# INSTITUTO DE FÍSICA

# preprint

IFUSP-P/321

MEASURE OF ABSORPTION IN MULTISTEP COMPOUND PROCESSES

B.V.Carlson and M.S.Hussein

Instituto de Física - U.S.P.



UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

MEASURE OF ABSORPTION IN MULTISTEP COMPOUND PROCESSES†

B.V. Carlson and M.S. Hussein
Instituto de Física, Universidade de São Paulo
C. Postal 20516, São Paulo, S.P., Brasil

ABSTRACT

The Moldauer-Simonius theorem, that relates the modulus of the determinant of the average, optical, S-matrix, to the average width and spacing of the compound nucleus resonances, is generalized to the multiclass resonances situation encountered in pre-equilibrium reactions. Corrections to the generalized M/S theorem are seen to be connected primarily to the width distribution of the widest doorway class.

†Supported in part by FAPESP and CNPq

February/1982



## 1. Introduction

The recent upsurge of theoretical interest[1-3] in multistep compound processes has brought into focus several important questions related to the statistical theory of the compound nucleus.

One particular aspect of the statistical theory, namely the distribution of level widths, $P(\Gamma)$, has recently been discussed by several authors[4-7], in connection with the conventional, one-class of overlapping resonances, model of the equilibrated compound system. It was discussed in Refs. 7) that the S-matrix auto-correlation function, $C^S(\varepsilon)$, should carry some information about $P(\Gamma)$. However such information would be experimentally difficult to disentangle. The calculated $C^S(\varepsilon)$ with a specific $P(\Gamma)$ was found to differ little from the one-pole approximation to $C^S(\varepsilon)$, as long as the correlation width, $\Gamma^{corr}$ was identified with $\frac{D}{2\pi}$ Tr $\underset{\sim}{P}$ where D is the mean level spacing and $\underset{\sim}{P}$ , the optical transmission matrix.

Clearly the above questions become even more subtle in the case of the multi-class resonance model of pre-equilibrium processes, since, as was demonstrated in Ref. 3) the fluctuation cross-section and the S-matrix auto-correlation function are not simply related. The relation is implicit, in the sense that $\sigma^{fl}_{cc'} = \sum_n \sigma^{fl}_{n,cc'}$ and $C^S_{cc'}(\varepsilon) = \sum_n \sigma^{fl}_{n,cc'} / 1 + i\frac{\varepsilon}{\Gamma^{corr}_n}$ and therefore $C^S_{cc'}(\varepsilon)/\sigma^{fl}_{cc'} \equiv F_{cc'}(\varepsilon)$ depend on the channels. This makes the discussion of the $P_n(\Gamma_n)$ through considerations of the generalized cross-section autocorrelation function more difficult.

Another quantity of theoretical interest which involves explicitly the consideration of the level width distribution is the average amount of absorption present in the system and its relation to $\bar{\Gamma}/D$. This is quantitatively described through a relation involving the modulus of the average (optical) S-matrix, $\bar{S}$ and the ratio $\bar{\Gamma}/D$. This relation carries the name of the Moldauer-Simonius (M/S) theorem[8].

It would be quite instructive to generalize the M/S theorem to the case of multistep compound processes (MSCP). This generalization would help in furthering our understanding of the role of the level width distribution of the different classes of doorways, in fixing the degree of absorption in the system and, accordingly, in relating observable physical quantities such as $\bar{S}$ to the inherently unobservable average doorway widths.

In the present paper, we demonstrate that the generalization of the M/S theorem to MSCP involves very simply the consideration of the ratios $\bar{\Gamma}_n/D_n$. Further we show that in the limit of the well nested sequence of doorway classes discussed in 3), the first correction to the M/S theorem involves the width distribution of the widest width class of doorways.

The paper is organized as follows: In section II the M/S theorem is discussed in the context of MSCP. In section III we introduce a particular explicit form



fo $P_n(\Gamma_n)$ and accordingly calculate $\bar{\Gamma}_n$ which are needed to obtain the corrections to the M/S theorem. Finally in section IV the consequences of the generalized MS theorem are discussed and several concluding remarks are made.

## II. The Moldauer/Simonius Theorem For MSCP

In its original form, the M/S theorem valid for a single class of overlapping resonances system reads*

$$\mathrm{Re}\,\ln\det\bar{S} = -\pi\,\bar{\Gamma}/D \qquad (1)$$

Though Eq. (1) relates the modulus of the determinant of $\bar{S}$ to $\bar{\Gamma}/D$, one may obtain the corresponding relation for $\left|(\bar{S})_{cc}\right|$ in the case of m equivalent channels

$$\left|\bar{S}\right| = \exp\left(-\pi\,\bar{\Gamma}/mD\right) \qquad (2)$$

Upon insertion into the unitarity relation, this gives the following value for the transmission coefficient P

$$P = 1-\exp\left(-2\pi\,\bar{\Gamma}/mD\right) \qquad (3)$$

In the more realistic case of non equivalent channels, simple relations such as (2) and (3) are not obtained. Nevertheless qualitative statements containing similar

* All formulae refer to a given partial wave.

physics as in Eqs. (2) and (3) may be made as was done in Ref. 9).

To generalize the M/S theorem to the case of N classes of overlapping resonances, we start with the usual sum-over-poles form of the S-matrix

$$\underset{\sim}{S} = \underset{\sim}{B} - i \sum_{n,\mu} \frac{g_{\mu}^{n} \; g_{\mu}^{n}}{E - E_{n,\mu} + i\,\Gamma_{n,\mu}/2} \tag{4}$$

where $\underset{\sim}{B}$ is the, unitary, background matrix, $BB^{+}=1$.

Since the sum over classes, $\Sigma$, is just another label, it may be considered on the same footing as $\mu$.

The background-plus- sum-over-poles representation of $\underset{\sim}{S}$ given in Eq. (4) may not guarantee the absence, in the energy-averaged cross section, of terms connected with the interference between compound (fluctuation) and direct processes.

One may, however, construct an alternative form for $\underset{\sim}{S}$ where these interference terms average out to zero. This was explicitly done in Ref. 3) using the optical background representation of Kawai, Kerman and McVoy[10], appropriately generalized to the multiclass resonances case. For our present purposes, however, Eq. (4) is more appropriate.

The average S-matrix, $\overline{\underset{\sim}{S}}$, (average over an energy interval I), may be obtained from (4) by merely adding to the imaginary part of the denominators, the factor $\frac{I}{2}$.



We then obtain the following for the determinant of $\overline{\underset{\sim}{S}}$

$$\det \overline{\underset{\sim}{S}} = \overline{\det \underset{\sim}{S}} = e^{2i\varphi} \prod_{n,\mu} \frac{E - E_{n,\mu} + iI/2 - i\Gamma_{n,\mu}/2}{E - E_{n,\mu} + iI/2 + i\Gamma_{n,\mu}/2} \tag{5}$$

where

$$\varphi = \frac{1}{2i} \det \underset{\sim}{B}$$

We consider now the real part of the logarithm of Eq. (5), which may be written as

$$\mathrm{Re}\,\ln \det \overline{\underset{\sim}{S}} = \ln \left|\det \overline{\underset{\sim}{S}}\right| = \mathrm{Re} \sum_{n,\mu} \ln \left( \frac{1 - i(\Gamma_{n,\mu}/2)/(E - E_{n,\mu} + iI/2)}{1 + i(\Gamma_{n,\mu}/2)/(E - E_{n,\mu} + iI/2)} \right) \tag{6}$$

Expanding (6) in powers of $\quad x_{n,\mu} \equiv \dfrac{i\,\Gamma_{n,\mu}/2}{E - E_{n,\mu} + iI/2}$

we finally obtain

$$\ln \left|\det \overline{\underset{\sim}{S}}\right| = - \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \left(\frac{\partial}{\partial I}\right)^{2j} \sum_{n,\mu} \frac{(I/2)(\Gamma_{n,\mu})^{2j+1}}{(E - E_{n,\mu})^2 + I^2/4} \tag{7}$$

The sums

$$\sum_{\mu} \frac{(I/2)(\Gamma_{n,\mu})^{2j+1}}{(E - E_{n,\mu})^2 + I^2/4} = \pi \frac{\left\langle (\Gamma_{n,\mu})^{2j+1} \right\rangle_{\mu \in I}}{D_n} \tag{8}$$

define the average of powers of the width $\Gamma_{n,\mu}$.
Calling $\overline{\Gamma}_n \equiv \langle \Gamma_{n,\mu} \rangle_\mu$ we then write

$$\ln|\det \underset{\sim}{\overline{S}}| = -\pi \sum_{n=1}^{N} \overline{\Gamma}_n / D_n - \pi \sum_{j=1}^{\infty} \frac{1}{(2j+1)!} \left(\frac{\partial}{\partial I}\right)^{2j} \sum_{n=1}^{N} \frac{\langle \Gamma_{n,\mu}^{2j+1} \rangle_\mu}{D_n} \tag{9}$$

Equation (9) is the principal result of this section. To continue further, we have to specify the distribution of level widths $P_n(\Gamma_n)$ which are needed to evaluate $\langle \Gamma_{n,\mu}^{2j+1} \rangle_\mu$.

We might mention that if the assumption that $\frac{\langle \Gamma_{n,\mu}^{2j+1} \rangle_\mu}{D_n}$ independent of the averaging interval I, is made, then we obtain immediately the simple generalization of the M/S theorem

$$|\det \underset{\sim}{\overline{S}}| = \exp\left[-\pi \sum_{n=1}^{N} (\overline{\Gamma}_n / D_n)\right] \tag{10}$$

where the sum extends over all doorway classes.

## III. The Level width Distribution and the Corrections To The M/S Theorem

Recently several authors[4),6),7)] have discussed the distribution of widths of overlapping resonances. Most of these studies result in a numerical histogram distribution which is not convenient for analytic discussion. In Ref. 7), however, an attempt was made to



actually construct $P(\Gamma)$ subject to several constraints motivated by unitarity, the uncorrected M/S theorem, Eq. (10) and an expression for the coherence width of Ericson fluctuations obtained from an analysis of the S-matrix auto-correlation function

$$\Gamma^{corr} = \langle \Gamma_\mu^{-1} \rangle_\mu / \langle \Gamma_\mu^{-2} \rangle_\mu \tag{11}$$

The distribution $P(\Gamma)$ was then constructed by use of the maximum entropy condition subjected to the above three constraints. The resulting $P(\Gamma)$ has the form

$$P(\Gamma) = \exp\left[-\left[0.429 + 1.25\,\Gamma + 0.16\left(\frac{\Gamma^{corr}}{\Gamma^2} - \frac{1}{\Gamma}\right)\right]\right] \tag{12}$$

where the numerical factors appearing in Eq. (12) were found by treating the 20-channel example discussed by Moldauer[4)].

Although $P(\Gamma)$ of Eq. (12) fits very well the numerically generated histogram, it is quite cumbersome to deal with in analytical studies. For the purpose of evaluating the corrections to the M/S expression, Eq. (9), we therefore use a simplified version of $P(\Gamma)$ which guarantees the finiteness of $\Gamma^{corr}$ as defined in Eq. (11).

$$P(\Gamma) = \frac{27}{2} \frac{1}{\overline{\Gamma}} \left(\frac{\Gamma}{\overline{\Gamma}}\right)^2 \exp\left(-3\frac{\Gamma}{\overline{\Gamma}}\right) \tag{13}$$

With $P(\Gamma)$ above, plotted in Fig. (1), the correlation width calculated using the de defining equation (11), comes out to be

$$\Gamma^{corr} = \bar{\Gamma}/3 \tag{14}$$

This is close to the value extracted from Moldauer's histogram[4].

To further exhibit the reasonableness of our distribution, Eq. (13), we calculate below the ratio of the S-matrix autocorrelation function to $\sigma^{fl}_{cc'}$ for the single class resonance case[7],

$$\frac{C^{S}_{cc'}(\varepsilon)}{\sigma^{fl}_{cc'}} = \left\langle \frac{1}{\Gamma_\mu + i\varepsilon} \right\rangle_\mu \left\langle \Gamma_\mu^{-1} \right\rangle_\mu^{-1} \tag{15}$$

$$= 1 - i3\varepsilon/\bar{\Gamma}$$

$$- (3\varepsilon/\bar{\Gamma})^2 \exp(i3\varepsilon/\bar{\Gamma})\, E_1(i3\varepsilon/\bar{\Gamma})$$

where $E_1(x)$ is the exponential integral[10]. The closed expression for $C^{S}_{cc'}(\varepsilon)/\sigma^{fl}_{cc'}$ given above has the correct behaviour at $\varepsilon=0$ $(=1)$ and $\varepsilon = \infty$ $(= 0)$.



In fig. (2) we show the cross-section auto-correlation function $\left| C^{S}_{cc'}(\varepsilon)/\sigma^{fl}_{cc'} \right|^2$, plotted vs. the quantity $\frac{3\varepsilon}{\bar{\Gamma}}$. The extracted correlation with is $\sim \frac{5}{4}\frac{\bar{\Gamma}}{3}$, slightly larger than that given in Eq. (14). For comparison, we also show the results obtained with the one-pole approximation to $C^{S}_{cc'}(\varepsilon)/\sigma^{fl}_{cc'}$ i.e. $\frac{\Gamma^{corr}}{\Gamma^{corr}+i\varepsilon}$, with $\Gamma^{corr}$ obtained from the exact result, Eq. (15). It is clear that the one-pole expression approximates very well the exact one in the small-$\varepsilon$ region. Of course this is the region accessible to unambiguous experimental studies.

The above findings agree with those of Ref.7) where $P(\Gamma)$ of Eq. (12) was used. Further, the result of our calculation shown in Fig. (2) are quite close to those of Ref. 7), indicating clearly that our approximate $P(\Gamma)$, Eq. (13), is quite reasonable.

Having thus given arguments to justify the form of $P(\Gamma)$ employed here, we turn now to the calculation of the corrections to M/S relation, Eq. (10). We assume similar width distributions for all classes of overlapping resonances, obtaining thus

$$\left\langle (\Gamma_{n,\mu})^{2j+1} \right\rangle_\mu = \frac{(2j+3)!}{2\,(3)^{2j+1}} \left(\bar{\Gamma}_n(I)\right)^{2j+1} \tag{16}$$

where we have purposely inserted a possible I-dependence in $\bar{\Gamma}_n$. Inserting (16) into (9) we finally obtain

$$\ln \left| \det \underset{\sim}{\bar{S}} \right| = -\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n(I)}{D_n} - \frac{\pi}{2} \sum_{j=1}^{\infty} \frac{(2j+3)(2j+2)}{(3)^{2j+1}} \left(\frac{\partial}{\partial I}\right)^{2j} \sum_{n=1}^{N} \frac{\left(\bar{\Gamma}_n(I)\right)^{2j+1}}{D_n} \tag{17}$$

At this point we assume that the average widths, $\bar{\Gamma}_n$, of the different classes of doorways satisfy the nested condition [3,11)]

$$\bar{\Gamma}_1 \gg \bar{\Gamma}_2 \gg - - - - \gg \bar{\Gamma}_N \tag{18}$$

Further, to unambiguously define an average width for a given class, one has to introduce a heirarchy of averaging intervals $I_n$, such that

$$\bar{\Gamma}_n \ll I_n < \bar{\Gamma}_{n-1} \tag{19}$$

with $I_1 \equiv I$

For convenience, we assume that the degree of "nestedness", $\frac{I_n}{I_{n+1}}$, is given by

$$\frac{I_n}{I_{n+1}} = \frac{\bar{\Gamma}_n}{\bar{\Gamma}_{n+1}} \tag{20}$$



With the above assumption, the averaging interval I, which is, by assumption, larger than all widths, is written as

$$I = \frac{\bar{\Gamma}_1}{\bar{\Gamma}_n} I_n \equiv \alpha_n I_n \tag{21}$$

With the help of the above assumptions and definitions, we may now write Eq. (17) in a more natural form

$$\ln \left| \det \underset{\sim}{\bar{S}} \right| = -\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n}{D_n} - \frac{\pi}{2} \sum_{j=1}^{\infty} \frac{(2j+3)(2j+2)}{(3)^{2j+1}} \sum_{n=1}^{N} \frac{1}{\alpha_n^{2j}} \left(\frac{\partial}{\partial I_n}\right)^{2j} \frac{\bar{\Gamma}_n^{2j+1}}{D_n} \tag{22}$$

As a result of the nested-doorway condition, Eq. (18), $\alpha_n \gg 1$, and therefore of all terms appearing in the n-sum of Eq. (17), n=1 would give the dominant contribution. To lowest order in the I-variation of $\bar{\Gamma}_1$ we obtain finally

$$\ln \left| \det \underset{\sim}{\bar{S}} \right| = -\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n}{D_n} - \pi \frac{10}{27} \left(\frac{\partial}{\partial I}\right)^2 \frac{\bar{\Gamma}_1^3}{D_1} \tag{23}$$

Equation (23) is the principal result of this paper. It supplies the measure of absorption in a nuclear reaction, due to multistep compound processes. It also dictates how the width distribution of the resonances enters in the determination of the average, optical S-matrix in terms of the average resonance parameters.

Though Eq. (23) deals with det $\bar{\underset{\sim}{S}}$, one may obtain a similar relation for the elements of $\bar{\underset{\sim}{S}}$ in the idealized case of m equivalent channels coupled equally to all doorway classes. Ignoring the correction factor in Eq.(23), we obtain

$$|\bar{S}_{cc}| = \exp\left[-\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n}{m D_n}\right] \tag{24}$$

from which the transmission factor $P_c \equiv 1 - |\bar{S}_{cc}|^2$ is obtained immediately

$$P_c = 1 - \exp\left[-2\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n}{m D_n}\right] \tag{25}$$

Assuming $\frac{D_n m}{\bar{\Gamma}_n} \gg 1$, and summing over c, we find

$$\sum_c P_c \cong 2\pi \sum_{n=1}^{N} \frac{\bar{\Gamma}_n}{D_n} \tag{26}$$

Since in the limit, $\frac{D_n m}{\bar{\Gamma}_n} \gg 1$, considered above one expects the correlation widths to coincide with the average width[13], we may rewrite Eq. (26) in the following form

$$\sum_c P_c = 2\pi \sum_{n=1}^{N} \frac{\Gamma_n^{corr}}{D_n} \tag{27}$$



Equation (27) is a sum rule relating the trace of the optical transmission matrix, obtainable from optical model analysis, to the correlation widths extracted from Ericson fluctuation analysis. The sum rule above has recently been discussed by one of us[14] in connection with preequilibrium reactions. We view our discussion above as a further support to the conclusions reached in Ref. 14).

## IV. Conclusions

In this paper we have generalized the Moldauer/Simonius theorem to the multi-class resonances situation. In the course of assessing the nature of the corrections to the generalized M/S theorem, we have examined the distribution of level widths of the different classes of doorways. It was found that in the limit of well-nested doorways, the first, and presumably dominant, correction to the generalized M/S theorem, involves the distribution of the level widths of the widest class of doorways.

REFERENCES

1) D. Agassi, H.A. Weidenmüller and G. Mantzouranis Phys. Rep. 22C (1975) 145.

2) H. Feshbach, A.K. Kerman and S.E. Koonin, Ann. Phys.(N.Y.) 125 (1980) 429.

3) W.A. Friedman, M.S. Hussein, K.W. McVoy and P.A. Mello Phys. Rep. 77C (1981) 47.

4) P.A. Moldauer, Phys. Rev. C11 (1975) 426.

5) T.A. Brody, J.Flores, J.B. French, P.A. Mello, A. Pandey and S.S.M. Wong, Rev. Mod. Phys. 53 (1981) 385.

6) H.M. Hofmann, T. Mertelmeier and H.A. Weidenmüller, Phys. Rev. C24 (1981) 1884.

7) K.W. McVoy, P.A. Mello and X.T. Tang, Ericson fluctuations revisited, MPI H-1981 - V30, to be published.

8) P.A. Moldauer, Phys. Rev. Lett. 19 (1967) 1047; Phys. Rev. C11 (1975) 426; M. Simonius, Phys. Lett. 52B (1974) 279.

9) K.W. McVoy and P.A. Mello, Nucl. Phys. A315 (1979) 391.

10) M. Kawai, A.K. Kerman and K.W. McVoy, Ann. Phys. (N.Y.) 75 (1973) 156.

11) M. Abramowitz and I.A. Stegun; Handbook of Mathematical Functions (Dover Publications Inc., New York, 1965) 227.



12) M.S. Hussein and K.W. McVoy, Phys. Rev. Lett. 43 (1979) 1645.

13) M. Bauer and P.A. Mello, J. Phys. G11 (1979) 1991; M. Bauer, P.A. Mello and K.W. McVoy, Zeit. Phys. A293 (1979) 151.

14) M.S. Hussein, Phys. Lett. 107B (1981) 307.

Figure captions

Figure 1 The width distribution $P(\Gamma) \times \frac{2}{27}\bar{\Gamma}$ plotted vs. $\Gamma/\bar{\Gamma}$ . The arrow indicates the correlation width $\Gamma^{corr} \equiv \frac{\langle\Gamma^{-1}\rangle}{\langle\Gamma^{-2}\rangle}$

Figure 2 The Cross-section auto correlation function, Eq. (15), plotted vs. $3\varepsilon/\bar{\Gamma}$ (solid curve). The dashed curve represents the one-pole approximation to $C(\varepsilon)$.

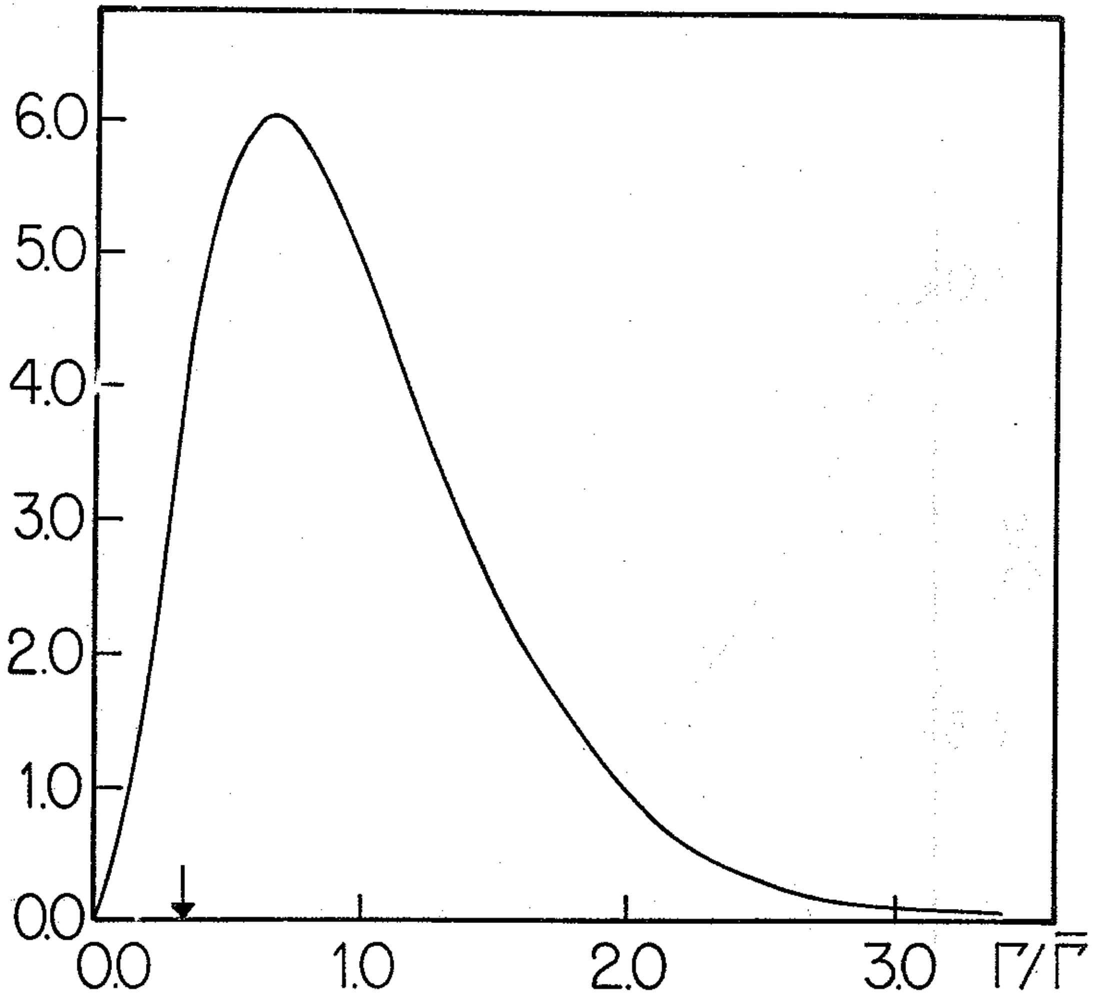


Figure 1

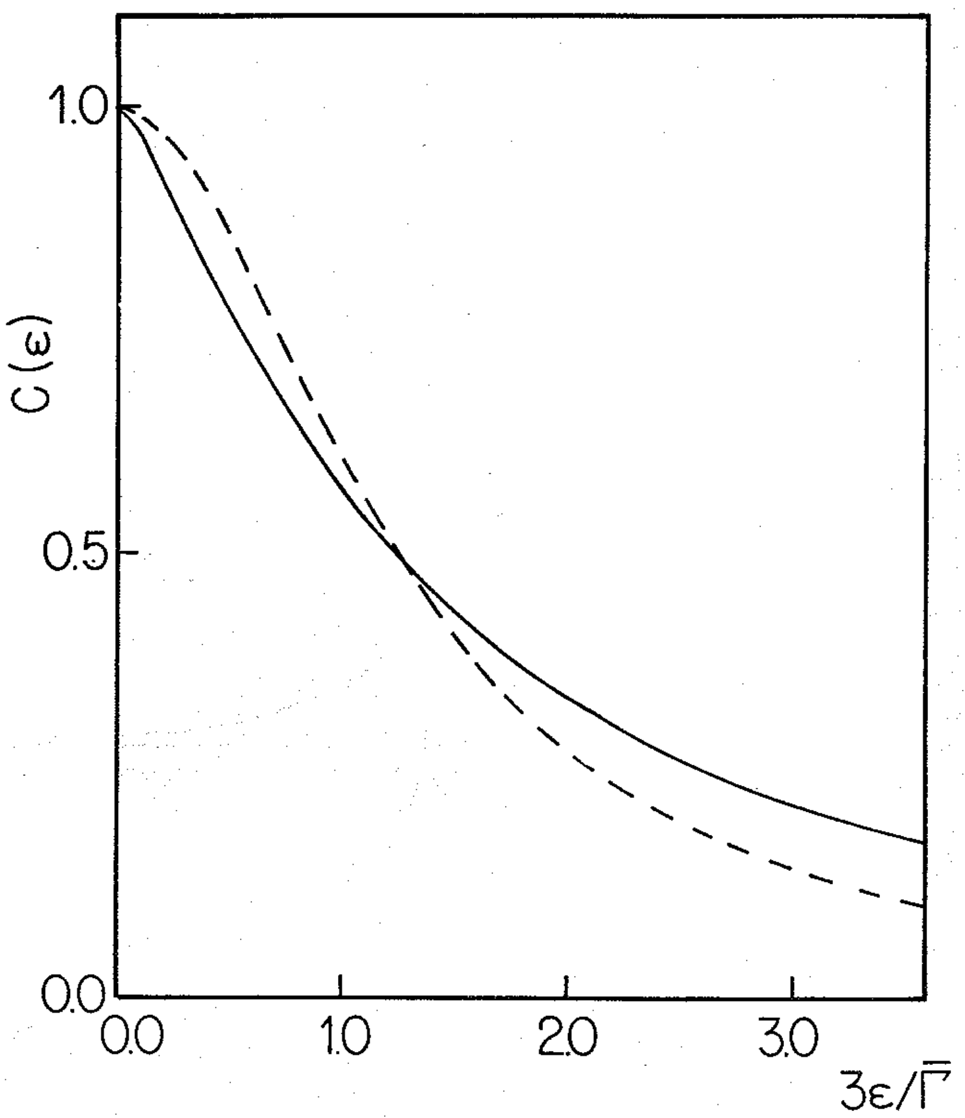


Figure 2