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COUNTING THE PEAKS IN THE EXCITATION FUNCTION FOR PRECOMPOUND PROCESSES[†]

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ABSTRACT

The "counting of maxima" method of Brink and Stephen, conventionally used for the extraction of the correlation width of statistical (compound nucleus) reactions, is generalized to include precompound processes as well. It is found that this method supplies an important independent check of the results obtained from auto-correlation studies. An application is made to the reaction $^{25}Mg(^{3}He,p)$.

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At low incident energies, the discrete part of the spectrum of emitted particles in a light-ion induced nuclear reaction is usually accounted for by the compound-nucleus picture. As the energy is increased, the probability of particle emission while the composite system is in its way towards the formation of the compound nucleus, becomes important. Consequently, the strength of a given transition is interpreted as arising from compound as well as "precompound" (multistep compound in the language of Ref. 1) processes. At still higher energies, the direct contribution becomes dominant.

Recently, several measurements of excitation functions for discrete transitions in the reactions[2] $^{27}Al(^{3}He,p)$, $^{25}Mg(^{3}He,p)$ and $^{25}Mg(^{3}He,\alpha)$ have been performed and subsequently analyzed and interpreted in terms of the precompound model referred to above. The main argument behind this interpretation has been the well evidenced symmetry of the angular distribution about 90° (which excludes direct contributions), as well as the discovery of more than one "correlation width" in the analysis of the cross-section auto-correlation function. It would be greatly advantageous if one could subject the above conclusions to further independent checks.

Within the conventional theory of Ericson fluctuations[3], based on the compound nucleus model, Brink and Stephen[4] (BS) have shown that the correlation width can be very simply related to the average number of maxima exhibited by the statistically fluctuating excitation function in a certain energy interval.

It is the purpose of this letter to generalize the BS result to include the possibility of precompound processes, thus providing an independent check of the results based on the auto-correlation function, as mentioned above.

We first summarize the method used in Ref. 4. It is first recognized that the average number of maxima, K, per unit energy range can be written as[5]

$$K = -\int_{-\infty}^{0} \sigma'' \, p(0,\sigma'') \, d\sigma'' \,, \qquad (1)$$

where $p(\sigma',\sigma'')$ is the joint probability distribution of the first and second derivatives of the cross section σ with respect to the energy. We now specifically consider the case in which only one channel is open : we shall denote the corresponding average number of maxima by K_1. The characteristic function (i.e. the Fourier transform) $\varphi(s,t)$ of $p(\sigma',\sigma'')$ is then found from the key assumption that the real and imaginary parts, x,y, of the a b element of the S-matrix, S_{ab}, are independent Gaussian variables with zero mean (in the absence of direct reactions, which is assumed throughout) and with the same variance. This result, which was assumed by Ericson and Brink and Stephen, has, later on, been fully proven by Agassi et. al.[6].

The result for $\varphi(s,t)$ is

$$\varphi(s,t) = \left[1 + \alpha^2 s^2 + \beta^2 t^2 + i\gamma^3 t^3\right]^{-1}, \qquad (2)$$

where α,β and γ can be expressed in terms of the auto-correlation function $C_x(\varepsilon)$ of the quantity x

$$C_x(\varepsilon) \equiv \langle x(E)\, x(E+\varepsilon)\rangle \qquad (3)$$

and its derivatives, as:

$$\alpha^2 = -4\, C_x(0)\, C_x''(0) \qquad (4a)$$

$$\beta^2 = 4\left[C_x(0)\, C_x^{IV}(0) + 3\left(C_x''(0)\right)^2\right] \qquad (4b)$$

$$\gamma^3 = 16\, C_x''(0)\left[C_x(0)\, C_x^{IV}(0) - \left(C_x''(0)\right)^2\right]. \qquad (4c)$$

The quantity $C_x(\varepsilon)$ can be expressed as $\frac{1}{2}\mathrm{Re}\, C^{(S)}(\varepsilon)$, where $C^{(S)}(\varepsilon)$ is the more familiar S-matrix auto-correlation function,

$$C^{(S)}(\varepsilon) = \langle S^*(E)\, S(E+\varepsilon)\rangle . \qquad (5)$$

The quantity K_1 is then found from Eq. (1) to be

$$K_1 = \frac{\beta}{2\pi\alpha}\, I_1(\gamma^3/\beta^3) , \qquad (6a)$$

where

$$I_1(a) \equiv \frac{1}{2}\int_{-\infty}^{\infty} \frac{1 + \frac{3}{2} i a z}{(1 + z^2 + i a z^3)^{3/2}}\, dz , \qquad (6b)$$

which can easily be shown to be real.

For $C^{(S)}(\varepsilon)$, Brink and Stephen[4] adopted Ericson's model[3]

$$C^{(S)}(\varepsilon) = \frac{\Gamma}{\Gamma - i\varepsilon}\, \bar{\sigma}, \qquad (7)$$

which is based on the assumption of a narrow distribution of the resonance widths. This restriction has been removed in Ref. 6.

Inserting Eq. (7) into Eq. (4) one finds

$$\alpha = \frac{\sqrt{2}}{\Gamma}\bar{\sigma} \quad , \beta = \frac{6}{\Gamma^2}\bar{\sigma} \quad , \gamma = -\frac{2\times 10^{1/3}}{\Gamma^2}\bar{\sigma}, \qquad (8)$$

from which we obtain (see Eq. (6))

$$K_1 = \frac{0.65}{\Gamma}\, I_1(-0.37). \qquad (9)$$

The average number of maxima in an energy interval ΔE is then

$$\bar{n} = K_1\, \Delta E \qquad (10a)$$

The width of the distribution of the statistical variable n is estimated to be

$$r.m.s.\ n = \sqrt{\mathrm{var}\ n} \approx \sqrt{\bar{n}} . \qquad (10b)$$

The integral I_1 of Eq.(6) was estimated as =1 in Ref. 4, since $I_1(0)=1$; we, however, calculated it numerically, and the results are presented in Table I : for the particular case $a = -0.37$, $I_1 = 1.04$. We note in passing that our results, summarized

in our Eq. 8, differ from those found in Ref. 4, where one finds, instead

$$\alpha = \frac{1}{\Gamma}\,\bar{\sigma} \quad , \quad \beta = \frac{2\sqrt{3}}{\Gamma^2}\,\bar{\sigma} \quad , \quad \gamma = \frac{2}{\Gamma^2}\,\bar{\sigma} \quad , \tag{11}$$

leading, in Eq. (9), to the factor 0.55 instead of our 0.65. The different sign of γ is immaterial, since $I_1(a) = I_1(-a)$.

The above analysis can be extended to the case in which these are N (>1) independent channels. The result for K_N can be expressed just as in Eq. (9), with the integral I_1 replaced by I_N, which in turn can be shown numerically to have the property $1 < I_N < I_1$, making I_N even closer to unity.

We now examine how the above method can be generalized to include precompound processes. Eq. (1) is certainly independent of the model. The key assumption that x,y are independent Gaussian variables with zero mean and equal variance, was shown to apply, even in this case, in Ref. 6. An alternative way to see this is within the "nested-doorway model" of Ref. 8, where S is written as a sum of contributions of the various doorway classes n

$$S_{ab} = \sum_n S^{ff}_{n,ab} \quad , \qquad (\overline{S}_{ab} = 0 \; , \; a \neq b). \tag{12}$$

If each $S^{ff}_{n,ab}$ is assumed to be distributed just as in a one-class problem, and the various classes are assumed independent of one another, the resulting S_{ab} will have the characteristics mentioned in the above paragraph. As a result, Eqs. (2) to (6) will still hold. The $C^{(s)}(\varepsilon)$ to be used in these equations is, however, different from that given in Eq. (7). We take here the result provided by the "nested-doorway model"[8] where $C^{(s)}(\varepsilon)$ appears as a sum over the various doorway classes, i.e.

$$C^{(s)}(\varepsilon) = \sum_n \frac{\Gamma_n}{\Gamma_n - i\varepsilon}\,\overline{\sigma_n} \; , \tag{13}$$

from which one then finds

$$\alpha^2 = 2\bar{\sigma}\sum_n \frac{\overline{\sigma_n}}{\Gamma_n^2} \quad , \tag{14a}$$

$$\beta^2 = 4\left[6\bar{\sigma}\sum_n \frac{\sigma_n}{\Gamma_n^4} + 3\left(\sum_n \frac{\sigma_n}{\Gamma_n^2}\right)^2\right] \tag{14b}$$

$$\gamma^3 = -16\left(\sum_{n'} \frac{\sigma_{n'}}{\Gamma_{n'}^2}\right)\left[6\bar{\sigma}\sum_m \frac{\sigma_m}{\Gamma_m^4} - \left(\sum_n \frac{\sigma_n}{\Gamma_n^2}\right)^2\right]. \tag{14c}$$

We remark here that Eq. (13) for the multiclass $C^{(s)}(\varepsilon)$ may also be obtained without the use of the nested-doorway condition, $\Gamma_1 >> \Gamma_2 >> \ldots$. This more general consideration of $C^{(s)}(\varepsilon)$ has been discussed in Ref. 9, where it was shown that the AWM result[6),10)] for the multi-class $C^{(s)}(\varepsilon)$ may be rewritten in exactly the same form as Eq. (13).

We give below the explicit result for the average number of maxima, $\bar{n}$, that would appear in an energy interval ΔE, in the special case of two classes of overlapping resonances

$$\bar{n} = \frac{\beta}{2\pi\alpha}\, I_N(\gamma^3/\beta^3)\,\Delta E$$

$$\cong \frac{\sqrt{6}}{2\pi}\,\frac{\Delta E}{\sqrt{\Gamma_1'\,\Gamma_2'}}\left[2\,\frac{X_1\left(\frac{\Gamma_2'}{\Gamma_1'}\right)^2 + X_2\left(\frac{\Gamma_1'}{\Gamma_2'}\right)^2}{X_1\left(\frac{\Gamma_2'}{\Gamma_1'}\right) + X_2\left(\frac{\Gamma_1'}{\Gamma_2'}\right)} + X_1\left(\frac{\Gamma_1'}{\Gamma_1'}\right) + X_2\left(\frac{\Gamma_1'}{\Gamma_2'}\right)\right]^{1/2}, \tag{15}$$

where X_n is defined as $X_n \equiv \frac{\bar{\sigma}_n}{\bar{\sigma}}$, with $\bar{\sigma} = \Sigma_n \bar{\sigma}_n$, and I_N was set equal to unity.

In contrast to Eq. (10a), Eq. (15) does not supply a direct mean of extracting the Γ_n's and the σ_n's ; it does provide, though, a check of the values of these parameters extracted from other sources, e.g., cross-section auto-correlation analysis.

In order to apply the above discussion to analyze experimental excitation functions, several corrections have to be introduced, as has been discussed extensively in the literature[11)]

in connection with the one-class problem. The first one deals with the finite size of the energy step ε_0 used in the construction of the experimental excitation function. We now have a maximim at a certain energy E, when the cross section at that energy is larger than the ones at the two neighbouring points $E+\varepsilon_0$ and $E-\varepsilon_0$; K of Eq.(1) has to be redefined accordingly, and for a large number of open channels Ref. 11b gives for $\bar{n}$, the following

$$\bar{n} = \frac{\Delta E}{\pi \varepsilon_0} \tan^{-1}\left(\sqrt{4\,\frac{C(0) - C(\varepsilon_0)}{C(0) - C(2\varepsilon_0)} - 1}\right) \qquad (16)$$

where $C(\varepsilon_0)$ is the cross-section autocorrelation function, related to $C^{(s)}(\varepsilon_0)$ of Eq.(5) by

$$C(\varepsilon_0) = \left| C^{(s)}(\varepsilon_0) \right|^2 \qquad (17)$$

in the absence of direct reactions. Again just as in the above generalization of the BS analysis, Eq.(16) is also valid in the multiclass case, provided we use the appropriate form for $C^{(s)}(\varepsilon)$ (see Eq.(13)). Relation (16) clearly shows the dependence of the average number of maxima on the value ε_0 of the energy step, and, as $\varepsilon_0 \to 0$, one recovers the BS result.

In the one-class case it is customary to write Eq.(16) as the BS result of Eq.(10a) times a correction factor $b(K)$ which depends on the ratio $K \equiv \varepsilon_0 / \Gamma$.

The second correction, associated with the possibility of counting false maxima in the experimental excitation function due to the finite error bar γ, is also accounted for by means of an extra factor $b'(K,\gamma)/b(K)$ [11b]. This analysis can again be generalized to the multiclass case.

The third correction is associated with the finite energy resolution of the incident beam, which is also considered in Ref. 11c.

We have applied our formula for $\bar{n}$, Eq.(15), to the reaction $^{25}Mg(^3He,p)$ mentioned earlier[2]. The correction factor $b(K)$ was calculated exactly with the help of Eq.(16), and $\frac{b'(K,\gamma)}{b(K)}$, which involves the numerical evaluation of a complicated integral, was estimated with the help of the tabulated one-class results of Ref.11b. This, however, requires a knowledge of the value of an "effective" K_{eff} for the multiclass case. The value of $b(K_{eff})$ was calculated as the ratio of $\bar{n}$ of Eq.(16) to that of Eq.(15), and the extracted value of K_{eff} was then introduced into the one-class curves for $\frac{b'(K,\gamma)}{b(K)}$ of Ref.11b. Finally the third correction, was found to be quite small[11a].

The results for $\bar{n}$ are shown in Table II, together with the number of maxima outside of the error bar extracted from the experimental excitation function of Ref.2b. We observe that the experimental results lie between the two extremes associated with just one value of Γ (see caption of Table II), although they are closer to the value of $\bar{n}$ associated with the smaller correlation width (50 kev). A similar behaviour is found for the theoretical $\bar{n}$.

The fact that the theoretical result, Eq.(15), lies between the two extremes just mentioned and is not, for instance, simply the algebraic sum of maxima associated with each class individually, may be seen qualitatively by analyzing the superposition of two sinusoidal functions of different periods.

In conclusion, we have generalized the one-class BS result to precompound reactions. In contrast to the one-class case, Eq.(15) does not supply a direct mean of extracting the Γ_n's and the σ_n's : It does provide, though, a check on the values of the parameters extracted from other sources, e.g., cross-section auto-correlation analysis.

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a	I_1
0.0	1.000
0.1	1.00726
0.2	1.0199
0.3	1.0344
0.4	1.0494
0.5	1.0646
0.6	1.0795
0.7	1.09425
0.8	1.1086
0.9	1.1226
1.0	1.1363

Table I: The function $I_1(a)$ of Eq. (6b) tabulated for several values of $a \equiv \gamma^3/\beta^3$

Transition	n_{exp}	$(\bar{n} \pm \delta n)_{th}$
p_o	15	19 ± 4
p_1	16	21 ± 5
p_2	22	22 ± 5
p_3	21	20 ± 4
p_4	21	20 ± 4

Table II: Number of maxima extracted from the excitation functions of $^{25}Mg(^3He,p)$ of Ref. 2b, in the energy range $E_{^3He} = 8$ to 16 MeV (Column 2), compared with the theoretical predictions (Column 3). The values of $\Gamma_1, \Gamma_2, \sigma_1, \sigma_2$ used in Eq.(15) are those indicated in Ref. 2. If only the larger Γ (200 kev) would be present, $\bar{n}$ would be 6 ; on the other hand, for Γ = 50 keV one would obtain $\bar{n} = 25$.