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AN SU(2) ⊗ U(1) MODEL WITH MASSIVE NEUTRINOS AND CP VIOLATION IN THE LEPTONIC SECTOR

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ABSTRACT

We examine an SU(2) x U(1) model incorporating massive neutrinos and CP violation for the leptons. The model has Dirac and Majorana mass terms for the neutrinos and two doublets and one triplet of Higgs bosons. We consider the implications of our model for the neutrino oscillations and for the electric dipole moment of the leptons.

1. INTRODUCTION

Recently there have been theoretical as well as experimental motivations for considering massive neutrinos. On the theoretical side is the result of some grand unified models [1] as for instance SO(10) where it is natural to have massive Majorana neutrinos. On the other side, from the experimental point of view, the possibility of neutrino oscillations [2] and the possible experimental measurement of a mass for the electron neutrino [3], have encouraged further investigations on the question of lepton number violation and neutrino masses. Assuming the neutrinos to be massive leads us to ask if it is a Majorana or a Dirac particle. In this paper we examine in detail a possible modification of the standard SU(2) x U(1) model [4]. The model we consider has both Majorana and Dirac mass terms, therefore lepton number is not conserved. The particle content is the following:

(i) the left-handed fermions are put into SU(2) doublets as usual,

(ii) the right-handed component of the charged leptons are SU(2) singlets and the right-handed neutrinos are singlets of both SU(2) and U(1), with zero hypercharge so that they are sterile having only a superweak Yukawa coupling,

(iii) two Higgs doublets of SU(2) giving Dirac masses for the leptons in the usual way, the reason why we choose two such doublets will be made clear further down in this section,

(iv) a Higgs triplet of SU(2) in order to give a Majorana mass to the left-handed neutrino[1] [7],

(v) the Majorana masses for the right-handed neutrinos are achieved by a bare mass term [7].

With such modifications of the standard model we can come close to the experimental limits on lepton number violating processes [8] such as $\mu \to e\gamma$, $\mu \to eee$ double beta neutrinoless decay, etc.. The eventual observation of such rare processes, while not directly indicating one particular grand unification scheme, would give us some hints as to the minimal scheme necessary for such unification. Therefore it is of some value to explore the full extent of a model built within the framework of $SU(2) \times U(1)$. On the other hand experimental indications of lepton number non-conservation will help to solve the well-known ambiguities on the nature of the partially conserved muon number; i.e. additive or multiplicative [9]. In this paper we are not concerned with such processes but only investigate neutrino oscillations and the electric dipole moment of the leptons, further applications of this model will be presented elsewhere [8].

The Higgs triplet has hypercharge $Y = 2$ (we use the definition $Q = T_3 + Y/2$) and can be represented by a 2×2 matrix,

$$\vec{\tau} \cdot \vec{\eta} = \begin{pmatrix} \eta^+ & \sqrt{2}\,\eta^{++} \\ \sqrt{2}\,\eta^0 & -\eta^+ \end{pmatrix} \tag{1.1}$$

We do not consider the case of spontaneous violation of lepton number, since no lepton number is attributed to the triplet as in the model of Gelmini and Roncadelli [10]. Lepton number is violated explicitly by the coupling of leptons with the triplet. This is more in line with our approach of relaxing natural flavour conservation (NFC) in the doublet sector, as will be explained in the following.

The general mass term in our model, with both Dirac and Majorana masses, is of the form[2] ,

$$L_m = D\overline{\psi}_L\psi_R + A\overline{\psi}^c_L\psi_L + B\overline{\psi}^c_R\psi_R + \text{h.c.} \tag{1.2}$$

The Majorana mass matrix A, for the left-handed neutrinos originates from the coupling with the Higgs triplet while B is a bare mass and D is a Dirac mass matrix coming from the Higgs doublets. A motivation for considering the general mass term is that rare processes such as $\mu \to e\gamma$ are incremented as close as possible to the experimental limit [11]. On the other hand this is the situation in grand unified models, like $O(10)$ with a <u>126</u>-Higgs which contains both triplet and singlet $SU(2)$ representations with possible non-vanishing vacuum expectation values. To impose a vanishing vacuum expectation value at the three level is not a natural procedure [7].

An important feature of our model is the possibility of having CP violation in the leptonic sector, as we will introduce two Higgs doublets. There is no reason a priori to expect CP to be a good symmetry for the leptons, especially if we invoke the symmetry between quarks and leptons.

There are two main alternatives to introduce CP violation in gauge theories. One possibility is through complex couplings in the weak charged current for more than two generations of fermions, as first pointed out by Kobayashi and Maskawa |12|[3]. The other alternative is to introduce a more complex Higgs sector and have CP violation via Higgs exchange [14]. It is clear that CP violation ã la Kobayashi-Maskawa would have very few, if any, observable consequences for the lepton sector since there is no analog of the $K^0 - \overline{K}^0$ system, apart, perhaps, from neutrino oscillations, in which case CP violating effects would be extremely difficult to detect anyway [13]. The other CP violating quantity of interest is the electric dipole moment (EDM), which in the Kobayashi-Maskawa model vanishes identically at the two-loop level for a single elementary fermion (quark or lepton) [15]. For quarks this does not imply a zero EMD, at that order in the weak coupling, since there is the possibility that strong (QCD) radiative corrections modify this result [16] or that bound state effects give a non-vanishing EDM for a di-quark subsystem, thus resulting in an EDM for the neutron [17][4]. These two possibilities are absent for the leptons and therefore the lepton EDM would vanish to fourth-order in the weak coupling in the Kobayashi-Maskawa model. This provides one of the motivations for exploring CP violation via Higgs exchange, where the EDM is not necessarily small and occurs at the one loop level [14].

There are at least two ways of introducing CP violation via Higgs exchange. The possibility suggested by Weinberg [14] relies on three or more doublets and is characterized by the property of natural flavour conservation [19]. In this model we can start with a lagrangean which is CP symmetric or not, the minimum number of three Higgs doublets is necessary in order to guarantee the existence of at least one CP violating parameter, in this case a complex phase in the mixing matrix for the charged Higgs[5]. T.D. Lee's model |14|, on the other hand, makes use of two Higgs doublets [21] but does not have NFC and CP violation is spontaneously broken. As discussed in Section 2, there are no compelling reasons for imposing NFC for the leptons, so that we adopt in this paper a model of CP violation with two Higgs doublets and one Higgs triplet, without NFC. Our model, is in a sense, a blend of Weinberg's and Lee's models, CP violation is spontaneous and occurs via the exchange of charged and neutral Higgs, but the presence of the triplet is important for the mixing of the charged Higgs, characteristic of the way CP violation is parametrized in a model like Weinberg's.

We now give an outline of the paper. In Section 2 we present the details of the model. Section 2.1 contains the discussion of the Higgs potential and the mixing of the charged Higgs fields. Section 2.2 presents the Yukawa couplings. Section 3 is devoted to neutrino oscillations in our model. Section 4 discusses the EDM of the leptons. Our conclusions are in Section 5. In the Appendix we analyse

the structure of charged and neutral currents.

2. THE MODEL

2.1. The Higgs Potential

As mentioned in the Introduction we need at least two Higgs doublets and one triplet in order to have both CP violation and a Majorana mass, without NFC and without having lepton number as a global symmetry [10]. Our hypercharge assignment implies that the triplet is non-hermitian. Therefore we have fourteen real fields, eight for the two doublets and six for the triplet. The most general potential is of the form,

$$
\begin{aligned}
V(\Phi_1,\Phi_2,\vec{\eta}) = & - \mu_1^2\Phi_1^{+}\Phi_1 - \mu_2^2\Phi_2^{+}\Phi_2 - \mu^2\vec{\eta}^{*}.\vec{\eta} + h_1(\Phi_1^{+}\Phi_1)^2 + \\
& + h_2(\Phi_2^{+}\Phi_2)^2 + h_3(\Phi_1^{+}\Phi_1)(\Phi_2^{+}\Phi_2) + h_4(\Phi_1^{+}\Phi_2)(\Phi_2^{+}\Phi_1) + \\
& + h_5(\Phi_1^{+}\Phi_2)^2 + h_5^{*}(\Phi_2^{+}\Phi_1)^2 + \left[h_6(\Phi_1^{+}\Phi_2) + h_6^{*}(\Phi_2^{+}\Phi_1)\right](\Phi_1^{+}\Phi_1) + \\
& + \left[h_7(\Phi_1^{+}\Phi_2) + h_7^{*}(\Phi_2^{+}\Phi_2)\right](\Phi_2^{+}\Phi_2) + f_0(\vec{\eta}^{*}.\vec{\eta})^2 + \\
& + f_1(\Phi_1^{+}\Phi_1)(\vec{\eta}^{*}.\vec{\eta}) + f_2(\Phi_2^{+}\Phi_2)(\vec{\eta}^{*}.\vec{\eta}) + \\
& + f_3(\Phi_1^{+}\Phi_2)(\vec{\eta}^{*}.\vec{\eta}) + f_3^{*}(\Phi_2^{+}\Phi_1)(\vec{\eta}^{*}.\vec{\eta}) + f_4(\Phi_1^{T}\epsilon\vec{\tau}\Phi_1) . \vec{\eta}^{*} + \\
& + f_4^{*}(\Phi_1^{+}\epsilon\vec{\tau}^{*}\Phi_1^{*}) . \vec{\eta} + f_5(\Phi_2^{T}\epsilon\vec{\tau}\Phi_2) . \vec{\eta}^{*} + f_5^{*}(\Phi_2^{+}\epsilon\vec{\tau}^{*}\Phi_2^{*}) . \vec{\eta} + \\
& + f_6(\Phi_1^{T}\epsilon\vec{\tau}\Phi_2) . \vec{\eta}^{*} + f_6^{*}(\Phi_1^{+}\epsilon\vec{\tau}^{*}\Phi_2^{*}) . \vec{\eta} \quad ,
\end{aligned}
\qquad (2.1)
$$

where $\epsilon = i\,\tau_2$ and $\vec{\tau}$ are the Pauli matrices.
Up this point we have not yet imposed any restrictions on the constants of the potential, but as we will require spontaneous CP violation, then these constants will ultimately be made real.

Expanding around the vacuum expectation values of the neutral Higgs fields,

$$\phi_1^0 = \frac{\lambda_1}{\sqrt{2}}\left(1 + \frac{H_1 + i\chi_1}{|\lambda_1|}\right) \quad ; \quad \phi_2^0 = \frac{\lambda_2}{\sqrt{2}}\left(1 + \frac{H_2 + i\chi_2}{|\lambda_2|}\right) \quad ; \quad \eta^0 = \frac{\lambda}{\sqrt{2}}\left(1 + \frac{\sigma + i\rho}{|\lambda|}\right) \tag{2.2}$$

we obtain,

$$\begin{aligned} V_{c.n_0.} = & -\frac{1}{2}\mu_1^2|\lambda_1|^2 - \frac{1}{2}\mu_2^2|\lambda_2|^2 - \frac{1}{2}\mu^2|\lambda|^2 + \frac{1}{4}h_1|\lambda_1|^4 + \frac{1}{4}h_2|\lambda_2|^4 + \\ & + \frac{1}{4}(h_3+h_4)|\lambda_1|^2|\lambda_2|^2 + \frac{1}{2}\mathrm{Re}\left[h_5(\lambda_1^*\lambda_2)^2\right] + \frac{1}{2}|\lambda_1|^2\,\mathrm{Re}(h_6\lambda_1^*\lambda_2) + \\ & + \frac{1}{2}|\lambda_2|^2\,\mathrm{Re}(h_7\lambda_1^*\lambda_2) + \frac{1}{4}f_0|\lambda_3|^2 + \frac{1}{4}f_1|\lambda_1|^2|\lambda|^2 + \\ & + \frac{1}{4}f_2|\lambda_2|^2|\lambda|^2 + \frac{1}{2}|\lambda|^2\,\mathrm{Re}(f_3\lambda_1^*\lambda_2) - \mathrm{Re}(f_4\lambda_1^2\lambda^*) - \mathrm{Re}(f_5\lambda_2^2\lambda^*) - \\ & - \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*) \quad . \end{aligned} \tag{2.3a}$$

Although we do not show it explicitly, it can be shown that the minimum of $V_{c.n_0.}$ will violate CP, as pointed out by Lee |14|. We also have the auxiliary conditions,

$$\begin{aligned} & -\mu_1^2|\lambda_1|^2 + h_1|\lambda_1|^4 + \frac{1}{2}(h_3+h_4)|\lambda_1|^2|\lambda_2|^2 + \mathrm{Re}\left[h_5(\lambda_1^*\lambda_2)^2\right] + \\ & + \frac{3}{2}|\lambda_1|^2\,\mathrm{Re}(h_6\lambda_1^*\lambda_2) + \frac{1}{2}|\lambda_2|^2\,\mathrm{Re}(h_7\lambda_1^*\lambda_2) + \frac{1}{2}f_1|\lambda_1|^2|\lambda|^2 + \\ & + \frac{1}{2}|\lambda|^2\,\mathrm{Re}(f_3\lambda_1^*\lambda_2) - 2\,\mathrm{Re}(f_4\lambda_1^2\lambda^*) - \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*) = 0 \quad , \end{aligned} \tag{2.3b}$$

$$\begin{aligned} & -\mu_2^2|\lambda_2|^2 + h_2|\lambda_2|^4 + \frac{1}{2}(h_3+h_4)|\lambda_1|^2|\lambda_2|^2 + \mathrm{Re}\left[h_5(\lambda_1^*\lambda_2^2)^2\right] + \frac{1}{2}|\lambda_1|^2\,\mathrm{Re}(h_6\lambda_1^*\lambda_2) + \\ & + \frac{3}{2}|\lambda_2|^2\mathrm{Re}(h_7\lambda_1^*\lambda_2) + \frac{1}{2}f_2|\lambda_2|^2|\lambda|^2 + \frac{1}{2}|\lambda|^2\mathrm{Re}(f_3\lambda_1^*\lambda_2) - 2\mathrm{Re}(f_5\lambda_2^2\lambda^*) - \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*) = 0 \end{aligned} \tag{2.3c}$$

$$\begin{aligned} & -\mu^2|\lambda|^2 + f_0|\lambda|^4 + \frac{1}{2}f_1|\lambda_1|^2|\lambda\;|^2 + \frac{1}{2}f_2|\lambda_2|^2|\lambda|^2 + |\lambda|^2\mathrm{Re}(f_3\lambda_1^*\lambda_2) - \\ & - \mathrm{Re}(f_4\lambda_1^2\lambda^*) - \mathrm{Re}(f_5\lambda_2^2\lambda^*) - \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*) = 0 \end{aligned} \tag{2.3d}$$

$$\begin{aligned} & \mathrm{Im}\left[h_5(\lambda_1^*\lambda_2)^2\right] + \frac{1}{2}|\lambda_1|^2\,\mathrm{Im}(h_6\lambda_1^*\lambda_2) + \frac{1}{2}|\lambda_2|^2\,\mathrm{Im}(h_7\lambda_1^*\lambda_2) + \\ & + \frac{1}{2}|\lambda|^2\,\mathrm{Im}(f_3\lambda_1^*\lambda_2) + 2\,\mathrm{Im}(f_4\lambda_1^2\lambda^*) + \mathrm{Im}(f_6\lambda_1\lambda_2\lambda^*) = 0 \end{aligned} \tag{2.3e}$$

$$\begin{aligned} & -\mathrm{Im}\left[h_5(\lambda_1^*\lambda_2)^2\right] - \frac{1}{2}|\lambda_1|^2\,\mathrm{Im}(h_6\lambda_1^*\lambda_2) - \frac{1}{2}|\lambda_2|^2\,\mathrm{Im}(h_7\lambda_1^*\lambda_2) - \\ & - \frac{1}{2}|\lambda|^2\,\mathrm{Im}(f_3\lambda_1^*\lambda_2) + 2\,\mathrm{Im}(f_5\lambda_2^2\lambda^*) + \mathrm{Im}(f_6\lambda_1\lambda_2\lambda^*) = 0 \end{aligned} \tag{2.3f}$$

Of the fourteen real scalar fields, three are eaten by the Higgs mechanism[6],

$$\begin{aligned} & \left\{|\lambda_1|(\phi_1^0 - \phi_1^{0*}) + |\lambda_2|(\phi_2^0 - \phi_2^{0*}) + \sqrt{2}|\lambda|(\eta^0 - \eta^{0*})\right\} / \sqrt{|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda|^2} \\ & \left\{|\lambda_1|\phi_1^- + |\lambda_2|\phi_2^- - |\lambda|\eta^-\right\} \quad \sqrt{|\lambda_1|^2 + |\lambda_2|^2 + |\lambda|^2} \end{aligned} \tag{2.4}$$

and hermitean conjugate.

The mass matrix for the charged Higgs is given by,

$$\begin{array}{c} \\ \phi_1^- \\ \\ \phi_2^- \\ \\ \eta^- \\ \\ \eta^{--} \end{array} \begin{array}{c} \begin{array}{cccc} \quad\phi_1^+ \qquad & \qquad \phi_2^+ \qquad & \qquad \eta^+ \qquad & \eta^{++} \end{array} \\ \left(\begin{array}{ccc|c} \dfrac{A_2 + A_3}{|\lambda_1|^2} & -\dfrac{A_3 - iB}{\lambda_1^*\lambda_2} & -\dfrac{A_2 + iB}{\sqrt{2}\lambda_1^*\lambda_2} & 0 \\ -\dfrac{A_3 + iB}{\lambda_1\lambda_2^*} & \dfrac{A_1 + A_2}{|\lambda_2|^2} & -\dfrac{A_1 - iB}{\sqrt{2}\lambda_2^*\lambda} & 0 \\ -\dfrac{A_2 - iB}{\sqrt{2}\lambda_1\lambda_2^*} & -\dfrac{A_1 + iB}{\sqrt{2}\lambda_2\lambda^*} & \dfrac{A_1 + A_2}{2|\lambda|^2} & 0 \\ \hline 0 & 0 & 0 & \dfrac{A_1 + A_2}{2|\lambda|^2} \end{array}\right) \end{array} \tag{2.5}$$

with

$$A_1 = 2\,\mathrm{Re}(f_5\lambda_1^2\lambda^*) + \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*)$$

$$A_2 = 2\,\mathrm{Re}(f_4\lambda_1^2\lambda^*) + \mathrm{Re}(f_6\lambda_1\lambda_2\lambda^*)$$

$$A_3 = -\frac{1}{2}h_4\,|\lambda_1|^2|\lambda_2|^2 - \mathrm{Re}\left[h_5(\lambda_1^*\lambda_2)^2\right] - \frac{1}{2}|\lambda_1|^2\,\mathrm{Re}(h_6\lambda_1^*\lambda_2) -$$
$$-\frac{1}{2}|\lambda_2|^2\,\mathrm{Re}(h_7\lambda_1^*\lambda_2) - \frac{1}{2}|\lambda|^2\,\mathrm{Re}(f_3\lambda_1^*\lambda_2)$$

$$B = -2\,\mathrm{Im}(f_4\lambda_1^2\lambda^*) - \mathrm{Im}(f_6\lambda_1\lambda_2\lambda^*) \qquad (2.6)$$

B is the parameter characterizing CP violation, $B=0$ would imply CP conservation. Apparently there is another parameter B', given by

$$B' = 2\,\mathrm{Im}(f_5\lambda_2^2\lambda^*) + \mathrm{Im}(f_6\lambda_1\lambda_2\lambda^*)$$

which would also control the amount of CP violation in our model, however, from eqs. (2.3e) and (2.3f) we have the constraint,

$$\mathrm{Im}(f_4\lambda_1^2\lambda^*) + \mathrm{Im}(f_5\lambda_2^2\lambda^*) + \mathrm{Im}(f_6\lambda_1\lambda_2\lambda^*) = 0$$

which implies $B' = B$.

In (2.5), the 3 x3 block matrix gives the mass matrix for the singly charged Higgs. From (2.5) we also see that there is no mixing involving the doubly charged Higgs, η^{++}, as expected. We can diagonalize the 3 x3 submatrix in order to obtain the mass eigenstates. When performing this, there will appear phases of the same type as those pointed out by Kobayashi and Maskawa [22]. An alternative is to find the expressions for the transition propagators $<\phi_1^-\phi_2^+>$, $<\phi_1^-\eta^+>$, $<\phi_2^-\eta^+>$. As the mass matrix (2.5) still

contains the would be Goldstone bosons, it is a singular matrix and in order to eliminate the singularity we can add the term $\lambda_o S^- S^+$ to this matrix, where S^- is the second expression in eq. (2.4). For zero momentum transfer, we have |23|

$$<\psi_i^-\psi_j^+> = -\lim_{\lambda_o\to 0}\frac{\mathrm{cofactor}\,(M+\lambda_o)_{ij} - \mathrm{cofactor}(M)_{ij}}{\det\,(M+\lambda_o)} \qquad (2.7)$$

where M is the 3x3 submatrix in (2.5)[7]. For instance, $<\phi_1^-\phi_2^+>$ is given by

$$\frac{<\phi_1^-\phi_2^+>}{\lambda_1^*\lambda_2} = -\frac{|\lambda_1|^2|\lambda_2|^2|\lambda|^2}{v^4\,\Delta}\left\{\frac{|\lambda|^2}{|\lambda_1|^2|\lambda_2|^2}(A_3+iB) - \frac{1}{|\lambda_1|^2}(A_2-iB) - \right.$$
$$\left. -\frac{1}{|\lambda_2|^2}(A_1+A_2) - \frac{1}{|\lambda_2|^2}(A_1-iB)\right\} \qquad (2.8)$$

where

$$v = (|\lambda_1|^2 + |\lambda_2| + |\lambda|^2)^{1/2} \qquad (2.9)$$

and

$$\Delta = A_1A_2 + A_2A_3 + A_3A_1 - B^2 \qquad (2.10)$$

Therefore, we have for $\mathrm{Im}\left[\frac{<\phi_1^-\phi_2^+>}{\lambda_1^*\lambda_2}\right]$ the expression,

$$\mathrm{Im}\left[\frac{<\phi_1^-\phi_2^+>}{\lambda_1^*\lambda_2}\right] = -\frac{|\lambda|^2}{v}\,\frac{B}{\Delta} \qquad (2.11)$$

For non-zero momentum transfer the generalization of the above expression is straightforward as given by Anselm and Ural'tsev

[22]. In the case of the neutral Higgs bosons, the mass matrix that we have to diagonalize is 6x6 , the procedure is similar but cumbersome, however the transition propagators as $\langle H_1 X_1 \rangle$ are always proportional to the parameter B , and we also have CP violation through neutral Higgs exchange [20].

2.2. The Yukawa Couplings

With several Higgs doublets giving Dirac masses to the neutrinos and charged leptons, plus a number of Higgs triplets responsible fo the Majorana masses of left-handed neutrinos, we have the following Yukawa couplings in the theory,

$$\mathcal{L}_y = \bar{\ell}_R \, \Gamma^{\alpha} (\phi^{-}_{\alpha} \nu_L + \phi^{o}_{\alpha} \ell_L) + \bar{\nu}_R \, \Gamma^{\beta} (\phi^{o}_{\beta} \nu_L - \phi^{+}_{\beta} \ell_L) + B \bar{\nu}^{c}_{R} \nu_R +$$

$$+ \kappa^{\gamma} \left[\sqrt{2} \, \eta^{o}_{\gamma} \, \bar{\nu}^{c}_{L} \nu_L - \eta^{+}_{\gamma} (\bar{\nu}^{c}_{L} \ell_L + \bar{\ell}^{c}_{L} \nu_L) - \sqrt{2} \, \eta^{++}_{\gamma} \, \bar{\ell}^{c}_{L} \ell_L \right] + \text{h.c.} \qquad (2.12)$$

where we have shown for completeness, a bare mass term for the right-handed neutrinos.

The coupling with the doublets is as in the quark sector and we could, if desired, impose natural flavour conservation (NFC) [19]. For example, with just two Higgs doublets, one of them giving a mass to the charged leptons while the other gives a mass to the neutrinos, we would have NFC. However we see no forceful reasons for imposing NFC for the leptons, since:

(i) the mass difference of the neutrinos is of the same order of magnitude of their own mass. This is not the situation for hadrons where $\frac{\Delta m_k}{m_k} \sim 10^{-14}$, therefore a mechanism has to be found in this case in order to suppress virtual corrections of order $G_F^2 \alpha$ that spoil this relation [19].

(ii) the processes that violate lepton number are much supressed by the very mass scale of the leptons [24], the neutrinos are very stable unlike kaons and we would like to know in what conditions an extended Higgs sector could push up the rate for processes such as $\mu \to e\gamma$, neutrinoless 2β decay, etc., as close as possible to the present experimental limits.

For the above reasons we will not impose NFC in the couplings of the leptons with the Higgs doublets.

In order to relax NFC, it is enough to couple several Higgs with ℓ_R and ν_R in such a way that the coupling matrices $\Gamma^{(\alpha)}$ in eq. (2.12) be not simultaneously diagonalizable by the same bi-unitary transformation which diagonalizes the mass matrix, $M = \sum_{\alpha} \Gamma^{\alpha} \lambda_{\alpha}$, with $\lambda_{\alpha}/\sqrt{2} = \langle \phi^{\alpha} \rangle$. As we want to introduce a minimum number of Higgs multiplets, with CP violation and without NFC, it is enough to work with just two Higgs multiplets [14]. Recall that for hadrons, the requirement of NFC implies at least three multiplets. We have from the beginning a triplet in order to introduce Majorana masses and one doublet for the Dirac masses, but due to our hypercharge assignment for the triplet, there is no CP violation. This forces us to introduce another doublet, which will also allows for CP violation even if it turns out that neutrinos are Dirac particles with or without masses and the triplet absent.

The consequences of not imposing NFC for the leptons are that, (i) the established results relating NFC and non-calculability of the mixing angles |25| are not valid anymore; (ii) the argument by which NFC plus spontaneous breakdown of CP imply that the mixing mass matrix is strictly real for any number of generations and Higgs doublets [26] is no longer valid and therefore in principle we could have complex couplings in the charged current. As we are interested in CP violation via Higgs, we will assume the generalized Cabibbo matrix to be real.

With these remarks, the Yukawa term in our lagrangian reads,

$$L_y = \bar{\ell}_R \left\{\Gamma^{(1)} \left(\phi_1^- \nu_L + \bar{\phi}_1^0 \ell_L\right) + \Gamma^{(2)} \left(\phi_2^- \nu_L + \bar{\phi}_2^0 \ell_L\right)\right\} +$$

$$+ \bar{\nu}_R \Lambda^{(2)} \left(\phi_2^0 \nu_L - \phi_2^+ \ell_L\right) + B \bar{\nu}_R^c \nu_R + \kappa \left\{\sqrt{2}\, \eta^0 \bar{\nu}_L^c \nu_L - \eta^+ \left(\bar{\nu}_L^c \ell_L + \bar{\ell}_L^c \nu_L\right)\right.$$

$$\left. - \sqrt{2}\, \eta^{++} \bar{\ell}_L^c \ell_L \right\} + h.c. \qquad (2.13)$$

We only allow the coupling of ν_R with the second doublet for reasons to be explained below. Expanding the neutral Higgs as in eq. (2.2), the Yukawa term splits into three parts, the mass term, and the neutral and charged parts,

$$L_y = L_M + L_N + L_c \qquad (2.14)$$

Let us first consider the mass term:

$$L_M = \bar{\ell}_R \left[\Gamma^{(1)} \frac{\lambda_1^*}{\sqrt{2}} + \Gamma^{(2)} \frac{\lambda_2^*}{\sqrt{2}}\right] \ell_L + \bar{\nu}_R \Lambda^{(2)} \frac{\lambda_2}{\sqrt{2}} \nu_L + B \bar{\nu}_R^c \nu_R +$$

$$+ \frac{\lambda}{\sqrt{2}} \kappa\, \bar{\nu}_L^c \nu_L + h.c. \qquad (2.15)$$

Up to now our formulation has been valid for any number of generations, however in the following we will restrict our discussion to two generations (electron and muon), for the sake of simplicity.

Already specifying for the case of two generations, let us now rotate the Dirac fields to the physical basis, by the following unitary transformation[8]

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \end{pmatrix}_{L,R} = U_{L,R}\, \nu_{L,R} \quad , \quad \begin{pmatrix} e' \\ \mu' \end{pmatrix}_{L,R} = V_{L,R}\, \ell_{L,R} \qquad (2.16)$$

The mass matrix for the charged leptons is brought to the diagonal form by a bi-unitary transformation,

$$V_R \left[\Gamma^{(1)} \frac{\lambda_1^*}{\sqrt{2}} + \Gamma^{(2)} \frac{\lambda_2^*}{\sqrt{2}}\right] V_L^{-1} \equiv M_1 = \begin{pmatrix} m_e & 0 \\ 0 & m_\mu \end{pmatrix} \qquad (2.17)$$

For the neutrinos, the mass term is as in eq. (1.2). Introducing the Majorana fields χ and ω,

$$\chi = \nu_L + \nu_L^c \quad ; \quad \omega = \nu_R + \nu_R^c \qquad (2.18)$$

the mass term can be written as,

$$(\bar{\chi} \quad \bar{\omega}) \begin{pmatrix} \frac{\lambda}{\sqrt{2}} \kappa & \frac{\lambda_2}{2\sqrt{2}} \Gamma^{(2)} \\ \frac{\lambda_2^* \Gamma^{(2)T}}{2\sqrt{2}} & B \end{pmatrix} \begin{pmatrix} \chi \\ \omega \end{pmatrix} \qquad (2.19)$$

With only a Dirac mass term, the situation would be simple,

$$U_R \Gamma^{(2)} \frac{\lambda_2}{\sqrt{2}} U_L^{-1} \equiv M = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \qquad (2.20)$$

In eq. (2.19) we have a 4x4 matrix , which can be diagonalized as,

$$W \mathcal{M} W^T = M_D \qquad (2.21)$$

where W is a 4x4 unitary matrix. A convenient parametrization for the matrix W is the following [11],

$$W = \begin{pmatrix} \tilde{c} & \tilde{s} \\ -\tilde{s} & \tilde{c} \end{pmatrix} \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix} \qquad (2.22)$$

With this form we can diagonalize first the off-diagonal terms of eq. (2.19) using (2.20). In eq. (2.22) $\tilde{c}$ and $\tilde{s}$ are 2x2 matrices whose elements are $c_{ij} = \cos\theta_i \delta_{ij}$ and $s_{ij} = \mathrm{sen}\theta_i \delta_{ij}$. This is a consequence of the fact that already with only one generation a Dirac neutrino splits into two Majorana neutrinos having different masses, in the presence of both Majorana and Dirac mass terms [7].

We now have,

$$W M W^T = \begin{pmatrix} \tilde{c} & -\tilde{s} \\ \tilde{s} & \tilde{c} \end{pmatrix} \begin{pmatrix} U_L \frac{\lambda \kappa U_L^{-1}}{\sqrt{2}} & M_2 \\ M_2 & U_R B U_R^{-1} \end{pmatrix} \begin{pmatrix} \tilde{c} & \tilde{s} \\ -\tilde{s} & \tilde{c} \end{pmatrix} \qquad (2.23)$$

As in general $U_L \kappa U_L^{-1}$ and $U_R B U_R^{-1}$ are not diagonal, we have two possibilities. The first is to impose that κ and B be each proportional to the identity matrix;

$$\frac{\lambda \kappa_{ij}}{\sqrt{2}} = A\, \delta_{ij} \qquad ; \qquad B_{ij} = B\, \delta_{ij} \qquad (2.24)$$

which implies the same Majorana mass for all neutrinos of a given handedness. The second alternative is to assume that κ and B are diagonalizable by a similarity transformation via the matrices U_L and U_R respectively, it then follows from (2.23) that the Majorana mass matrices are proportional to the square of the Dirac ones[9]. We will adopt in the following the first alternative since it is less restrictive and because we are interested in the general features of a model with both Dirac and Majorana masses independently of the specific values of masses and mixing angles, which after all are not well known experimentally.

After the rotation to the physical basis we must consider the form of the mixing matrix appearing in the charged current. This is done in the Appendix.

Returning to the lagrangean given in eq. (2.14), the neutral and charged parts are given by,

$$L_N = \bar{\ell}_R \left\{ \frac{\lambda_1^*}{\sqrt{2}} \frac{(H_1 - i\chi_1)}{|\lambda_1|} \Gamma^{(1)} + \frac{\lambda_2^*}{\sqrt{2}} \frac{(H_2 - i\chi_2)}{|\lambda_2|} \Gamma^{(2)} \right\} \ell_L +$$

$$+ \frac{\lambda_2}{\sqrt{2}} \frac{(H_2 + i\chi_2)}{|\lambda_2|} \bar{\nu}_R \Lambda \nu_L + \frac{\lambda}{\sqrt{2}} (\sigma + i\rho) \bar{\nu}_L^c \nu_L + \text{h.c.} \qquad (2.25)$$

$$L_C = \bar{\ell}_R \left\{ \Gamma^{(1)} \phi_1^- + \Gamma^{(2)} \phi_2^- \right\} \nu_L - \bar{\nu}_R \phi_2^+ \ell_L - \kappa \eta^+ (\bar{\nu}_L^c \ell_L + \bar{\ell}_L^c \nu_L) -$$

$$- \sqrt{2}\, \kappa \eta^{++} \bar{\ell}_L^c \ell_L + \text{h.c.} \qquad (2.26)$$

Since by eq. (2.17), it is the sum $\lambda_1^* \Gamma^{(1)} + \lambda_2^* \Gamma^{(2)}$ that has to be diagonalized instead of each of them separately, the parametrization of Γ_1 or Γ_2 are rather arbitrary and is not of the familiar generalized Cabibbo form. As a simple way of reducing this arbritariness and obtaining a Cabbibo form, we require $\Gamma^{(2)}$ and Λ to be proportional. However, allowing for an arbritary constant of proportionality between $\Gamma^{(2)}$ and Λ , then the scale of flavour non-conservation would no longer be controlled by the neutrino mass scale. For this reason we use in the following an equality[10],

$$\Gamma^{(2)} = \Lambda \qquad (2.27)$$

With $U_{L,R}$ and $V_{L,R}$ defined in eq. (2.16), the Cabibbo matrices are written as,

$$V_R\, U_R^{-1} = \begin{pmatrix} c_R\, e^{i\alpha_R} & s_R\, e^{i\beta_R} \\ -s_R\, e^{i\gamma_R} & c_R\, e^{i\delta_R} \end{pmatrix}$$

$$V_L\, U_L^{-1} = \begin{pmatrix} c_L & s_L\, e^{i\beta_L} \\ -s_L\, e^{-i\beta_L} & c_L \end{pmatrix} \tag{2.28}$$

where $c_{L,R} = \cos\theta_{L,R}$ and $s_{L,R} = \sin\theta_{L,R}$. In the Appendix we show how to arrive at the above result. Notice that in L_N and L_c, eqs. (2.24) and (2.25) there will be both left and right mixing angles. Since the right-handed neutrinos are sterile, a mixing among them has no observable consequences in the weak charged current. On the other hand the right mixing angles will only appear in the neutral and charged Yukawa couplings always multiplied by the neutrino masses and together with the corresponding left mixing angle, therefore even if θ_R is non-vanishing it will be a difficult task to detect its presence. For these reasons we make θ_R and the right phases vanishing.

With respect to the phase β_L in (2.28), we notice that it is a potential source of CP violation. For the reasons explained before we also make $\beta_L = 0$.

With these remarks L_c and L_N in eqs. (2.25) and (2.26) can be written as,

$$L_c = \left(\frac{\lambda_1^*}{\sqrt{2}}\right)^{-1} \phi_1^- \left[(m_e\cos\theta - D_1)\,\bar{e}_R\,\nu_{eL} + m_e\sin\theta\,\bar{e}_R\nu_{\mu L} - m_\mu\sin\theta\,\bar{\mu}_R\nu_{eL} + \right.$$
$$\left. + (m_\mu\cos\theta - D_2)\,\bar{\mu}_R\nu_{\mu L}\right] + \left(\frac{\lambda_2}{\sqrt{2}}\right)^{-1} \phi_2^- \left[D_1\,\bar{e}_R\nu_{eL} + D_2\,\bar{\mu}_R\nu_{\mu L}\right] -$$

$$- \left(\frac{\lambda_2}{\sqrt{2}}\right)^{-1} \phi_2^+ \left[D_1\cos\theta\,\bar{\nu}_{eR}\,e_L - D_1\sin\theta\,\bar{\nu}_{eR}\mu_L + D_2\sin\theta\,\bar{\nu}_{\mu R}e_L + \right.$$
$$\left. + D_2\cos\theta\,\bar{\nu}_{\mu R}\mu_L\right] - \left(\frac{\lambda}{\sqrt{2}}\right)^{-1} A\,\eta^+ \left[\bar{\nu}^c_{eL}e_L + \bar{\nu}^c_{\mu L}\mu_L + \bar{e}^c_L\nu_{eL} + \bar{\mu}^c_L\nu_{\mu L}\right] -$$
$$- \sqrt{2}\left(\frac{\lambda}{\sqrt{2}}\right)^{-1} A\eta^{++}\left[\bar{e}^c_L e_L + \bar{\mu}^c_L\mu_L\right] + \text{h.c.} \tag{2.29}$$

and

$$L_N = \frac{H_1}{|\lambda_1|}\left[(m_e - D_1\cos\theta)\bar{e}e + (m_\mu - D_2\cos\theta)\bar{\mu}\mu + D_1\sin\theta(\bar{e}_R\mu_L + \bar{\mu}_L e_R) - \right.$$
$$\left. - D_2\sin\theta(\bar{\mu}_R e_L + \bar{e}_L\mu_R)\right] + \frac{i\,\chi_1}{|\lambda_1|}\left[(m_e - D_1\cos\theta)\bar{e}\gamma_5 e + (m_\mu - D_2\cos\theta)\bar{\mu}\gamma_5\mu - \right.$$
$$\left. - D_1\sin\theta(\bar{e}_R\mu_L - \bar{\mu}_L e_R) + D_2\sin\theta(\bar{\mu}_R e_L - \bar{e}_L\mu_R)\right] + \frac{H_2}{|\lambda_2|}\left[D_1\bar{e}e + \right.$$
$$\left. + D_2\bar{\mu}\mu - D_1\sin\theta(\bar{e}_R\mu_L + \bar{\mu}_L e_R) + D_2\sin\theta(\bar{\mu}_R e_L + \bar{e}_L\mu_R)\right] -$$
$$- \frac{i\,\chi_2}{|\lambda_2|}\left[D_1\cos\theta\bar{e}\gamma_5 e + D_2\cos\theta\bar{\mu}\gamma_5\mu - D_1\sin\theta(\bar{e}_R\mu_L - \bar{\mu}_L e_R) + D_2\sin\theta(\bar{\mu}_R e_L - \right.$$
$$\left. - \bar{e}_L\mu_R)\right] + \frac{H_2}{|\lambda_2|}(D_1\bar{\nu}_e\nu_e + D_2\bar{\nu}_\mu\nu_\mu) - \frac{i\,\chi_2}{|\lambda_2|}(D_1\bar{\nu}_e\gamma_5\nu_e + D_2\bar{\nu}_\mu\gamma_5\nu_\mu) +$$
$$+ \left(\frac{\lambda}{\sqrt{2}}\right)^{-1} A\sigma(\bar{\nu}^c_{eL}\nu_{eL} + \bar{\nu}^c_{\mu L}\nu_{\mu L} + \bar{\nu}_{eL}\nu^c_{eL} + \bar{\nu}_{\mu L}\nu^c_{\mu L}) +$$
$$+ i\left(\frac{\lambda}{\sqrt{2}}\right)^{-1} A\rho(\bar{\nu}^c_{eL}\nu_{eL} + \bar{\nu}^c_{\mu L}\nu_{\mu L} - \bar{\nu}_{eL}\nu^e_{eL} - \bar{\nu}_{\mu L}\nu^c_{\mu L}) \,. \tag{2.30}$$

Coming back to eq. (2.19) and remembering the restriction on the matrices given in eq. (2.28), we can identify the generalized Cabibbo matrix W^T from eq. (2.22). In this way, we can express the phenomenological neutrinos (unprimed fields) in terms of the mass eigenstates (primed) as follows,

$$\begin{pmatrix} X_e \\ X_\mu \\ \omega_e \\ \omega_\mu \end{pmatrix} = \begin{pmatrix} cc_1 & sc_2 & cs_1 & ss_2 \\ -sc_1 & cc_2 & -ss_1 & cs_2 \\ -s_1 & 0 & c_1 & 0 \\ 0 & -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} X'_e \\ X'_\mu \\ \omega'_e \\ \omega'_\mu \end{pmatrix} \qquad (2.31)$$

Using A and B defined in (2.24), the mass eigenvalues are given by

$$m_i = \frac{B+A}{2} - \frac{1}{2}\left[D_i^2 + (B-A)^2\right]^{1/2}$$
$$m'_i = \frac{B+A}{2} + \frac{1}{2}\left[D_i^2 + (B-A)^2\right]^{1/2} \qquad (2.32)$$

and $\tan 2\theta_i = - D_i/(A-B)$ with $i = 1,2$. The angles θ_i could be called Majorana mixing angles since they are characteristic of the splitting of a Dirac neutrino into two Majorana ones [7].

3. NEUTRINO OSCILLATIONS

We now explore the consequences of the mixing matrix (2.31) for neutrino oscillations. For two generations the phenomenological neutrinos (the weak eigenstates) will evolve in time as

$$|\nu_{jL}(t)\rangle = \sum_{k=1}^{4} W^T_{jk}\, e^{-iE_k t} \left\{ \sum_{p=1}^{2} W_{kp}\, |\nu_{pL}\rangle + \sum_{q=3}^{4} W_{kq} |\nu^c_{qL}\rangle \right\} \qquad (3.1)$$

where W is taken from eq. (2.31) and E_k is the energy of the k-th mass eigenstate ($k = 1,2,3,4$ for 2 generations)[11].

We show the expressions for some of the possible transitions that will occur in our model:

$$P_{\nu_{eL}\to\nu_{eL}}(t) = c^4\left[c_1^4+s_1^4+2c_1^2s_1^2\cos(E_1-E_3)t\right] + s^4\left[c_2^4+s_2^4+2c_2^2s_2^2\cos(E_2-E_4)t\right] +$$
$$+ c^2s^2\left[2c_1^2c_2^2\cos(E_1-E_2)t + 2c_2^2s_1^2\cos(E_2-E_3)\ t\ +\right.$$
$$\left. + 2c_1^2s_2^2\cos(E_1-E_4)t + 2s_1^2s_2^2\cos(E_3-E_4)t\right] \qquad (3.2a)$$

$$P_{\nu_{eL}\to\nu_{\mu L}}(t) = c^2s^2\left[c_1^2+s_1^4+2c_1^2s_1^2\cos(E_1-E_3)t + c_2^4 + s_2^4 + 2c_2^2s_2^2\cos(E_2-E_4)t\ -\right.$$
$$- 2c_1^2c_2^2\cos(E_1-E_2)t - 2c_2^2s_1^2\cos(E_2-E_3)t - 2c_1^2s_2^2\cos(E_1-E_4)t\ -$$
$$\left. - 2s_1^2s_2^2\cos(E_3-E_4)t\right] \qquad (3.2b)$$

$$P_{\nu_{eL}\to\nu^c_{eL}}(t) = 2c^2c_1^2s_1^2(1-\cos(E_1-E_3)t) \qquad (3.2c)$$

$$P_{\nu_{eL}\to\nu^c_{\mu L}}(t) = 2s^2c_2^2s_2^2(1-\cos(E_2-E_4)t) \qquad (3.2d)$$

There are also the analogous expressions for the oscillations $\nu_{\mu L} \to \nu^c_{\mu L}$ and $\nu_{\mu L} \to \nu^c_{eL}$. Notice that even in the absence of mixing between the two generations ($\theta=0$), there will occur a depletion of flux intensity for ν_e beams due to the oscillation within the same generation ($\nu_{eL} \to \nu^c_{eL}$), in which case equations (3.2) become,

$$P_{\nu_{eL}\to\nu_{eL}}(t,\theta=0) = 1 - \frac{1}{2}\sin^2 2\theta_1\left[1-\cos(E_1-E_3)t\right] \qquad (3.3a)$$

$$P_{\nu_{eL}\to\nu^c_{eL}}(t,\theta=0) = \frac{1}{2}\sin^2 2\theta_1\left[1-\cos(E_1-E_3)t\right] \qquad (3.3b)$$

We consider now some particular values for the Cabibbo and Majorana angles and present the probabilities of oscillation in terms of R, the distance from source to detector and L_{ij}, the oscillation length [2] expressed in meters as:

$$L_{ij} = \frac{2.53\ p/\mathrm{MeV}}{\delta m^2_{ij}/(\mathrm{eV})^2}\ \mathrm{m} \tag{3.4}$$

where p is the neutrino momentum and $\delta m^2_{ij} = |m_i^2 - m_j^2|$ with m_i the neutrino masses given in Eq. (2.32). A first possibility is to choose for θ the same value as the Cabibbo angle for the quarks, as is suggested in some models of grand-unification [27] as well as by experimental analysis of reactor data [28]. Other phenomenologically suggested value for θ is $\theta=50^0$ [29]. For the Majorana mixing angle, for each generation we use a value $\theta_i \approx 10^{-2}$ rad., following the analysis given in [11]. This value for θ_i leads to a branching ratio B.R. $(\mu \to e\gamma) \sim 10^{-13}$ which is below, but not so far from the present experimental upper limit, 1.7×10^{-10} [30].

For the first set of angles, $\theta=\theta_c=13^0$ and $\theta_i=10^{-2}$ rad.,

$$P_{\nu_{eL}\to\nu_{eL}}(R) = 0.90 + 0.10 \cos\frac{2\pi R}{L_{12}} \tag{3.5a}$$

$$P_{\nu_{eL}\to\nu_{\mu L}}(R) = 0.10\ (1 - \cos\frac{2\pi R}{L_{12}}) \tag{3.5b}$$

while for $\theta=50^0$ and $\theta_i=10^{-2}$ rad. we obtain,

$$P_{\nu_{eL}\to\nu_{eL}}(R) = 0.52 + 0.48 \cos\frac{2\pi R}{L_{12}} \tag{3.6a}$$

$$P_{\nu_{eL}\to\nu_{\mu L}}(R) = 0.48\ (1 - \cos\frac{2\pi R}{L_{12}}) \tag{3.6b}$$

We have only shown the dominant oscillations. It is important to notice that the oscillations $\nu_{eL} \to \nu^c_{eL}$ and $\nu_{\mu L} \to \nu^c_{\mu L}$ which could in

principle lead to an apparent non-conservation of probability are, for the above values of θ and θ_i, completely negligible. For typical reactor energies of 5 MeV and R of about 10 m, the maximal effect would occur for $\delta m^2_{12} = 0.32\ (\mathrm{eV})^2$, which is not an unreasonable value for the mass difference to which reactor experiments may be sensitive [28].

The value used above for the Majorana mixing angles imply an hierarchy for the mass parameters A, B and D_i which appear in (2.32). A, B and D_i are of order 10^{-1}, 10^3 and 10 GeV, respectively, as pointed out by Cheng and Li [11]. We call attention to the fact that within this scheme it is possible to obtain for δm^2_{ij} values of the order of a few tenth of $(\mathrm{eV})^2$, keeping the Majorana angles for each generation equal up to 10^{-20}. With this choice of parameters, m'_i, the other mass eigenvalues in eq. (2.32), are much larger than m_i, being of the same order as B.

4. THE ELECTRIC DIPOLE MOMENT OF THE CHARGED LEPTONS

The most immediate consequence of CP violation in the leptonic sector is a non-vanishing electric dipole moment for the charged leptons. As a matter of fact the experimental upper limit on the EDM of the electron is almost as good as for the neutron [31],

$$D_e < 2 \times 10^{-24}\ \mathrm{ecm} \tag{4.1}$$

while for the muon the experimental limit is [32],

$$D_\mu < 1 \times 10^{-18}\ \mathrm{ecm}\ . \tag{4.2}$$

In our model, the EDM for the charged leptons is given by the diagrams in Figs. 1a and b, which can be calculated using the couplings given in Eqs. (2.29) and (2.30). The neutral Higgs contribution from Fig. 1a gives the following result,

$$D_e(\text{neutral Higgs}) = \frac{\langle H_1X_1\rangle}{|\lambda_1|^2} m_e(m_e - D_1\cos\theta)^2 F(m_e) + \frac{\langle H_1X_1\rangle}{|\lambda_1|^2} m_\mu D_1D_2\sin^2\theta\, F(m_\mu) -$$

$$- \frac{\langle H_2X_2\rangle}{|\lambda_2|^2} m_e D_1^2\cos\theta\, F(m_e) + \frac{\langle H_2X_2\rangle}{|\lambda_2|^2} m_\mu D_1D_2\sin^2\theta\, F(m_\mu) \qquad (4.3a)$$

$$D_\mu(\text{neutral Higgs}) = \frac{\langle H_1X_1\rangle}{|\lambda_1|^2} m_\mu(m_\mu - D_2\cos\theta)^2 F(m_\mu) - \frac{\langle H_1X_1\rangle}{|\lambda_1|^2} m_e D_1D_2\sin^2\theta\, F(m_e) -$$

$$- \frac{\langle H_2X_2\rangle}{|\lambda_2|^2} m_\mu D_2^2\cos\theta\, F(m_\mu) - \frac{\langle H_2X_2\rangle}{|\lambda_2|^2} m_e D_1D_2\sin^2\theta\, F(m_e) \qquad (4.3b)$$

In the above equations $F(x)$ is given by,

$$F(x) = \frac{1}{8\pi^2} \ell n\, m_H^2/x^2 \qquad (4.4)$$

where m_H is a typical mass for a neutral Higgs boson. Notice that we have not diagonalized the mass matrix for the neutral Higgs scalars and have used instead the transition propagator $\langle H_iX_i\rangle$ as mentioned at the end of section 2.1.

For the charged Higgs contribution to the EDM (Fig. 1b), we have to be careful since the intermediate fermions lines are of two types, X_i and ω_i, associated with mass eigenvalues m_i and m_i', respectively. There are four contributions to this diagram. For the diagrams involving a X-line, we use the approximation $m_H^2/m_i^2 >> 1$, while for those involving an ω-state, we use $\frac{m_i'^2}{m_{e,\mu}^2} >> \frac{m_i'^2}{m_H^2} >> 1$.

This is in the spirit of our choice of θ_i, made in the last section, where an hierarchy $B >> D_i >> A$ was used. With these approximations we obtain for the charged Higgs contributions,

$$D_e(\text{charged Higgs}) = -\,\mathrm{Im}\,A \cdot \Big[(m_e\cos\theta - D_1)D_1\cos^2\theta\,\cos\theta_1\,\sin\theta_1 +$$

$$+ m_e D_2\sin^2\theta\,\cos\theta_1\,\sin\theta_1\Big] \cdot (m_1 G(m_1) - m_1' G'(m_1')) +$$

$$+ \mathrm{Im}\,A \cdot \Big[(m_e\cos\theta - D_1)D_2\sin^2\theta\,\cos\theta_2\,\sin\theta_2 +$$

$$+ m_e D_2\sin^2\theta\,\cos\theta\,\cos\theta_2\,\sin\theta_2\Big] \cdot (m_2 G(m_2) - m_2' G'(m_2')) \qquad (4.5a)$$

$$D_\mu(\text{charged Higgs}) = \mathrm{Im}\,A \cdot \Big[(m_\mu\cos\theta - D_2)D_1\sin^2\theta\,\sin\theta_1\,\cos\theta_1 +$$

$$+ m_\mu D_1\sin^2\theta\,\cos\theta \times \cos\theta_1\,\sin\theta_1\Big] \cdot (m_1 G(m_1) - m_1' G'(m_1')) +$$

$$+ \mathrm{Im}\,A \cdot \Big[(m_\mu\cos\theta - D_2)D_2\cos^2\theta\,\cos\theta_2\,\sin\theta_2 -$$

$$- m_\mu D_2\cos\theta\,\sin^2\theta\,\cos\theta_2\,\sin\theta_2\Big] \cdot (m_2 G(m_2) - m_2' G'(m_2')) \qquad (4.5b)$$

With the approximations mentioned before, $G(m_i)$ and $G'(m_i')$ are given by

$$G(m_i) = -\frac{1}{32\pi^2 m_H^2}$$

and (4.6)

$$G'(m_i') = \frac{1}{8\pi^2 m_i'^2}\left[\frac{5}{4} - 2\ell n\frac{m_i'^2}{m_H^2}\right]$$

In the above expressions $\mathrm{Im}\,A$ means $\mathrm{Im}\,\frac{\langle\phi_1^-\phi_2^+\rangle}{\lambda_1^*\lambda_2}$, as in eq. (2.11), but with the restriction that the momentum dependence of the transition propagator has already been taken into account in the loop integration, therefore Im A has dimensions of inverse mass squared and in it the only dimensional quantity is $(\lambda_1^*\lambda_2)^{-1}$. However, it is important to remember, that Im A contains the mixing parameters from the transition propagator. We have not displayed the contributions from the mixing between $\phi_{1,2}$ and η since they are less important being supressed by the smallness of the Majorana mixing angles chosen at the end of section 3.

Now, for the numerical estimates of (4.3) and (4.5). For the charged Higgs contribution, the dominant term, will be the same for the muon and the electron,

$$D_{e,\mu}(\text{charged Higgs}) = \left[\text{Im } A/(\text{GeV}^{-2})\right] \times 4 \times 10^{-18} \text{ ecm} \quad . \tag{4.7}$$

The experimental upper limit on the EDM of the electron [31] implies,

$$\text{Im } A < 2.5 \times 10^{-7} \text{ GeV}^{-2} \tag{4.8}$$

We should not be worried about the smallness of this value for Im A since it does not necessarilly imply a large mass scale in the Higgs sector (remember that we have been assuming a Higgs mass around 10 GeV). The reason for this is that, as remarked before, Im A contains mixing angles for the charged Higgs and these can provide additional suppresion factors. Furthermore if $|\lambda_i|$ takes the maximum value allowed by their contribution to the masses of the W and Z bosons, then $|\lambda_i| \sim 600$ GeV and we only need a suppresion factor from the mixing angles of the order of 10^{-1} , which is quite reasonable a value.

The neutral Higgs contribution to the EDM is of the same order of magnitude as the charged one, provided $|\lambda_i|$ has just

the magnitude mentioned above, $|\lambda_i| \sim 600$ GeV , in which case our scheme is consistent with a Higgs mass around 10 GeV.

5. CONCLUSIONS

We have investigated in this paper an SU(2)xU(1) model for the leptonic sector, displaying both massive neutrinos and CP violation. We have explored the consequences for neutrino oscillations and the electric dipole moment of the charged leptons. It has been shown that for a reasonable range of the values of the Yukawa couplings and parameters of the Higgs potential, the model could have testable consequences for neutrino oscillations and for the EDM. One of the characteristics of our model is that the electron and muon turn out to have comparable electric dipole moments. In particular, we think it is worth trying to improve the experimental upper limit on the lepton EDM, since with this kind of model, it seems natural to obtain for the EDM, values around 10^{-25} ecm. For the neutrinos, since the mass eigenstates are Majorana particles, they do not have static electromagnetic form factors, however they do have electromagnetic transition moments. As our model exhibits CP violation, the magnetic and electric transition dipole moments will be of the same order of magnitude and this could be a distinguishing feature of this type of model. This last point will be explored elsewhere.

APPENDIX

We summarize here the consequences, for the charged and neutral currents, of having a $(2n \times 2n)$ mass matrix, where n is the number of generations.

We first deal with the charged current, which is of the form,

$$\bar{\ell}_L \gamma_\mu \nu_L \tag{A.1}$$

where ℓ_L is an array matrix containing the n-charged left-handed lepton fields and ν_L the n-phenomenological left-handed neutrinos.

After rotating to the physical basis, (A.1) becomes,

$$\bar{\ell}_L \gamma_\mu U \psi_L \tag{A.2}$$

Before examining the meaning of (A.2) for the case of Dirac and Majorana mass terms, we briefly recall the pure cases: either Dirac or Majorana masses. In both cases the neutrino fields which appear in the charged (neutral) current are those in the mass term, i.e., ψ_L denotes n-mass eigenstates and U is the $(n \times n)$ mixing matrix, as usual. The basic difference between Dirac and Majorana masses is now the number of phases in U, since in the Dirac case there are $\frac{1}{2}(n-1)(n-2)$ phases as is well known, while in the Majorana case there are $\frac{1}{2}n(n-1)$ such phases [13]. However, in most models it is not possible to observe this difference in the number of phases in phenomena like oscillations. Nevertheless, in certain types of models à la Konopinski-Mahmoud-Zel'dovich [33], or in phenomena other than oscillations, it is possible, in principle, to observe this difference [34].

We now return to the general case in which we have

both Majorana and Dirac mass terms. There are now $\frac{1}{2}(3n^2-n)$ phases and $\frac{1}{2}(3n^2-n)$ mixing angles [35]. The matrix U in (A.2) is rectangular, $(n \times 2n)$. In our model, however, the matrix $U = W^{-1}$, has a particular form, according to eq. (2.22). From the beginning we consider a real Majorana mass matrix so that the only phase must come from the sub-matrix U_L in eq. (2.16). This is $(n \times n)$ but there are only n-charged lepton fields to absorb phases, then despite our matrix being $(2n \times 2n)$ the number of phases is only $\frac{1}{2}n(n-1)$. This explains the form of $V_L U_L^{-1}$ in eq. (2.28). For the right-mixing matrix, $V_R U_R^{-1}$, as we do not have right-handed charged currents, it is not possible to absorb the phases in the charged leptons.

For the neutral currents the situation is similar. In the physical basis, it is given by,

$$\bar{\Psi}'_L \gamma_\mu U' \psi_L \tag{A.3}$$

where U' is a $(2n \times 2n)$ matrix, $U'_{ij} = W_{ik} W^T_{kj}$, $i,j = 1,\ldots 2n$ and $k = 1,\ldots n$. U has, in general, non-vanishing off-diagonal elements, implying the existence of flavour-changing neutral currents. For instance, with (2.22) we have,

$$U' = \begin{pmatrix} c_1^2 & 0 & c_1 s_1 & 0 \\ 0 & c_2^2 & 0 & c_2 s_2 \\ c_1 s_1 & 0 & s_1^2 & 0 \\ 0 & c_2 s_2 & 0 & s_2^2 \end{pmatrix} \tag{A.4}$$

The phenomenology of such flavour-changing neutral currents has been already analysed in the literature either in the case where the rectangular character arises because of the simultaneous occurence of Dirac and Majorana mass terms [36] or because of the presence of $m(m<n)$ right-handed neutrinos [37].

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FOOTNOTES

1. Of course, we could give a Majorana mass with a singlet Higgs, but only via radiative corrections, as shown by Zee [5]. The phenomenology of this model has been explored by Wolfenstein [6].

2. Notation: $\psi^c = C\gamma^0\psi^* = C\bar{\psi}^T$, where $C = i\gamma^2\gamma^0$

$$\psi_L^c = (\psi_L)^c = (\psi^c)_R \ .$$

3. In the case of Majorana neutrinos, the CP violating phases in the Kobayashi-Maskawa matrix, already occur for two generations [13].

4. It is possible, however, to have a significant contribution to the EDM, at the two-loop level if there are flavour-changing neutral Higgs boson couplings and all CP violation coming from the charged current [18]. We do not consider this possibility in the following and assume that the charged current couplings are real.

5. It is also possible to have CP violation through the exchange of neutral Higgs bosons [20].

6. The vacuum expectation values of $\phi^0_{1,2}$ and η^0 give masses to the $W^\pm$ and Z^0 bosons. We now have,

$$\rho \equiv \left(\frac{M_W}{M_Z\cos\theta_W}\right)^2 = \frac{|\lambda_1|^2 + |\lambda_2|^2 + |\lambda|^2}{|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda|^2} \ ,$$

so that $\frac{1}{2} \lesssim \rho \lesssim 1$. To be consistent with the data on ρ , within one standard deviation, we must impose, $\frac{|\lambda|}{|\lambda_1| + |\lambda_2|} < \frac{1}{4}$ [7].

7. ψ_i^- and ψ_j^+ may be members of the doublets or of the triplet.

8. The primed fields denote the mass eigenstates.

9. A relation like this, between Majorana and Dirac mass matrices, appear in some models of grand-unification [27].

10. This is also the reason why we have not coupled the ν_R and the Φ_1-doublet.

11. In our notation, k = 1,3 are related to the electron neutrino and k = 2,4 to the muon neutrino.

FIGURE CAPTION

FIG. 1 - One loop Higgs exchange diagrams contributing to the EDM of a charged lepton: a) neutral Higgs, b) charged Higgs.

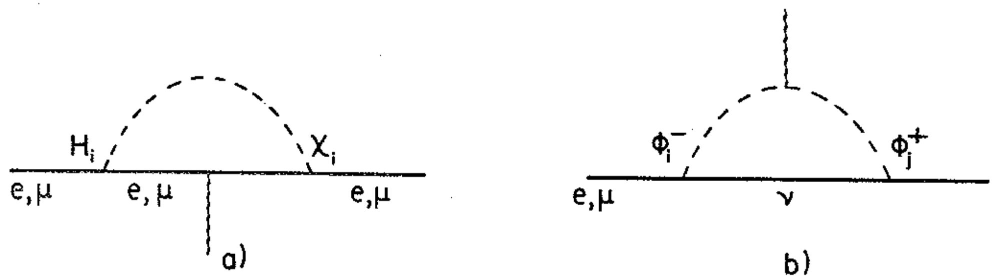

FIG. 1