\frac{i\alpha}{\sqrt{N}}(\eta^2 - \frac{N}{2f}) +$$

$$+\frac{i}{2\sqrt{N}}(\bar{\eta}\,\bar{c}\,\psi + \bar{\psi}\,c\,\eta) - \frac{i\sqrt{f}}{N}(\bar{\psi}\psi)\phi - i\frac{\phi^2}{N}\Big] \qquad (4)$$

* We will subsequently omit any index, unless necessary.

and rescaling G_μ,

$$G_\mu \to G'_\mu = \sqrt{N}\,G_\mu$$

gives for (3):

$$Z(\bar{J},\ldots) = \int[d\psi]\ldots[de^\mu_a]\exp\int d\hat{x}^2\sqrt{-g}\Big[-\frac{1}{2}\bar{\eta}\,\Delta_B\eta -$$

$$-\frac{1}{2}\bar{\psi}\,\Delta_F\psi + \frac{i\bar{\psi}}{2\sqrt{N}}\big[c + \gamma^\mu\gamma^\nu G_\mu\partial_\nu\big]\eta +$$

$$+\frac{i\bar{\eta}}{2\sqrt{N}}\big[-\bar{G}_\nu\gamma^\mu\gamma^\nu\partial_\mu + \bar{c}\big]\psi - \frac{i\sqrt{N}}{2f}\alpha - i\frac{\phi^2}{N} + \text{source terms}\Big] \qquad (5)$$

where:

$$\Delta_B = \frac{i}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu) + im^2 - \frac{2i\alpha}{\sqrt{N}} \qquad (6a)$$

$$\Delta_F = \not{D} - iM + \frac{2i\sqrt{f}}{N}\phi + \frac{i}{2N}(\bar{G}_\mu\gamma^\nu\gamma^\mu G_\nu) \qquad (6b)$$

After performing functional integration on the ψ and η field, we obtain:

$$Z(\bar{J},\ldots) = \int[dc]\ldots[de^\mu_a]\exp\Big\{i\,S_{eff} +$$

$$+\int d\hat{x}^2\sqrt{-g}\Big[-\frac{1}{2}\bar{\xi}\Delta_F\xi - \frac{1}{2}(\bar{J} + \bar{\xi}\,\Delta_F^{-1}/\sqrt{N}\,c')$$

$$(\Delta_B - \bar{c}'\frac{\Delta_F^{-1}}{N}c')(J + \bar{c}'\,\Delta_F^{-1}\xi/\sqrt{N})\Big]\Big\} \qquad (7)$$

The effective action is:

$$S_{eff} = -i\frac{N}{2}\Big\{ \mathrm{Tr}\ \log\sqrt{-g}\ \Big[\not\partial - iM + \frac{2i\sqrt{f}}{N}\phi + $$

$$+ \frac{i}{2N}(\bar{G}_\mu \gamma^\nu \gamma^\mu G_\nu)\Big] - \mathrm{Tr}\ \log\sqrt{-g}\ \Big[\frac{i}{\sqrt{-g}}\partial_\mu (g^{\mu\nu}\sqrt{-g}\,\partial_\nu) +$$

$$+ im^2 - \frac{2i\alpha}{\sqrt{N}} - \bar{c}'\frac{\Delta_F^{-1}}{N}c'\Big]\Big\} - \int d^2x\sqrt{-g}\Big[\frac{\sqrt{N}}{2f}\alpha + \frac{\phi^2}{N}\Big] \qquad (8)$$

where $c' = i(c + \gamma^\nu\gamma^\mu G_\nu \partial_\mu)$.

III. MASS GENERATION AND QUADRATIC TERM FOR THE FIELDS IN 1/N EXPANSION SCHEME

First, the $\sqrt{N}$ order terms in the effective action lead mass generation. The mass term breaks Weyl invariance and this fact has important consequences for our theory*. The ϕ field generates the fermionic mass (M). Rescaling ϕ,

$$\phi \to \phi' = \sqrt{\frac{f}{N}}\ \phi$$

and since,

$$\phi' = \phi_0' - \sqrt{N}\,M/2 \qquad (9)$$

The $\sqrt{N}$ order ϕ field term is:

$$\sqrt{N}\ \Big[\mathrm{Tr}(F^{-1}\phi_0') + \frac{M}{f}\int d^2x\ \phi_0'\Big] \qquad (10)$$

$$(F^{-1} = (\gamma^a\partial_a - iM)^{-1})$$

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* See physical interpretation, sec.IV.

which as $N\to\infty$ diverges, unless the brachet expression is equal to zero. This gives:

$$\frac{1}{2f} = i\int\frac{d\kappa^2}{\kappa^2 - M^2} \qquad (11)$$

With a Pauli-Villars regularization,

$$\frac{1}{2f} = \pi\log\Lambda^2/M^2 \qquad (12)$$

In an analogous way, we have for the α field the $\sqrt{N}$ order term conditon,

$$\frac{1}{2f} = \pi\log\Lambda^2/m^2 \qquad (13)$$

This shows that m=M, which is expected to occur, since we are dealing with a supersymmetric theory.

The quadratic terms for the fields are calculated in the 1/N expansion (that is, are given by the $\mathcal{O}(1)$ terms). Particularly, we define the quantum "tetrad" field as (9,10):

$$h_a^\mu = -\frac{\eta_a^\mu - e_a^\mu}{\kappa} \qquad (14)$$

where $\kappa^2 = 16\pi G$ (G is the Newtonian) gravitational coupling constant) and η_a^μ is the flat space "tetrad". In terms of the quantum "tetrad" field we writte the metric field:

$$g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab} = \eta^{\mu\nu} + \kappa(h^{\mu\nu} + h^{\nu\mu}) + \kappa^2 h_a^\mu h^{a\nu} \qquad (15)$$

Rescaling h_a^μ,

$$h^{\mu}_{a} \to h'^{\mu}_{a} = (N)^{-1/2}\, h^{\mu}_{a}$$

such that $\kappa^2 N=\lambda$, where λ is fixed[18], makes κ proportional to $1/\sqrt{N}$, enabling us to obtain the quadratic term for h^{μ}_{a}, in the 1/N expansion scheme. First, the Gravitino quadratic part is calculated, in approximating the metric and tetrad fields by its corresponding flat space terms*. In this approximation the effective action reads:

$$S_{ef} \cong -\frac{iN}{2}\Big\{ \mathrm{Tr}\,\log\Big[\gamma^a\partial_a - iM + 2i\frac{\phi'}{\sqrt{N}} + \frac{i}{2N}(\bar{G}_a\gamma^b\gamma^a G_b)\Big] -$$

$$- \mathrm{Tr}\,\log\Big[i(\Box+m^2) - \frac{2i\alpha}{\sqrt{N}} - \frac{1}{N}(\bar{G}_a\gamma^b\gamma^a\partial_b - \bar{c})(\gamma^a\partial_a - iM)^{-1}$$

$$\times(c+\gamma^c\gamma^d G_c\partial_d)\Big] + \ldots \qquad (16)$$

Writting $F = \gamma^a\partial_a - iM$ and $B = -i[\Box+m^2]$, we have for the Gravitino pure quadratic part:

$$\frac{1}{2}\int d^2x\,d^2y\, G_a(x)\,\Gamma^{\bar{G}/G}_{ab}(x-y)\,G_b(y) =$$

$$= -\frac{i}{2}\Big[\frac{i}{2}\ \mathrm{Tr}\,F^{-1}\,\bar{G}_a\gamma^b\gamma^a G_b - \mathrm{Tr}\,\bar{G}_a\gamma^b\gamma^a\partial_b F^{-1}\gamma^c\gamma^d G_c\partial_b B^{-1}\Big] \qquad (17)$$

which is formally written as:

$$-\frac{i}{2}\Big[-\int d^2x\,d^2y\,\bar{G}_a(x)\,\gamma^b\gamma^a\partial^x_b\langle x|F^{-1}|y\rangle\,\gamma^c\gamma^d\partial^y_d\,\langle y|B^{-1}|x\rangle G_c(y) \ +$$

* The "Graviton-other-fields" vertixes appear only in the subsequent orders.

$$+\frac{i}{2}\ \mathrm{Tr}\int d^2x\ \bar{G}_a(x)\,\gamma^b\gamma^a G_b(x)\,\langle x|F^{-1}|x\rangle\Big] \qquad (18)$$

with the corresponding Feynman Graphs:

I

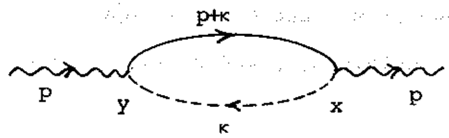


FIG. 1

II

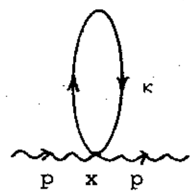
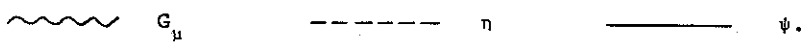


FIG. 2



The flat space ψ and η fields propagators are given by:

$$\langle x|F^{-1}|y\rangle = \frac{1}{(2\pi)^2 i}\int \frac{d\kappa^2\ e^{i\kappa(x-y)}}{\not{\kappa} - M + i\varepsilon} \qquad (19a)$$

and

$$\langle x|B^{-1}|y\rangle = \frac{1}{(2\pi)^2 i}\int \frac{d\kappa^2\ e^{i\kappa(x-y)}}{\kappa^2 - m^2 + i\varepsilon} \qquad (19b)$$

Taking the Fourier transform of part I in (18) gives:

$$\text{I:}\quad +\frac{i}{2}\int \frac{d^2p\, d^2\kappa\, \tilde{\tilde{G}}_a(p)\ \gamma^b\gamma^a(p_b+\kappa_b)(\not\kappa+\not p+M)\gamma^c\gamma^d\kappa_d G_c(p)}{\left[(\kappa+p)^2 - M^2\right]\left[\kappa^2-m^2\right]} \qquad (20)$$

where the $i\varepsilon$ is absorbed in M^2, and $G_a(x) = \int d^2p\, e^{ipx}\tilde{G}_a(p)$, etc. Since* $\bar{G}_a\gamma_b G_c=0$, terms containing $\not p$ and $\not\kappa$ do not contribute. Since $m=M$,

$$\int\frac{d\kappa^{2+\varepsilon}\ (p_b+\kappa_b)\kappa d}{\left[(\kappa+p)\ -M\right]\left[\kappa^2-M^2\right]} = i\pi\left\{pbpd\left[-\frac{1}{p^2} - \frac{2M^2}{p^2}\ \frac{1}{\sqrt{p^4-4M^2p^2}}\right.\right.$$

$$\left.\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right] - \frac{1}{2}\eta_{bd}\left[-\ell n\,\pi - \ell n(-M^2) + \Gamma(-\varepsilon/2) + \right.$$

$$\left.\left. + 2 - \frac{\sqrt{p^4-4M^2p^2}}{p^2}\ \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right]\right\} \qquad (21)$$

Analogously for part II we get:

$$\text{II:}\quad \frac{1}{4i}\,\mathrm{Tr}\int\frac{d\kappa^2\ \tilde{\tilde{G}}_a(p)\gamma^b\gamma^a G_b(p)M}{\kappa^2 - M^2} \qquad (22)$$

Since

* See Appendix B for detailed explanation of the "G_μ Gauge".

$$\int\frac{d\kappa^{2+\varepsilon}}{\kappa^2-M^2} = -i\pi\left[\Gamma(-\varepsilon/2) - \ell n\,\pi - \ell n(-M^2)\right] \qquad (23)$$

$$\tilde{\tilde{G}}_a\gamma^b\gamma^a\gamma^c\gamma^d\tilde{G}_c = 4\eta^{ab}\eta^{cd}\tilde{\tilde{G}}_b\tilde{G}_a \qquad (24a)$$

and

$$\tilde{\tilde{G}}_a\gamma^b\gamma^a G_b = 2\eta^{ab}\tilde{\tilde{G}}_b\tilde{G}_a \qquad (24b)$$

the sum of part I and II is independent of $\Gamma(-\varepsilon/2)$ and the remaining finite part is given by:

$$\frac{1}{2}\tilde{\Gamma}^{\bar{G}/G}_{bd}(p^2,M^2) = \pi M\left\{2pbpd\left[\frac{1}{p^2} + \frac{2M^2}{p^2}\ \frac{1}{\sqrt{p^4-4M^2p^2}}\right.\right.$$

$$\left.\left.\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2\ p^2}}\right) + \eta_{bd}\left[2 - \frac{\sqrt{p^4-4M^2p^2}}{p^2}\ \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right]\right\} \qquad (25)$$

We have to add to (25) the term corresponding to the Gauge fixing term:

$$\mathcal{L}^{G_\mu}_{fix} = -\frac{1}{2}\alpha\ \bar{G}^\mu\gamma_\mu\gamma_\nu G^\nu \qquad (26)$$

so that

$$\frac{1}{2}\ \Gamma^{\bar{G}_\mu/G_\mu}_{bd}(p^2,M^2) + \frac{1}{2}\ \tilde{\Gamma}^{\bar{G}_\mu/G_\mu}_{bd} - \frac{1}{2}\alpha\ \gamma_b\gamma_d \qquad (27)$$

For the $\underline{c}$ field the pure quadratic part is:

$$\frac{1}{2}\,\tilde{\Gamma}^{\bar{c}/c}(p^2,M^2) = \frac{M\pi}{\sqrt{p^4-4M^2p^2}}\,\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) \tag{28}$$

and the ϕ field quadratic part:

$$\frac{1}{2}\,\tilde{\Gamma}^{\bar{\phi}/\phi}(p^2,M^2) = 2\pi\frac{[p^2-4M^2]}{\sqrt{p^4-4M^2p^2}}\,\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) \tag{29}$$

But to obtain the complete Gravitino and $\underline{c}$ field quadratic part we have to calculate the mixed term for these fields. It is given by:

$$\tilde{\Gamma}_b^{\bar{G}_\mu/c}(p^2,M^2) = \frac{\pi i}{\sqrt{p^4-4M^2p^2}}\;\frac{1}{p^2}\;\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\times$$

$$\left[\,2p_bMp^2 + \not{p}p_b(p^2-4M^2)\right] - \frac{2\pi\not{p}p_{bi}}{p^2} \tag{30}$$

In the same way as the Gravitino field has no kinetic term, the Graviton has no free propagator in two dimensions. It is described by the symmetric part of the quantum tetrad field. Writting the metric field in terms of the symmetric ($S_{\mu\nu}$) and antisymmetric ($a_{\mu\nu}$) parts of the quantum tetrad field gives:

$$g^{\mu\nu} = \eta^{\mu\nu} + 2\kappa S^{\mu\nu} + \kappa^2 S^\mu_a S^{\nu a} + \kappa^2[S^\mu_a\, a^{a\nu} + a^\mu_a S^{\nu a} + a^\mu_a\, a^{\nu a}] \tag{31}$$

We fix the Gauge by adding to the lagrangian:

$$\mathcal{L}^{S_{\mu\nu}}_{fix} = -\frac{1}{2}\sqrt{-g}\,[\partial_\mu S^{\mu\nu} - \frac{1}{2}\partial^\nu S^\alpha_\alpha]^2 \tag{32a}$$

$$\mathcal{L}^{a_{\mu\nu}}_{fix} = -\frac{1}{2}\sqrt{-g}\,[a_{\mu\nu}]^2 \tag{32b}$$

(which corresponds to have $\partial_\mu S^{\mu\nu} = \frac{1}{2}\partial^\nu S^\alpha_\alpha$ and $a_{\mu\nu}=0$). The effective action, with special attention to Graviton field ocurrence, is:

$$S_{eff} = -\frac{iN}{2}\Big\{\,\mathrm{Tr}\,\log\sqrt{-g} + \mathrm{Tr}\,\log[\kappa\mathcal{K}^\mu\partial_\mu + F + \ldots.] -$$

$$- \mathrm{Tr}\,\log\,[i\partial_\mu(g^{\mu\nu}\sqrt{-g}\,\partial_\nu) + i\,g^{\mu\nu}\sqrt{-g}\,\partial_\mu\partial_\nu + im^2\sqrt{-g} + \ldots] \tag{33}$$

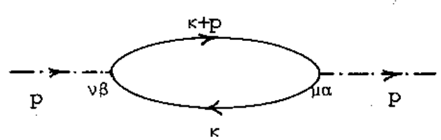
Using (31) and $a_{\mu\nu}=0$ gives the quadratic term for $S_{\mu\nu}$:

$$\frac{1}{2}\int d^2x\,d^2y\;S^{\mu\nu}(x)\,\Gamma^{SS}_{\mu\nu\rho\lambda}(x-y)\,S^{\rho\lambda}(y) =$$

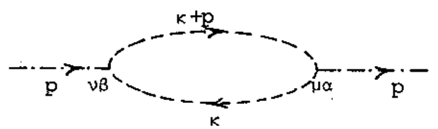
$$= -\frac{i}{2}\Big\{\mathrm{Tr}\;\underbrace{-\frac{1}{2}[\not{g}^\mu\partial_\mu\,F^{-1}]^2}_{\text{III}} - \underbrace{2[S^{\mu\nu}\partial_\mu\partial_\nu\,B^{-1}]^2}_{\text{IV}} +$$

$$+ \underbrace{i\,S^\mu_a S^{\nu a}\partial_\mu\partial_\nu B^{-1}}_{\text{V}} - \frac{1}{2}(S^{\mu\nu}S_{\mu\nu})\Big\} \tag{34}$$

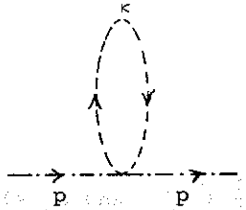
Diagrammaticaly,

III. 

$A^{F\alpha\beta}_{\mu\nu}(p^2,M^2)$

IV. 

$A^{B}_{\mu\nu\alpha\alpha}\;(p^2,M^2)$

V. 

$$A^{B}_{\mu\nu\alpha\alpha}\quad (p^2,M^2)$$

where

$$A^{F\alpha\beta}_{\mu\nu}\ (p^2,M^2) = -\frac{1}{2}\int\frac{d\kappa^2\ \kappa_\mu\kappa_\lambda(\kappa_\nu+p_\nu)(\kappa_\rho+p_\rho)\,\mathrm{Tr}\ [\gamma^\alpha\gamma^\lambda\gamma^\beta\gamma^\rho]}{[(\kappa+p)^2-M^2][\kappa^2-M^2]} -$$

$$-\frac{M^2}{2}\ \mathrm{Tr}\ [\gamma^\alpha\gamma^\beta]\int\frac{d\kappa^2\ \kappa_\mu(\kappa_\nu+p_\nu)}{[(\kappa+p)^2-M^2]\,[\kappa^2-M^2]} \qquad (35a)$$

$$A^{B}_{\mu\nu\alpha\beta}(p^2,M^2) = 2\int\frac{d\kappa^2\kappa_\mu\kappa_\alpha(\kappa_\nu+p_\nu)(\kappa_\beta+p_\beta)}{[(\kappa+p)^2-M^2]\,[\kappa^2-M^2]} \qquad (35b)$$

$$A^{B}_{\mu\nu\alpha\beta}\ (p^2,M^2) = -\eta_{\alpha\beta}\int\frac{d\kappa^2\ \kappa_\mu\kappa_\nu}{[\kappa^2-\ M^2]}$$

Adding (35a), (35b) and (35c) gives:

$$\int\frac{d\kappa^2}{[(\kappa+p)^2-M^2]\,[\kappa^2-M^2]}\Big[\ \eta_{\alpha\beta}\ \kappa_\mu\kappa^2p_\nu + \eta_{\alpha\beta}\ \kappa_\mu\kappa p p_\nu + \kappa_\mu\kappa_\nu\kappa_\alpha p_\beta +$$

$$+\kappa_\mu\kappa_\alpha p_\nu p_\beta - \kappa_\mu\kappa_\beta\kappa_\nu\ p_\alpha - \kappa_\mu\kappa_\beta\ p_\nu p_\alpha - M^2\ \eta_{\alpha\beta}\ \kappa_\mu p_\nu -$$

$$-\eta_{\alpha\beta}\ \kappa_\mu\kappa_\nu\kappa p - \eta_{\alpha\beta}\kappa_\mu\kappa_\nu p^2\ \Big] \qquad (36)$$

Using dimensional regularization[19] (in 2+ε dimensions) gives the divergent part of (36):

$$\frac{1}{2}\,\Gamma^{SSdiv}_{\mu\nu\alpha\beta}(p^2,M^2) = \frac{p^2\pi}{8}\left[\frac{1}{2}\ \eta_{\mu\alpha}\eta_{\nu\beta}-\eta_{\mu\nu}\eta_{\alpha\beta}\right]\Gamma(-\varepsilon/2) \qquad (37)$$

and the finite part:

$$\frac{1}{2}\Gamma^{SSfin}_{\mu\nu\alpha\beta}\quad (p^2,M^2) = \frac{\pi}{2}\Bigg[\ (\eta_{\alpha\beta}\eta_{\mu\nu} - \frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta}\ )$$

$$\left(\frac{1}{4}\ \sqrt{p^4-4M^2p^2}\quad \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) - 2p^2\right) +$$

$$+\ \frac{\eta\mu\alpha\eta\nu\beta p^2M^2}{4\sqrt{p^4-4M^2p^2}}\quad \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\ \sqrt{p^4-4M^2p^2}}\right)\Bigg] \qquad (38)$$

Taking into account that two dimensions is conformally flat[20] we have as a consequence that $h_{\mu\nu}(x)=h(x)\eta_{\mu\nu}$, and both the infinite part (37) as well as the first term in (38) equal zero.

APPENDIX A

Metric:

$\eta_{\mu\nu}$ = diag (1,-1).

γ_μ matrices representation

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi} = \psi^T \gamma_0$$

Fierz transformation:

$$\delta_{\alpha\beta}\,\delta_{\lambda\rho} = \frac{1}{2}\left[\delta_{\alpha\rho}\delta_{\lambda\beta} + \gamma_{5\alpha\rho}\gamma_{5\lambda\beta} + \gamma_{\mu\alpha\rho}\gamma^{\mu}_{\lambda\beta}\right]$$

APPENDIX B

Proof a local supersymmetry of the Lagrangian (1)

The global supersymmetric (GSS) non-linear O(N)σ model given by the lagrangian:

$$\mathcal{L}_0 = \frac{1}{2}\,\partial^\mu\eta\partial_\mu\eta + \frac{i}{2}\,\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{1}{8}(\bar{\psi}\psi)^2 \qquad \text{(B1)}$$

($\bar{\psi}\eta=0$ and $\eta^2=1$)

which is invariant under the GSS transformation law for the fields:

$$\delta^{GSS}_\eta = \bar{\varepsilon}\psi \qquad \text{(B2a)}$$

$$\delta^{GSS}\psi = -i\gamma^\mu\varepsilon\partial_\mu\eta + \frac{1}{2}\,\varepsilon\eta(\bar{\psi}\psi) \qquad \text{(B2b)}$$

Making ε local, gives the local supersymmetry (LSS):

$$\delta^{LSS}\mathcal{L}_0 = J^\mu\partial_\mu\varepsilon + \partial^\mu(\bar{\varepsilon}\psi\partial_\mu\eta\,) \qquad \text{(B3)}$$

where $J^\mu = \bar{\psi}\gamma^\mu\gamma^\nu\partial_\nu\eta$ is the supersymmetric Noether current.

If we introduce a spinorial vector field G_μ (Gravitino), such that

$$\delta^{LSS}\,G_\mu = -\partial_\mu\varepsilon \qquad \text{(B4)}$$

and add to $\mathcal{L}_0$

$$\mathcal{L}_1 = \bar{\psi}\gamma^\mu\gamma^\nu\partial_\nu\eta\,G_\mu \qquad \text{(B5)}$$

$\mathcal{L}' = \mathcal{L}_0 + \mathcal{L}_1$ is still not independent of $\partial_\mu \varepsilon$ terms under LSS. We have to add a second term:

$$\mathcal{L}_2 = -\frac{1}{4}(\bar{\psi}\psi)(\bar{G}_\nu \gamma^\mu \gamma^\nu G_\mu) \qquad \text{(B6)}$$

$\mathcal{L}'' = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$ is now independent of $\partial_\mu \varepsilon$ under LSS but does not correspond to a lagrangian invariant under LSS, because it has to be written in curved space-time. It can be trivialy seen by

$$[\delta_2, \delta_1]\eta = 2\bar{\varepsilon}_2 \gamma^\mu \varepsilon_1 \partial_\mu \eta \qquad \text{(B7)}$$

that the LSS corresponds to the "square root" of the local general coordinate transformation law. This means that the Gravitational interaction emerges naturally in a LSS theory. So we re-writte $\mathcal{L}''$ in curved space-time:

$$\mathcal{L}' = \sqrt{-g}\left\{\frac{1}{2}g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta + \frac{i}{2}\bar{\psi}\not{D}\psi + \frac{1}{8}(\bar{\psi}\psi)^2 + \right.$$

$$\left. + \bar{\psi}\gamma^\mu\gamma^\nu\partial_\nu\eta G_\mu - \frac{1}{4}(\bar{\psi}\psi)(\bar{G}_\nu\gamma^\mu\gamma^\nu G_\mu)\right\} \qquad \text{(B8)}$$

where

$$\sqrt{-g} = (-\det g_{\mu\nu})^{1/2}, \quad \gamma^\mu = e^\mu_a \gamma^a \quad \text{and} \quad D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu \gamma_5 .$$

Under LSS,

$$\delta^{LSS} e^\mu_a = \alpha \bar{G}^\mu \gamma_a \varepsilon \qquad \text{(B9a)}$$

$$\delta^{LSS} g^{\mu\nu} = \alpha[\bar{G}^\mu \gamma^\nu \varepsilon + \bar{G}^\nu \gamma^\mu \varepsilon] \qquad \text{(B9b)}$$

$$\delta\sqrt{-g} = -\sqrt{-g}\,\alpha(\bar{G}^\mu \gamma_\mu \varepsilon) \qquad \text{(B9c)}$$

with α a complex constant, subsequently fixed. Also, modifying (B2b) and (B4) to

$$\delta^{LSS}\psi = \delta^{GSS}\psi + \beta(\bar{G}_\mu\psi)\gamma^\mu\varepsilon \qquad \text{(B10a)}$$

and

$$\delta G_\mu = -D_\mu \varepsilon \qquad \text{(B10b)}$$

The Gravitino field possesses the Gauge freedom:

$$G_\mu \to G'_\mu = \tilde{G}_\mu + \gamma_\mu \phi(x) \qquad \text{(B11)}$$

where ϕ is a scalar field. Using the Gauge

$$\not{G} = 0 \qquad \text{(B12)}$$

implies that

$$\gamma_\mu G_\nu = \gamma_\nu G_\mu \qquad \text{(B13a)}$$

$$\gamma_5 G_\mu = \varepsilon_{\mu\nu} G^\nu \qquad \text{(B13b)}$$

$$\bar{G}_\mu G_\nu = \frac{1}{2} g_{\mu\nu} \bar{G}.G \qquad \text{(B13c)}$$

$$\bar{G}_\mu \gamma_\nu G_\beta = 0 \qquad \text{(B13d)}$$

So, under LSS $\mathcal{L}''$, given by (B8), takes the form:

$$\delta^{LSS}\mathcal{L}'' = \delta\sqrt{-g}\{\ \} + \sqrt{-g}\,\delta\{\ \} \qquad \text{(B14)}$$

But

$\delta\sqrt{-g} = 0$ ((B12) Gauge). Therefore,

$$\sqrt{-g}\,\delta\{\ \} = \sqrt{-g}\{\frac{\alpha}{2}[\underbrace{(\bar G^\mu\gamma^\nu\epsilon)}_{(1)} + \underbrace{(\bar G^\nu\gamma^\mu\epsilon)}_{(2)}]\partial_\mu\eta\partial_\nu\eta + \underbrace{\partial^\mu(\bar\epsilon\psi)\partial_\mu\eta}_{(3)} - \underbrace{\frac{1}{2}(\bar\epsilon\gamma^\nu\gamma^\mu\partial_\mu\psi)\partial_\nu\eta}_{(4)} +$$

$$+ \underbrace{i\frac{1}{4}(\bar\psi\psi)(\bar\epsilon\,\not\partial\psi)\eta}_{(5)} - \frac{\beta i}{2}(\bar\epsilon\gamma^\mu\gamma^\nu\partial_\nu\psi)(\bar\psi G_\mu) + \underbrace{\frac{i\alpha}{2}(\bar\psi\gamma^\rho\partial_\mu\psi)(\bar G^\mu\gamma_\rho\epsilon)}_{(7)} +$$

$$+ \underbrace{\frac{1}{2}(\bar\psi\gamma^\mu\partial_\mu\gamma^\nu\epsilon)\partial_\nu\eta}_{(8)} + \underbrace{\frac{1}{2}(\bar\psi\gamma^\mu\gamma^\nu\partial_\mu\epsilon)\partial_\nu\eta}_{(9)} + \underbrace{\frac{1}{2}(\bar\psi\gamma^\mu\gamma^\nu\epsilon)\partial_\mu\partial_\nu\eta}_{(10)} +$$

$$+ \underbrace{\frac{i}{4}(\bar\psi\psi)(\bar\psi\gamma^\mu\epsilon)\partial_\mu\eta}_{(11)} + \underbrace{\frac{i\beta}{2}\bar\psi\not\partial[(\bar G_\mu\psi)\gamma^\mu\epsilon]}_{(12)} + \underbrace{\frac{i}{2}(\bar\psi\psi)(\bar\epsilon\gamma^\mu\psi)\partial_\mu\eta}_{(13)} +$$

$$+ \underbrace{\frac{\beta}{2}(\bar\psi\psi)(\bar G_\mu\psi)(\bar\psi\gamma^\mu\epsilon)}_{(14)} + \underbrace{i(\bar\epsilon\gamma^\rho\gamma^\mu\gamma^\nu G_\mu)\partial_\nu\eta\partial_\rho\eta}_{(15)} -$$

$$- \underbrace{\beta(\bar\epsilon\gamma^\rho\gamma^\mu\gamma^\nu G_\mu)(\bar\psi G_\rho)\partial_\nu\eta}_{(16)} + \underbrace{\alpha(\bar\psi\gamma^\rho\gamma^\nu G_\mu)(\bar G^\mu\gamma_\rho\epsilon)\partial_\nu\eta}_{(17)} +$$

$$+ \underbrace{\alpha(\bar\psi\gamma^\mu\gamma^\rho G_\mu)(\bar G^\nu\gamma_\rho\epsilon)\partial_\nu\eta}_{(18)} - \underbrace{(\bar\psi\gamma^\mu\gamma^\nu\partial_\mu\epsilon)\partial_\nu\eta}_{(19)} -$$

$$- \underbrace{\frac{1}{2}\omega_\mu(\bar\psi\gamma^\mu\gamma^\nu\gamma_5\epsilon)\partial_\nu\eta}_{(20)} + \underbrace{(\bar\psi\gamma^\mu\gamma^\nu G_\mu)(\partial_\nu\bar\epsilon\psi)}_{(21)} +$$

$$+ \underbrace{(\bar\psi\gamma^\mu\gamma^\nu G_\mu)(\bar\epsilon\partial_\nu\psi)}_{(22)} - \underbrace{\frac{i}{2}(\bar\epsilon\gamma^\mu\psi)\partial_\mu\eta(\bar G_\rho\gamma^\lambda\gamma^\rho G_\lambda)}_{(23)} -$$

$$- \underbrace{\frac{\beta}{2}(\bar\psi\gamma^\mu\epsilon)(\bar G_\mu\psi)(\bar G_\rho\gamma^\lambda\gamma^\rho G_\lambda)}_{(24)} + \underbrace{\frac{1}{4}(\bar\psi\psi)(\partial_\nu\bar\epsilon\gamma^\mu\gamma^\nu G_\mu)}_{(25)} -$$

$$- \underbrace{\frac{1}{8}(\bar\psi\psi)(\bar\epsilon\gamma_5\gamma^\mu\gamma^\nu G_\mu)\omega_\nu}_{(26)} + \underbrace{\frac{1}{4}(\bar\psi\psi)(\bar G_\nu\gamma^\mu\gamma^\nu\partial_\mu\epsilon)}_{(27)} +$$

$$+ \underbrace{\frac{1}{8}(\bar\psi\psi)(\bar G_\nu\gamma^\mu\gamma^\nu\gamma_5\epsilon)\omega_\mu}_{(28)} - \underbrace{\frac{\alpha}{4}(\bar\psi\psi)(\bar G_\nu\gamma^\rho\gamma^\nu G_\mu)(\bar G^\mu\gamma_\rho\epsilon)}_{29)} -$$

$$- \underbrace{\frac{\alpha}{4}(\bar\psi\psi)(\bar G_\nu\gamma^\mu\gamma^\rho G_\mu)(\bar G^\nu\gamma_\rho\epsilon)\}}_{(30)} \qquad \text{(B15)}$$

Since $\gamma^\rho\gamma^\mu\gamma^\nu = + g^{\rho\mu}\gamma^\nu + g^{\mu\nu}\gamma^\rho - g^{\rho\nu}\gamma^\mu$, $i(\bar\epsilon\gamma^\rho\gamma^\mu\gamma^\nu G_\mu)\partial_\nu\eta\partial_\rho\eta =$

$-i(\bar\epsilon\gamma^\mu G_\mu)\partial^\mu\eta\partial_\mu\eta + i(\bar\epsilon\gamma^\nu G^\mu)\partial_\nu\eta\partial_\mu\eta + i(\bar\epsilon\gamma^\mu G^\nu)\partial_\nu\eta\partial_\mu\eta$, which compared to (1) and (2) (using $\not G=0$) yield $\alpha=2i$. Now, making a partial integration on (4):

$$-\frac{\sqrt{-g}}{2}(\bar\epsilon\gamma^\nu\gamma^\mu\partial_\mu\psi)\partial_\nu\eta = \underbrace{\frac{\partial_\mu\sqrt{-g}}{2}(\bar\epsilon\gamma^\nu\gamma^\mu\psi)\partial_\nu\eta}_{(4a)} + \sqrt{-g}\Big[\underbrace{\frac{1}{2}(\partial_\mu\bar\epsilon\gamma^\nu\gamma^\mu\psi)\partial_\nu\eta}_{(4b)} +$$

$$+ \underbrace{\frac{1}{2}(\bar\epsilon\partial_\mu(\gamma^\nu\gamma^\mu)\psi)\partial_\nu\eta}_{(4c)} + \underbrace{\frac{1}{2}(\bar\epsilon\gamma^\nu\gamma^\mu\psi)\partial_\mu\partial_\nu\eta}_{(4d)}\Big]$$

(4b) together with (9) cancels (19); (4d) summed with (3) gives

$\sqrt{-g}\;\partial^\mu(\bar\varepsilon\psi\partial_\mu\eta)$. Also, summing (5), (11) and (13) gives zero.

(31)

Using Fierz transformation, $\gamma^\rho\gamma^\mu\gamma_\rho=0$ and $\bar G^\mu\gamma_\rho G_\mu=0$ in (17), (29) and (30) gives zero for each term. (21), (25) and (27) sum up to zero. Using $\not{G}=0$ and Fierz transformation in (14) and (24) make them zero. Since $\bar\psi\gamma^\mu\gamma^\nu\partial_\nu\eta=\frac{1}{2}(\bar\psi\psi)\bar G_\nu\gamma^\mu\gamma^\nu$ the sum of (20), (26) and (28) is zero. With a partial integration on (12) gives:

$$\sqrt{-g}\;\frac{i\beta}{2}\;\bar\psi\gamma^\nu\partial_\nu\left[(\bar G_\mu\psi)\gamma^\mu\varepsilon\right]= -\frac{i\beta}{2}\partial_\nu\sqrt{-g}(\bar\psi\gamma^\nu\gamma^\mu\varepsilon)(\bar G_\mu\psi)\;-$$

(12a)

$$-\;\frac{i\beta}{2}\;\sqrt{-g}\left[(\partial_\nu\bar\psi\gamma^\nu\gamma^\mu\varepsilon)(\bar G_\mu\psi)+\;(\bar\psi\partial_\nu\gamma^\nu\gamma^\mu\varepsilon)(\bar G_\mu\psi)\right]$$

(12b) (12c)

Using Fierz transformation, $\bar\psi\;\gamma^\mu\gamma^\nu\partial_\nu\eta=\frac{1}{2}\;(\bar\psi\psi)\bar G_\nu\gamma^\mu\gamma^\nu$ for (12a) and summing this result with (4a) gives:

$$\frac{\partial_\mu\sqrt{-g}}{2}\;(1+\beta_i)(\bar\psi\gamma^\mu\gamma^\nu\varepsilon)\partial_\nu\eta$$

(32)

The sum of (12b),(6), (7) and (22):

$$(\beta_i-1)\left[(\bar\psi\partial_\mu\psi)(\bar G^\mu\varepsilon)\;+\;(\bar\psi\gamma_5\partial_\nu\psi)(\bar\varepsilon\gamma_5 G_\nu)\right]$$

(33)

In the same way for (16), (18) and (23) gives:

$$2(\beta+i)(\bar G_\nu\gamma_5 G_\mu)(\bar\psi\gamma^\mu\gamma_5\varepsilon)\partial^\nu\eta$$

(34)

Finally, the sum of (4c), (8) and (12c) yields:

$$[\bar\psi\partial_\mu(\gamma^\mu\gamma^\nu)\varepsilon]\partial_\nu\eta$$

(35)

The remaining terms are:

$$\sqrt{-g}\delta\{\;\} = \sqrt{-g}\Big\{\frac{1}{2\sqrt{-g}}\;\partial_\mu\sqrt{-g}(1+\beta i)(\bar\psi\gamma^\mu\gamma^\nu\varepsilon)\partial_\nu\eta\;+$$

$$+\;[\bar\psi\partial_\mu(\gamma^\mu\gamma^\nu)\varepsilon]\partial_\nu\eta\;+\;\partial^\mu(\bar\varepsilon\psi\partial_\mu\eta)\;+\;(i\beta-1)\left[(\bar\psi\partial_\mu\psi)(\bar G^\mu\varepsilon)\;+\right.$$

$$\left.+\;(\bar\psi\gamma_5\partial_\nu\psi)(\bar\varepsilon\gamma_5 G^\nu)\right]\;+\;2(\beta+i)(\bar G_\nu\gamma_5 G_\mu)(\bar\psi\gamma^\mu\gamma_5\varepsilon)\partial^\nu\eta\Big\} \qquad \text{(B15)}$$

If $\beta=-i$,

$$\sqrt{-g}\delta\{\;\} = \sqrt{-g}\;\Big\{\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}(\bar\psi\gamma^\mu\gamma^\nu\varepsilon)\partial_\nu\eta\;+\;\bar\psi\partial_\mu(\gamma^\mu\gamma^\nu)\varepsilon\partial_\nu\eta\;+$$

$$+\;\partial^\mu(\bar\varepsilon\psi\partial_\mu\eta)\;\;=\;\sqrt{-g}\;\Big\{\frac{1}{\sqrt{-g}}\;\partial_\mu\sqrt{-g}\;\varepsilon^{\mu\nu}\;+\;\partial_\mu\varepsilon^{\mu\nu}\Big\}(\bar\psi\gamma_5\varepsilon)\partial_\nu\eta\;=\;0$$

since $\varepsilon^{\mu\nu}=e^\mu_a e^\nu_b\varepsilon^{ab}$, $\frac{\partial_\mu\sqrt{-g}}{\sqrt{-g}}=\Gamma^\lambda_{\mu\lambda}$ and $\partial_\mu e^\nu_b=\omega^c_{\mu b}e^\nu_c-\Gamma^\nu_{\mu\lambda}\;e^\lambda_b$ (with $\omega^\mu_{bc}=\varepsilon_{bc}\;e^\mu_d\varepsilon^{\rho\lambda}\partial_\lambda e^d_\rho$).

IV. PHYSICAL INTERPRETATION

We verified, in lowest order, that our model is renormalizable. This is achieved due to unwaited cancelations between fermion and boson divergencies, as is clear in the case of the Gravitino propagator. However this feature seems to be general in supersymmetric theories[21].

We have already called attention to the mass generation. It is easy to see from expressions (25) and (38) that in the zero mass limit the quadratic part of the Graviton and Gravitino completely disappear from the theory. This is a consequence of the classical Weyl invariance[14] of the theory, in which we can rescale the fields as:

$$\psi \to \Lambda^{-1/2}\,\psi \qquad\qquad \eta \to \eta$$

$$e_a^\mu \to \Lambda^{-1} e_a^\mu \qquad\qquad G_\mu \to \Lambda^{1/2}\, G_\mu$$

where Λ is a local arbitrary parameter. This invariance is broken by the fermion mass term, so that quantum mechanically it no longer holds. The Weyl invariance enables us, in the classical theory, to eliminate completely the Graviton field. In this case also the Gravitino field could be eliminated[22].

The rich   of the structure carried by the generated mass does not yet stop. For zero momentum, we have a pole in the Graviton and Gravitino propagators:

$$\langle \tilde{G}_\mu(p)\ \tilde{G}_\nu(-p) \rangle \sim \eta_{\mu\nu} \frac{1}{p^2} \quad \cdots$$

$$\langle \tilde{h}_{\mu\nu}(p)\ \tilde{h}_{\rho\sigma}(-p) \rangle \sim \eta_{\mu\nu}\, \eta_{\rho\sigma} \frac{4}{p^2} + \cdots$$

Zero mass propagator, in two dimensions, leads to terrible infrared divergencies[23], and the theory could be even undefined. In general it means confinement as in the CP^{N-1} model. In that case we are able to construct gauge invariant bound states, such as $N(z_i \bar{z}_m)(x)$ which represent the (un confined) mesons[16]. However we were not able to do the same in our present problem, and we can not construct the "mesons" of our theory.

Also the important relation between 2 dimensional gravity and string models[14,24] turn out to be no longer valid - or at least screened, because to establish this relation we need to go to flat space ($g_{\mu\nu}=\eta_{\mu\nu}$), by a Weyl transformation, and there after the Gravitino field can be put in the form $G_\mu=\gamma_\mu$ which is made zero through a gauge transformation. We guess that the string collapses due to the long range forces involved.

At last, the model can be seen to be classicaly integrable, for it has a (classically) conserved non-local charge, namely:

$$Q = \int dy_1 dy_2\ \varepsilon(y_1-y_2) J_0(z,y_1) J_0(z,y_2) - \int dy\ [j_1(t,y)+2i_1(t,y)]$$

where:

$$J_\mu^{ij}(t,y) = \partial_\mu \eta^i \eta^j - \eta^i \partial_\mu \eta^j +$$

$$+ \frac{i}{2}\bar{\psi}^i \gamma_\mu \psi^j - \frac{i}{2}\bar{\psi}^j \gamma_\mu \psi^i + \bar{\psi}^i \gamma^\nu \gamma^\mu \eta^i G_\nu - \bar{\psi}^j \gamma^\nu \gamma^\mu \eta^i\ G_\nu$$

$$j_\mu^{ij} = \partial_\mu \eta^i \eta^j - \eta^i\ \partial_\mu \eta^j$$

$$i_\mu^{ij} = \frac{i}{2}\bar{\psi}^i \gamma_\mu\ \psi^j - \frac{i}{2}\bar{\psi}^j \gamma_\mu \psi^i$$

Using current conservation, it can be shown that Q is conserved, implying, at classsical level, integrability of the system. Quantum mechanically, if this non-local charge conservation is not spoiled by radiative corrections, it should imply a factorizable S-matrix. However, the existence of zero mass gauge particles, implying confinement of some degrees of freedom, can spoil the charge conservation, and the quantum matrix would no longer factorize.

V. CONCLUSIONS AND OUTLOOK

First, we should point out that it is very important to look for gauge invariant operators, which should display infrared finiteness. Local supersymmetry implies Poincaré symmetry, however it is not true in our algebra that

$$[\delta_{\varepsilon_1},\delta_{\varepsilon_2}]I \sim \bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 \partial_\mu I$$

for all operators I, where $\delta_\varepsilon I$ consists in the supersymmetric variation of I with the parameter ε.

Conservation of the non-local charge should give an exact S operator, at least in the unconfined sector. Here we have another problem, namely, the interpretation of the Gravitino and Graviton asymptotic fields. They are no more simply Lagrange multipliers. However we are not confident to say that they survire asymptotically.

Finally, we wonder if this structure carries over to the CP^{N-1} model, and what should be the consequence for θ vacuum and confinement in this case.

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