UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
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NON LINEAR σ MODEL, SUPERGRAVITY AND THE SPINNING STRING

by

E. Abdalla and R.S. Jasinschi
Instituto de Física, Universidade de São Paulo

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ABSTRACT

We discuss the locally supersymmetric O(N) non linear sigma model and its conexion to the spinning string.

I. INTRODUCTION

Local supersymmetry emmerged first in the context of the Neveu-Schwarz-Ramond (NSR) dual string model[1]. There, in addition to the reparametrization invariance with respect to the variables (σ,τ) which describe the string world sheet[2], the theory presented a symmetry between boson and fermion fields[3]. From a field theoretical point of view, Deser and Zumino[4] (DZ) showed that the NSR spinning string can be described by a 1+1 dimensional action, given by a free boson and fermion fields, which is invariant under local supersymmetry (including Lorentz and general coordinate transformation symmetries). This introduced in the theory a Graviton and a Gravitino fields, which due to Weyl symmetry, could be gauged away.

The existing field theory string models do not account for the string stability in a dynamical way. In the present work, we generalize the DZ lagrangian allowing the original fields to interact. Starting with the O(N) non linear sigma model with global supersymmetry[5] (where the boson and fermion fields have each N components, plus a geometric constraint on the scalar

superfield), upon imposing local supersymmetry, we are led to an invariant lagrangian. Taking advantage of the 1/N expansion[6] we obtain an effective action where the original boson and fermion fields aquire a mass dynamically[6],[7]. This fact breaks the Weyl symmetry so that the Gravitino and Graviton fields cannot be gauged away. They present, in calculating their propagators to lowest order, long range forces due to poles in the infrared region, that can be interpreted as Goldstone boson and Goldstino. It is worth mentioning that in 1+1 dimensions the Gravitino[4] and Graviton[8] fields do not present a free term, so that all their non trivial n-point functions arise from matter fields quantum fluctuations. On the other hand, taking into account the finiteness of the string, we restrain the domain of integration[9] of the longitudinal variable (x_1 or σ) so that momentum is quantized, generating mass for the Gravitino and Graviton fields. This mass is exponencially small with respect to the string lengh. In this way we obtain a natural explanation of the string stability.

Classically, it is possible to define a non-local conserved charge, as in general non-linear sigma models[10]. If this charge survives quantization, it should imply a factorizable S-matrix[11]. Because of the confining properties of the model, we guess that it shares some properties with the CP^{N-1} model, which presents an anomaly destroying non-local charge conservation[12].

Finally, it is also worth studying two dimensional world to mimic the reality. Non trivial behavior of four dimensional field theories, can be obtained in a very simple way using two dimensional toy models, such as confinement and θ-vacua[13]. Also non perturbative informations can be gathered, with techniques such as 1/N expansions[6] or instanton gas calculations[14], which are known to exist in four dimensions but from a technical point of view are not manageable. In two dimensional supergravity, we are particularly interested in the cancellation of infinite quantities[25] due to fermion (against) boson contributions.

In section II we perform the functional integration on the bosonic and fermionic fields obtaining an effective action, whereupon, in sec. III, through $1/N$ expansion, the quadratic Green functions of the Graviton and Gravitino are calculated. A discussion of the renormalization of higher order terms is given in sec. IV and in sec. V we discuss the finite version of the string. Sec. VI contains the physical interpretation and in sec. VII we draw conclusions.

II. THE EFFECTIVE ACTION AND THE 1/N EXPANSION

Imposing local supersymmetry on the globally supersymmetric (O(N) invariant) non-linear sigma model[5] it can be shown that the following Lagrangian is symmetric(*)(+)

$$L = \frac{N}{2f}\sqrt{-g}\,\Big[\frac{1}{2}g^{\mu\nu}\,\partial_\mu\eta_i\,\partial_\nu\eta_i + \frac{1}{2}i\bar{\psi}_i\,\not\partial\,\psi_i + \frac{1}{8}(\bar{\psi}_i\psi_i)^2 + \bar{\psi}_i\,\gamma^\mu\gamma^\nu\,\partial_\nu\eta_i\,G_\mu - \frac{1}{4}(\bar{\psi}_i\psi_i)\bar{G}_\mu\,\gamma^\nu\gamma^\mu\,G_\nu\Big] \qquad (II.1)$$

with additional constraints (whenever two indices repeat, sum is implied):

$$\eta_i\eta_i = 1 \quad ; \quad \bar{\psi}^i\eta_i = 0 \quad . \qquad (II.1a,b)$$

The action is:

$$S = \int d^2x\, L \qquad (II.2)$$

In the string model case, to be discussed in section V the action is replaced by[9]

(*) In ref. (16) a detailed proof of this fact is given.
(+) In order to take into account the dimension D of the space-time in which the string is immersed, we should, in all formulas replace N by $N(D-2)$.

$$S_L = \int_{-\infty}^{\infty} dx_0 \int_{-L/2}^{L/2} dx_1\; L \quad . \qquad (II.3)$$

The local supersymmetry transformations are given by

$$\delta\eta_i = \bar{\varepsilon}\,\psi_i \qquad (II.4a)$$

$$\delta\psi_i = -\,i\,\partial_\mu\eta_i\,\gamma^\mu\,\varepsilon - i(\bar{G}_\mu\,\psi_i)\gamma^\mu\,\varepsilon + \frac{i}{2}\,\eta_i(\bar{\psi}_j\,\psi_j)\varepsilon \qquad (II.4b)$$

$$\delta e^\mu_a = 2i\,\bar{G}^\mu\,\gamma_a\,\varepsilon \qquad (II.4c)$$

$$\delta G^a_\mu = -\,(D_\mu\varepsilon)^a = -\,(\partial_\mu\varepsilon^a + \frac{1}{2}\,\omega^{ab}_\mu\,\varepsilon^b) \qquad (II.4d)$$

where

i) $g^{\mu\nu} = e^\mu_a\, e^\nu_b\, \eta^{ab}$

with η^{ab} given by the flat space Minkowski metric(*)

ii) $\gamma^\mu(x) = \gamma^a\, e^\mu_a(x)$

e^μ_a is the "vierbein" (zweibein) gauge field associated to local general coordinate transformation

iii) G^a_μ is the real gravitino gauge field associated to local supersymmetry transformation

iv) ψ^a_i and η_i are N component real fermion and boson fields subject to constraints (II.1a,b)

(*) See appendix for notation.

v) $\omega_\mu^{ab} = \omega_\mu \gamma_5^{ab}$

ω_μ is the spin conexion. It does not appear in lagrangian (II.1) because in our representation it is true that

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}\gamma^\mu\gamma_5\psi = 0 \qquad \text{(II.5)}$$

We have also the following symmetry for the gravitino field:

$$G_\mu \to G_\mu + \gamma_\mu \phi \qquad \text{(II.6)}$$

and the Weyl symmetry:

$$\psi \to \Lambda^{-1/2}\psi \qquad \text{(II.7a)}$$

$$\eta \to \eta \qquad \text{(II.7b)}$$

$$G_\mu \to G_\mu \Lambda^{1/2} \qquad \text{(II.7c)}$$

$$e_\mu^a \to e_\mu^a \Lambda \qquad \text{(II.7d)}$$

The Introduction of quartic counterterms in h and G_μ fields to the lagrangian can modify the dynamical properties of the model, whenever these terms turn out to be important. These quartic interaction can through quantum fluctuations generate a mass, as in the case in Gross-Neveu[6] model. This mass, if it exists is exponentially small in the coupling of the interaction, i.e., if the induced counterterm has the form:

$$\delta L = \frac{K}{\sqrt{N}}(\bar{G}_\mu G^\mu)^2 + \frac{K}{\sqrt{N}} h^4 \qquad \text{(II.8)}$$

the dynamically generated masses have the form:

$$m \sim e^{-\frac{\sqrt{N}}{K}} . \qquad \text{(II.9)}$$

This could be important at energies comparable to the Planck mass, where the gravitational field becomes important[16].

Rescaling(*) η and ψ

$$\eta \to \eta' = \left(\frac{N}{2f}\right)^{-1/2}\eta \quad \text{and} \quad \psi \to \psi' = \left(\frac{N}{2f}\right)^{-1/2}\psi \qquad \text{(II.10)}$$

we can write the functional generator for the Green functions as:

$$Z(\bar{J},\ldots) = \int [d\psi] \ldots [de_a^\mu] \,\underset{x}{\delta}(\bar{\psi}\eta)\,\underset{x}{\delta}(\eta^2 - \tfrac{N}{2f}) \times$$
$$\times \exp i \int d^2x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta + \frac{i}{2}\bar{\psi}\not{\partial}\psi + \right.$$
$$+ \frac{f}{4N}(\bar{\psi}\psi)^2 + \bar{\psi}\gamma^\mu\gamma^\nu\partial_\nu\eta G_\mu - \frac{1}{4}(\bar{\psi}\psi)(\bar{G}_\mu\gamma^\nu\gamma^\mu G_\nu) +$$
$$\left. + \frac{\bar{J}\eta}{2} + \frac{\bar{\xi}\psi}{2} + \ldots \frac{\bar{j}^\mu}{2} G_\mu\right] \qquad \text{(II.11)}$$

The following identity holds

$$\underset{x}{\delta}(\eta^2 - \tfrac{N}{2f})\,\underset{x}{\delta}(\bar{\psi}\eta)\,\exp \frac{if}{4N}\int d^2x \sqrt{-g}\,(\bar{\psi}\psi)^2 =$$
$$= \int [dc][d\phi][d\alpha]\,\exp \int d^2x \sqrt{-g}\left[\frac{i\alpha}{\sqrt{N}}(\eta^2 - \tfrac{N}{2f}) + \right.$$
$$\left. + \frac{i}{2\sqrt{N}}(\bar{\eta}\,\bar{c}\,\psi + \bar{\psi}\,c\,\eta) - \frac{i\sqrt{f}}{N}(\bar{\psi}\psi)\phi - i\frac{\phi^2}{N}\right] \qquad \text{(II.12)}$$

(*) We will subsequently omit any index, unless necessary.

We also rescale the G_μ field

$$G_\mu \to G'_\mu = \sqrt{N}\, G_\mu$$

so that we can rewrite (II.11), using (II.12) to obtain

$$Z(\bar{J},\ldots) = \int [d\psi] \ldots [de^\mu_a] \exp \int d^2x \sqrt{-g} \left[-\frac{1}{2}\bar{\eta}\,\Delta_B \eta - \right.$$

$$- \frac{1}{2}\bar{\psi}\,\Delta_F \psi + \frac{i\bar{\psi}}{2\sqrt{N}} \left[c + \gamma^\mu \gamma^\nu G_\mu \partial_\nu \right] \eta +$$

$$\left. + \frac{i\bar{\eta}}{2\sqrt{N}} \left[-\bar{G}_\nu \gamma^\mu \gamma^\nu \partial_\mu + \bar{c} \right] \psi - \frac{i\sqrt{N}}{2f}\,\alpha - i\,\frac{\phi^2}{N} + \text{source terms} \right] \qquad \text{(II.13)}$$

where Δ_B and Δ_F are given by the following expressions (we have already included an m^2 in the boson part, which is completely irrelevant at this stage corresponding only to a reparametrization of the normalization term, and an M which appears due to symmetry breaking - see next section.

$$\Delta_B = \frac{i}{\sqrt{-g}}\,\partial_\mu (g^{\mu\nu} \sqrt{-g}\, \partial_\nu) + im^2 - \frac{2i\alpha}{\sqrt{N}} \qquad \text{(II.14a)}$$

$$\Delta_F = \not{\partial} - iM + \frac{2i\sqrt{f}}{N}\,\phi + \frac{i}{2N}\,(\bar{G}_\mu \gamma^\nu \gamma^\mu G_\nu) \qquad \text{(II.14b)}$$

After performing functional integration on the ψ and η fields, (II.13) turns out to be:

$$Z(\bar{J},\ldots) = \int [dc] \ldots [de^\mu_a] \exp \left\{ i\, S_{eff} + \right.$$

$$+ \int d^2x \sqrt{-g} \left[-\frac{1}{2}\,\bar{\xi}\Delta_F \xi - \frac{1}{2}\,(\bar{J} + \bar{\xi}\Delta_F^{-1}/\sqrt{N}\, c') \right.$$

$$\left. \left. (\Delta_B - \bar{c}'\,\frac{\Delta_F^{-1}}{N}\, c')(J + \bar{c}'\,\Delta_F^{-1}\,\xi/\sqrt{N} \right] + \frac{1}{2}\,\xi^\mu e^a_\mu + \frac{1}{2}\,\bar{j}_\mu G^\mu \right\} \qquad \text{(II.15)}$$

The effective action appearing in the above expression is obtained from the functional determinant of Δ_B and Δ_F resulting from ψ and η integration. It contains also the α and ϕ^2 terms already in (II.13), and reads:

$$S_{eff} = -i\,\frac{N}{2} \left\{ \mathrm{Tr}\,\log \sqrt{-g} \left[\not{\partial} - iM + \frac{2i\sqrt{f}}{N}\,\phi + \right. \right.$$

$$\left. + \frac{i}{2N}\,(\bar{G}_\mu \gamma^\nu \gamma^\mu G_\nu) \right] - \mathrm{Tr}\,\log \sqrt{-g} \left[\frac{i}{\sqrt{-g}}\,\partial_\mu\,(g^{\mu\nu} \sqrt{-g}\,\partial_\nu) + \right.$$

$$\left. \left. + im^2 - \frac{2i\alpha}{\sqrt{N}} - \bar{c}'\,\frac{\Delta_F^{-1}}{N}\,c' \right] \right\} - \int d^2x \sqrt{-g} \left[\frac{\sqrt{N}}{2f}\,\alpha + \frac{\phi^2}{N} \right] \qquad \text{(II.16)}$$

where $c' = i(c + \gamma^\nu \gamma^\mu G_\nu \partial_\mu)$.

III. MASS GENERATION AND QUADRATIC TERM FOR THE FIELDS IN 1/N EXPANSION SCHEME

The $\sqrt{N}$ order terms in the effective action lead mass generation. The mass term breaks Weyl invariance(+) and this fact has important consequences for this theory*. The ϕ field generates the fermionic mass (M). Rescaling it according to:

$$\phi' = \sqrt{\frac{f}{N}}\,\phi \qquad \text{(III.1)}$$

and subtracting a constant from it, which will turn out to be its vacuum expectation value, gives:

$$\phi' = \phi'_0 - \sqrt{N}\, M/2 \qquad \text{(III.2)}$$

The $\sqrt{N}$ order ϕ field term is:

(*) See physical interpretation, sec. VI
(+) There is no contradiction with Lüscher-Elitzur theorem[26], since it corresponds to a local transformation which does not have any gauge field associated to, because of (II.5).

$$\sqrt{N}\left[\mathrm{Tr}\,(F^{-1}\phi_0') + \frac{M}{f}\int d^2x\,\phi_0'\right] \qquad (III.3)$$

with the compact notation

$$F^{-1} = (\gamma^a\,\partial_a - iM)^{-1}$$

It is clear that (III.3) diverges as $N \to \infty$ and unless the bracket expression is equal to zero the $1/N$ expansion does not make sense. We impose

$$\frac{1}{2f} = \frac{i}{(2\pi)^2}\int\frac{d\kappa^2}{\kappa^2 - M^2} \qquad (III.4)$$

With a Pauli-Villars regularization, (III.4) implies

$$\frac{1}{2f} = \frac{1}{4\pi}\log\Lambda^2/M^2 \qquad (III.5)$$

In an analogous way, we have for the α field the condition that $\sqrt{N}$ order term should vanish, which gives us:

$$\frac{1}{2f} = \frac{1}{4\pi}\log\Lambda^2/m^2 \qquad (III.6)$$

This shows that $m=M$, which is expected to occur, since we are dealing with a supersymmetric theory.

The quadratic terms for the fields are calculated in the $1/N$ expansion (that is, are given by the lowest order terms). In particular, we define the quantum "tetrad" field as[17]:

$$h^\mu_a = \frac{e^\mu_a - \eta^\mu_a}{\kappa} \qquad (III.7)$$

where $\kappa^2 = 16\pi G$ (G is the Newtonian "gravitational" coupling constant) and η^μ_a is the flat space "tetrad". In terms of the quantum "tetrad" field we writte the metric field:

$$g^{\mu\nu} = e^\mu_a\, e^\nu_b\, \eta^{ab} = \eta^{\mu\nu} + \kappa(h^{\mu\nu} + h^{\nu\mu}) + \kappa^2\, h^\mu_a\, h^{a\nu} \qquad (III.8)$$

Rescaling h^μ_a,

$$h^\mu_a \to h'^\mu_a = (N)^{-1/2}\; h^\mu_a$$

such that $\kappa^2 N=\lambda$, where λ is fixed[18], makes κ proportional to $1/\sqrt{N}$, enabling us to obtain the quadratic term for h^μ_a, in the $1/N$ expansion scheme. The Gravitino quadratic part is calculated, approximating the metric and tetrad fields by its corresponding flat space terms*. In this approximation the effective action reads:

$$\begin{aligned} S_{eff} = -\frac{iN}{2}\Big\{&\mathrm{Tr}\,\log\Big[\gamma^a\partial_a - iM + 2i\,\frac{\phi'}{\sqrt{N}} + \frac{i}{2N}(\overline{G}_a\gamma^b\gamma^a G_b)\Big] - \\ &- \mathrm{Tr}\,\log\Big[i(\Box + m^2) - \frac{2i\alpha}{\sqrt{N}} - \frac{1}{N}(\overline{G}_a\gamma^b\gamma^a\partial_b - \overline{c})(\gamma^a\partial_a - iM)^{-1}\, x \\ &x\;(c + \gamma^c\gamma^d G_c\partial_d)\Big] + \dots\Big\} \end{aligned} \qquad (III.9)$$

Writting $F = \gamma^a\partial_a - iM$ and $B = -i\left[\Box + m^2\right]$, we have for the Gravitino pure quadratic part:

$$\frac{1}{2}\int d^2x\, d^2y\; G^\mu(x)\;\Gamma^{\overline{G}/G}_{\mu\nu}\,(x-y)\; G^\nu(y) =$$

$$= -\frac{i}{2}\left[\frac{i}{2}\,\mathrm{Tr}\,F^{-1}\,\overline{G}_a\gamma^b\gamma^a G_b - \mathrm{Tr}\,\overline{G}_a\gamma^b\gamma^a\partial_b F^{-1}\gamma^c\gamma^d G_c\partial_d B^{-1}\right] \qquad (III.10)$$

which is formally written as follows:

$$-\frac{i}{2}\Big[-\underbrace{\int d^2x\, d^2y\; \overline{G}_a(x)\gamma^b\gamma^a\partial^x_b\langle x|F^{-1}|y\rangle\; \gamma^c\gamma^d\partial^y_a\,\langle y|B^{-1}|x\rangle\; G_c(y)}_{I} +$$

$$+\frac{i}{2}\,\underbrace{\mathrm{Tr}\int d^2x\; \overline{G}_a(x)\gamma^b\gamma^a G_b(x)\;\langle x|F^{-1}|x\rangle}_{II}\Big] \qquad (III.11)$$

The corresponding Feynman graphs are displayed in figure 1.

(*) The "Graviton-other-fields" vertices appear only in the subsequent orders.

The flat space ψ and η fields propagators are given by:

$$\langle x|F^{-1}|y\rangle = \frac{1}{(2\pi)^2 i}\int \frac{d\kappa^2\, e^{i\kappa(x-y)}}{\not{\kappa} - M + i\varepsilon} \tag{III.12}$$

and

$$\langle x|B^{-1}|y\rangle = \frac{1}{(2\pi)^2 i}\int \frac{d\kappa^2\, e^{i\kappa(x-y)}}{\kappa^2 - m^2 + i\varepsilon} \tag{III.13}$$

Equation (III.11) contains two terms which can be handled in momentum space. The first term (I) is given by:

$$I = +\frac{i}{2}\int \frac{d\overset{2}{p}\, d\overset{2}{\kappa}\, \tilde{\bar{G}}_a(p)\ \gamma^b\gamma^a(p_b+\kappa_b)(\not{\kappa}+\not{p}+M)\ \gamma^c\gamma^d\kappa_d G_c(p)}{\left[(\kappa+p)^2 - M^2\right]\left[\kappa^2 - m^2\right]} \tag{III.14}$$

where the $i\varepsilon$ is absorbed in M^2, and $G_a(x) = \int d\overset{2}{p}\, e^{ipx}\tilde{G}_a(p)$, etc. Since(*) $\bar{G}_a\gamma_b G_c = 0$, terms containing $\not{p}$ and $\not{\kappa}$ do not contribute. Because $m = M$, we have:

$$\int \frac{d\kappa^{2+\varepsilon}\ (p_b+\kappa_b)\kappa d}{\left[(\kappa+p)^2-M\right]\left[\kappa^2-M^2\right]} = i\pi\left\{p_b p_d\left[-\frac{1}{p^2} - \frac{2M^2}{p^2}\,\frac{1}{\sqrt{p^4-4M^2p^2}}\ x\right.\right.$$

$$\left.\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right] - \frac{1}{2}\eta_{bd}\left[-\ell n\pi - \ell n(-M^2) + \Gamma(-\varepsilon/2) + \right.$$

$$\left.\left. + 2 - \frac{\sqrt{p^4 - 4M^2p^2}}{p^2}\ \ell n\left(\frac{p^2+\sqrt{p^2-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right]\right\} \tag{III.15}$$

(*) We use the symmetry (II.6) to restrict G_μ to obey $\gamma^\mu G_\mu = 0$.

Analogously for the second term of (III.11):

$$II:\ \frac{1}{4i}\,\mathrm{Tr}\int \frac{d\kappa^2\ \tilde{\bar{G}}_a(p)\gamma^b\gamma^a G_b(p) M}{\kappa^2 - M^2} \tag{III.16}$$

Now since

$$\int \frac{d\kappa^{2+\varepsilon}}{\kappa^2-M} = -i\pi\left[\Gamma(-\varepsilon/2) - \ell n\pi - \ell n(-M^2)\right] \tag{III.17}$$

and also

$$\tilde{\bar{G}}_a\gamma^b\gamma^a\gamma^c\gamma^d\tilde{G}_c = 4\eta^{ab}\eta^{cd}\tilde{\bar{G}}_b\tilde{G}_d \tag{III.18a}$$

$$\tilde{\bar{G}}_a\gamma^b\gamma^a G_b = 2\eta^{ab}\tilde{\bar{G}}_b\tilde{G}_a \tag{III.18b}$$

the sum of parts I and II is finite and can be written as:

$$\frac{1}{2}\tilde{\Gamma}^{\bar{G}/G}_{bd}(p^2,M^2) = \pi M\left\{2p_b p_d\left[\frac{1}{p^2} + \frac{2M^2}{p^2}\,\frac{1}{\sqrt{p^2-4M^2p^2}}\ x\right.\right.$$

$$\left.\left. x\ \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right] + \eta_{bd}\left[2 - \frac{\sqrt{p^4-4M^2p^2}}{p^2}\ \ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\right]\right\} \tag{III.19}$$

We have to add to (III.19) the term corresponding to the Gauge fixing

$$L^{G\mu}_{fix} = -\frac{1}{2}\alpha\ \bar{G}^\mu\gamma_\mu\gamma_\nu G^\nu \tag{III.20}$$

so that the Gravitino two point function turns into:

$$\frac{1}{2}\,\bar{\Gamma}_{bd}^{\bar{G}_\mu/G_\mu}(p^2,M^2) \to \frac{1}{2}\,\tilde{\Gamma}_{bd}^{\bar{G}_\mu/G_\mu} - \frac{1}{2}\,\alpha\,\gamma_b\gamma_d \tag{III.21}$$

The $\underline{c}$ field pure quadratic part is:

$$\frac{1}{2}\,\bar{\Gamma}^{\bar{c}/c}(p^2,M^2) = \frac{M\,\pi}{\sqrt{p^4-4M^2p^2}}\,\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) \tag{III.22}$$

The ϕ field quadratic part can be also be obtained by standard methods[19]

$$\frac{1}{2}\,\bar{\Gamma}^{\bar{\phi}/\phi}(p^2,M^2) = 2\pi\,\frac{[p^2-4M^2]}{\sqrt{p^2-4M^2p^2}}\,\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) \tag{III.23}$$

But to obtain the complete Gravitino and $\underline{c}$ field quadratic parts we have to calculate the mixed term for these fields. It is given by:

$$\tilde{\Gamma}_b^{\bar{G}_\mu/c}(p^2,M^2) = \frac{\pi i}{\sqrt{p^4-4M^2p^2}}\,\frac{1}{p^2}\,\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\times$$

$$\times\left[2p_bMp^2 + \not{p}p_b(p^2-4M^2)\right] - \frac{2i\pi\not{p}p_b}{p^2} \tag{III.24}$$

In the same way as the Gravitino field has no kinetic term, the Graviton has no free propagator in two dimensions. It is described by the symmetric part of the quantum tetrad field. Writting the metric field in terms of the symmetric ($S_{\mu\nu}$) and antisymmetric ($a_{\mu\nu}$) parts of the quantum tetrad field gives:

$$g^{\mu\nu} = \eta^{\mu\nu} + 2\kappa S^{\mu\nu} + \kappa^2 S^\mu_a S^{\nu a} + \kappa^2\left[S^\mu_a\, a^{a\nu} + a^\mu_a\, S^{\nu a} + a^\mu_a\, a^{\nu a}\right] \tag{III.25}$$

We fix the Gauge by adding to the lagrangian[15]:

$$L^{S_{\mu\nu}}_{fix} = -\frac{1}{2}\sqrt{-g}\left[\partial_\mu S^{\mu\nu} - \frac{1}{2}\partial^\nu S^\alpha_\alpha\right]^2 \tag{III.26}$$

$$L^{a_{\mu\nu}}_{fix} = -\frac{1}{2}\sqrt{-g}\left[a_{\mu\nu}\right]^2 \tag{III.26a}$$

(which corresponds to have $\partial_\mu S^{\mu\nu} = \frac{1}{2}\partial^\nu S^\alpha_\alpha$ and $a_{\mu\nu} = 0$).(*) The effective action, with special attention to Graviton field, is:

$$S_{eff} = -\frac{iN}{2}\left\{\mathrm{Tr}\,\log\sqrt{-g} + \mathrm{Tr}\,\log\left[\kappa\,\not{E}^\mu\partial_\mu + F + \ldots\right] - \right.$$

$$- \mathrm{Tr}\,\log\left[i\partial_\mu(g^{\mu\nu}\sqrt{-g}\,\partial_\nu) + i\,g^{\mu\nu}\sqrt{-g}\,\partial_\mu\partial_\nu + im^2\sqrt{-g} + \ldots\right] \tag{III.27}$$

Using (III.25) and $a_{\mu\nu} = 0$ gives the quadratic term for $S_{\mu\nu}$:

$$\frac{1}{2}\int d^2x\, d^2y\, S^{\mu\nu}(x)\,\Gamma^{SS}_{\mu\nu\rho\lambda}(x-y)S^{\rho\lambda}(y) =$$

$$= -\frac{i}{2}\,\mathrm{Tr}\left\{-\frac{1}{2}\left[\underset{III}{\not{S}^\mu\partial_\mu\, F^{-1}}\right]^2 - 2\left[\underset{IV}{S^{\mu\nu}\partial_\mu\partial_\nu\, B^{-1}}\right]^2 + \right.$$

$$\left. + \underset{V}{i\,S^\mu_a S^{\nu a}\partial_\mu\partial_\nu\, B^{-1}} - \frac{1}{2}\left(S^{\mu\nu}\,S_{\mu\nu}\right)\right\} \tag{III.28}$$

Diagrammaticaly, we have the terms III, IV and V in figure 2. The momentum space contributions are given by:

(*) Although there exist ghost contributions, they only occur at higher order in 1/N.

$$A^{F\alpha\beta}_{\mu\nu}(p^2,M^2) = -\frac{1}{2}\int \frac{d\kappa^2 \kappa_\mu \kappa_\lambda (\kappa_\nu+p_\nu)(\kappa_\rho+p_\rho)\,Tr\left[\gamma^\alpha\gamma^\lambda\gamma^\beta\gamma^\rho\right]}{\left[(\kappa+p)^2-M^2)\right]\left[(\kappa^2-M^2\right]}$$

$$-\frac{M^2}{2}\,Tr\left[\gamma^\alpha\gamma^\beta\right]\int \frac{d\kappa^2\kappa_\mu(\kappa_\nu+p_\nu)}{\left[(\kappa+p)^2-M^2\right]\left[\kappa^2-M^2\right]} \qquad (III.29a)$$

$$A^{B}_{\mu\nu\alpha\beta}(p^2,M^2) = 2\int \frac{d\kappa^2\kappa_\mu\kappa_\alpha(\kappa_\nu+p_\nu)(\kappa_\beta+p_\beta)}{\left[(\kappa+p)^2-M\right]\left[\kappa^2-M^2\right]} \qquad (III.29b)$$

$$A'^{B}_{\mu\nu\alpha\beta}(p^2,M^2) = -\eta_{\alpha\beta}\int \frac{d\kappa^2\kappa_\mu\kappa_\nu}{\left[\kappa^2-M^2\right]} \qquad (III.29c)$$

Adding (III.29a), (III.29b) and (III.29c) gives the result:

$$\int \frac{d\kappa^2}{\left[(\kappa+p)^2-M^2\right]\left[\kappa^2-M^2\right]}\Big[\eta_{\alpha\beta}\,\kappa_\mu\kappa^2 p_\nu + \eta_{\alpha\beta}\,\kappa_\mu\kappa\cdot pp_\nu + \kappa_\mu\kappa_\nu\kappa_\alpha p_\beta +$$

$$+\kappa_\mu\kappa_\alpha p_\nu p_\beta - \kappa_\mu\kappa_\beta\kappa_\nu p_\alpha - \kappa_\mu\kappa_\beta p_\nu p_\alpha - M^2\,\eta_{\alpha\beta}\,\kappa_\mu p_\nu -$$

$$-\eta_{\alpha\beta}\kappa_\mu\kappa_\nu\kappa\cdot p - \eta_{\alpha\beta}\kappa_\mu\kappa_\nu p^2\Big] \qquad (III.30)$$

We can use dimensional regularization[20] (2+ε dimensions) to get the divergent part of (III.30)

$$\frac{1}{2}\,\Gamma^{SSdiv}_{\mu\nu\alpha\beta}\;(p^2,M^2) = \frac{p^2\pi}{8}\left[\frac{1}{2}\,\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]\Gamma(-\varepsilon/2) \qquad (III.31)$$

The finite part of (III.30) can also be readily calculated:

$$\frac{1}{2}\,\Gamma^{SSfin}_{\mu\nu\alpha\beta}\;(p^2,M^2) = \frac{\pi}{2}\Big[(\eta_{\alpha\beta}\eta_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\alpha}\eta_{\nu\beta})\;x$$

$$x\left\{\frac{1}{4}\sqrt{p^4-4M^2p^2}\;\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right) - 2p^2\right\} +$$

$$+\frac{\eta_{\mu\alpha}\eta_{\nu\beta}p^2M^2}{4\sqrt{p^4-4M^2p^2}}\;\ell n\left(\frac{p^2+\sqrt{p^4-4M^2p^2}}{p^2-\sqrt{p^4-4M^2p^2}}\right)\Big] \qquad (III.32)$$

Taking into account that two dimensions is conformally flat[8] we can impose $h_{\mu\nu}(x) = h(x)\eta_{\mu\nu}$, and both the infinite part (III.31) as well as the first term in (II.32) equal zero.

We should call attention to the fact that Gravitino and Graviton propagators are massless, γ behaving as $1/p^2$ in the infrared region.

IV. BEYOND LOWEST ORDER IN 1/N EXPANSION

Up to lowest order in 1/N expansion of the effective action we showed that the theory is finite. But if we analise subsequent orders, in the effective action, we will find that non finite amplitudes occur an example of a finite amplitude we have the two Gravitino and one Graviton term.

The part of the effective action corresponding to this amplitude is given by:

$$S^{2G|1S}_{eff} = -\frac{i}{2N^{3/2}}\,Tr\,\Big\{\frac{i}{2}\Big[\underbrace{(\overline{G}_a\,g^b\,\gamma^a\,G_b)F^{-1}}_{I} + \underbrace{(\overline{G}_a\,\gamma^b\,g^a\,G_b)\,F^{-1}}_{II} -$$

$$-\underbrace{g^a\,\partial_a F^{-1}(\overline{G}_b\,\gamma^a\gamma^b\,G_a)F^{-1}}_{III}\Big] - \Big[\underbrace{\overline{G}_a\,g^b\,\gamma^a\partial_b\,F^{-1}\gamma^c\gamma^d\,G_c\partial d\,B^{-1}}_{IV} +$$

$$+\underbrace{\overline{G}_a\,\gamma^b\,g^a\partial_b\,F^{-1}\gamma^c\gamma^d\,G_c\,\partial d\,B^{-1}}_{V} + \underbrace{\overline{G}_a\gamma^b\gamma^a\partial_b F^{-1}\,g^c\,\gamma^d\,G_c\partial d\,B^{-1}}_{VI} +$$

$$+ \bar{G}_a\gamma^b\gamma^a\partial_b F^{-1}\gamma^c \not{g}^d G_c \partial d\, B^{-1} + 2\, i\, S^{\mu\nu}\partial_\mu\partial_\nu\, B^{-1}\, \bar{G}_a\gamma^b\gamma^a\partial_b F^{-1}\gamma^c\gamma^d G_c \times \partial d\, B^{-1} -$$

VII VIII

$$- \bar{G}_a\gamma^b\gamma^a\partial_b\, F^{-1}\, \not{g}^\lambda\, \partial_\lambda\, F^{-1}\gamma^c\gamma^d\, G_c\, \partial d\, B^{-1}\Big]\Big\} \quad . \qquad \text{(IV.1)}$$

IX

The corresponding Feynman Graphs are displayed in figure 3.

It can be shown that this amplitude is ultraviolet finite.

Next we have the two Gravitino-two Gravitons amplitude. The terms corresponding to the amplitude are given by:

$$S^{2G|2S}_{\text{eff}} = - \frac{i}{2N^2}\text{Tr}\Big\{ \frac{i}{2}\Big[(\bar{G}_a \not{g}^b \not{g}^a G_b)F^{-1} - \not{g}^\lambda\, \partial_\lambda F^{-1}((\bar{G}_a \not{g}^b\, \gamma^a\, G_b)F^{-1} +$$

I II

$$+ (\bar{G}_a\, \gamma^b \not{g}^a\, G_b)F^{-1}\Big] - \Big[\bar{G}_a \not{g}^b \not{g}^a\, \partial_b\, F^{-1}\, \gamma^c\gamma^d\, G_c\, \partial d\, B^{-1} +$$

III IV

$$+ \bar{G}_a\, \gamma^b\gamma^a\, \partial_b\, F^{-1} \not{g}^c \not{g}^d\, G_c\, \partial d\, B^{-1} + \bar{G}_a \not{g}^b\, \gamma^a\, \partial_b\, F^{-1}(\not{g}^c\, \gamma^d\, G_c\, \partial d\, B^{-1} +$$

V VI

$$+ \gamma^c \not{g}^d\, G_c\, \partial d\, B^{-1}) + \bar{G}_a\, \gamma^b \not{g}^a\, \partial_b\, F^{-1}(\not{g}^c\, \gamma^d\, G_c\, \partial d\, B^{-1} + \gamma^c \not{g}^d\, G_c\, \partial d\, B^{-1})\Big] +$$

VII VIII IX

$$+ (\bar{G}_a\, \gamma^b\gamma^a\, \partial_b\, F^{-1}\gamma^c\gamma^d\, G_c\, \partial d\, B^{-1})\Big[- i\partial_\mu(S^\mu_\alpha S^{\nu\alpha})\partial_\nu\, B^{-1} -$$

X

$$- i\, \partial^\nu(S^{\mu\alpha}S_{\mu\alpha})\, \partial_\nu\, B^{-1} - i\, S^\mu_\alpha\, S^{\nu\alpha}\, \partial_\mu\partial_\nu\, B^{-1}\Big] +$$

XI XII

$$+ (-2iS^{\mu\nu}\partial_\mu\partial_\nu B^{-1})\Big[\bar{G}_a \not{g}^b\, \gamma^a\partial_b F^{-1}\gamma^c\gamma^d\, G_c\partial d\, B^{-1} + \bar{G}_a\gamma^b \not{g}^a\partial_b F^{-1}\gamma^c\gamma^d G_c\partial d\, B^{-1} +$$

XIII XIV

$$+ \bar{G}_a\, \gamma^b\gamma^a\, \partial_b F^{-1} \not{g}^c\, \gamma^d\, G_c\, \partial d B^{-1} + \bar{G}_a\, \gamma^b\gamma^a\, \partial_b F^{-1}\, \gamma^c \not{g}^d\, G_c\, \partial d\, B^{-1}\Big] +$$

XV XVI

$$+ \bar{G}_a \not{g}^b\, \gamma^a\partial_b\, F^{-1} \not{g}^\lambda\partial_\lambda F^{-1}\gamma^c\gamma^d\, G_c\, \partial d\, B^{-1} + \bar{G}_a\, \gamma^b \not{g}^a\, \partial_b\, F^{-1} \not{g}^\lambda\partial_\lambda F^{-1}\gamma^c\gamma^d\, G_c\, \partial d\, B^{-1}$$

XVII XVIII

$$+ \bar{G}_a\, \gamma^b\gamma^a\, \partial_b\, F^{-1} \not{g}^\lambda\partial_\lambda\, F^{-1} \not{g}^c\, \gamma^d\, G_c\, \partial d\, B^{-1} + \bar{G}_a\, \gamma^b\gamma^a\, \partial_b\, F^{-} \not{g}^\lambda\partial_\lambda F^{-1}\gamma^c\gamma^d G_c\partial d B^{-1} -$$

XIX XX

$$- \bar{G}_a\, \gamma^b\gamma^a\, \partial_b F^{-1}(\not{g}^\lambda\partial_\lambda F^{-1})^2\, \gamma^c\gamma^d G_c\, \partial d\, B^{-1} + \frac{i}{2}\, (\not{g}^\lambda\partial_\lambda F^{-1})^2\, (\bar{G}_a\gamma^b\gamma^a\, G_b)F^{-1} +$$

XXI XXII

$$+ 4(S^{\mu\nu}\, \partial_\mu\partial_\nu\, B^{-1})^2\, (\bar{G}_a\, \gamma^b\gamma^a\, \partial_b F^{-1}\gamma^c\gamma^d\, G_c\, \partial d\, B^{-1})\Big\} \quad . \qquad \text{(IV.2)}$$

XXIII

Figure 4 contains the Feynman graphs associated with these amplitudes.

Through a lenghy calculation it can be shown that this amplitude is not finite. We have to add a counterterm to the lagrangean. Making analogous calculations for the quartic vertex of the Graviton and the Gravitino we conclude that their amplitudes contain also a non finite term which must be renormalized through the addition of new counterterms to the lagrangian.

V. THE STRING MODEL

As already mentioned in section II (eq. (II.3) the string action is obtained integrating the lagrangian over a finite region of space, i.e., the string lengh is finite. This implies quantization of the spatial component of the momentum. In particular, all formulas of chapter III must be reobtained, for the finite lengh case. For doing that task, we use a well know formula, very useful in the framework of finite temperature quantum field theory[21]:

$$\frac{\pi}{L} \sum_{n, p_n = \frac{n\pi}{L}} f(p_n) = \oint \frac{f(p')\, dp'}{e^{2iLp'} - 1} = \int f(p)\, dp + B \qquad (V.1)$$

where B is given by the sum of two integrals in complex plane:

$$B = \int_{-\infty - i\varepsilon}^{\infty - i\varepsilon} \frac{f(p)\, dp}{e^{2Lip} - 1} + \int_{-\infty + i\varepsilon}^{\infty + i\varepsilon} \frac{f(p)\, dp}{e^{-2Lip} - 1} \qquad (V.2)$$

We should calculate only the expression (V.2), because the remaining integral, corresponds to the infinite lengh case, which has been already treated. Because of the exponencial behavior of the integrands in (V.2), for $f(p)$ being any power of a rational function it can be calculated closing the integral with a circle in the infinite and calculating by residues.

For the case of mass generation we have no contribution since after integrating over the zeroth component of momentum (as in (III.4)) we are led to:

$$\int \frac{dk_o}{2\pi} \frac{1}{k_0^2 - k_1^2 - m^2} = \frac{1}{\sqrt{k_1^2 + m^2}} = f(k_1) \qquad (V.3)$$

This function has no pole, and does not contribute in (V.2). As a consequence mass generation is untouched in the case of η_i and ψ_i .

We now turn to the Gravitino propagator. The calculation is easierly performed in euclidian space. The terms to be studied are (see (III.14)):

$$- \int dk_o \sum_{k_1} \frac{(p+k)_\mu\, k_\nu}{[(p+k)^2 + m^2][k^2 + m^2]} + \int dk_o \sum_{k_1} \frac{1}{k^2 + m^2} \qquad (V.4)$$

The second term remains the same as before. The correction of the first term of (V.4) with respect to (III.15) is given by:

$$\Gamma_{\mu\nu}(p_0, p_1) = \int_{-\infty}^{\infty} dk_o \int_{-\infty - i\varepsilon}^{\infty - i\varepsilon} \frac{dk_1\, f_{\mu\nu}(k,p)}{e^{2Lik_1} - 1} + \int_{-\infty}^{\infty} dk_o \int_{-\infty + i\varepsilon}^{\infty + i\varepsilon} \frac{dk_1 f_{\mu\nu}(k,p)}{e^{-2Lik_1} - 1} \qquad (V.5)$$

where $f_{\mu\nu}(k,p)$ is given by the integrand of the first term in (V.5) (or (III.15)).

There are contributions to (V.5) from the following poles:

$$k_1 = - i \sqrt{K_0^2 + m^2} \qquad (V.6a)$$

$$k_1 = - p_1 - i \sqrt{(p_o + K_o)^2 + m^2} \qquad (V.6b)$$

to the first integral, and

$$k_1 = + i \sqrt{K_0^2 + m^2} \qquad (V.7a)$$

$$k_1 = - p_1 + i \sqrt{(k_o + p_o)^2 + m^2} \qquad (V.7b)$$

to the second one.

We are interested in the behavior of the pole in the Gravitino propagator, which in the infinite lengh limit corresponds to a massless state, so that we calculate (V.5) for p^2 near zero, and L very big. In this case it is easy to see that (V.6b) and (V.7b) produce a very much oscillating function. We turn to the contribution of (V.6a) and (V.7a). Because of the lack of Lorentz invariance, we calculate separately Γ_{00} and Γ_{11}, whose results are

$$\Gamma_{00} = -\frac{\pi}{2m^2}\int dk_o \frac{k_o^2}{\sqrt{k_o^2+m^2}} \frac{1}{e^{2L\sqrt{k_o^2+m^2}}-1} \qquad (V.8)$$

$$\Gamma_{11} = \frac{\pi}{2m^2}\int dk_o \frac{\sqrt{k_o^2+m^2}}{e^{2L\sqrt{k_o^2+m^2}}-1} \qquad (V.9)$$

The two point Gravitino functions are given by ($p^2 \simeq 0$):

$$\langle G_o(p)G_o(-p)\rangle = \frac{p^2}{8m^2} + \Gamma_{00} \qquad (V.10)$$

$$\langle G_1(p)G_1(-p)\rangle = -\frac{p^2}{8m^2} + \Gamma_{11} \qquad (V.11)$$

To calculate the inverse propagator, we should take into account mixed [G-c fields] contributions (see chapter III). However, (V.10) and (V.11) have a deep result: there is a mass gap exponencially small for big L, providing a breaking of the strong force for finite L . The same result follows for the Graviton.

VI. PHYSICAL INTERPRETATION

Finiteness of the string lengh led us to massive gravitational fields. This is a natural explanation for the stability

of the string: if it goes to infinity, the gravitino (graviton) mass tends quickly (exponencially) to zero, originating a long range force whose tendency is to collapse the string.

The quantum nature of our calcullation was of fundamental importance. Classically there is no mass generation, so that Weyl invariance is unbroken. This invariance permits to gauge away the graviton and the gravitino fields[22], and the only trace of their previous presence were the string conditions[4][23].

In the infinite lengh limit all fields are confined. Gauge invariant objects should survive. However, supersymmetry invariance implies Lorentz invariance, and gauge invariant objects are in this case space independent. Nevertheless it is a good supergravity laboratory, concerning divergence cancellations: unexpected finite Green functions appear in the theory, such as gravitino (graviton) two point functions, and two gravitino-one graviton vertex.

Classically, the model is yet integrable. A conserved non-local charge can be written, namely

$$Q = \int dy_1 dy_2 \varepsilon(y_1-y_2) J_o^{ik}(t,y_1) J_o^{kj}(t,y_2) - \int dy \left[J^{ij}(t,y) + 2i_1^{ij}(t,y)\right] \qquad (VI.1)$$

where

$$J_\mu^{ij} = \partial_\mu \eta^i \eta^j - \eta^i \partial_\mu \eta^j + \frac{i}{2}\bar\psi^i \gamma_\mu \psi^j - \frac{i}{2}\bar\psi^j \gamma_\mu \psi^i + \bar\psi^i \gamma^\nu \gamma^\mu \eta^j G_\nu - \bar\psi^j \gamma^\nu \gamma^\mu \eta^i G_\nu \qquad (VI.2)$$

$$j_\mu^{ij} = \partial_\mu \eta^i \eta^j - \eta^i \partial_\mu \eta^j \qquad (VI.3)$$

$$i_\mu^{ij} = \frac{i}{2}\bar\psi^i \gamma_\mu \psi^j - \frac{i}{2}\bar\psi^j \gamma_\mu \psi^i \quad . \qquad (VI.4)$$

Conservation of Q means classical integrability. However we guess that quantum fluctuations spoils charge conservation, because of the existence of many candidates to the anomaly(24), and to the non-renormalizability (section IV).

VII. CONCLUSIONS AND OUTLOOK

We displayed a mechanism for string stability. It is natural to ask if this mechanism holds for other quantum string models, for example, other symmetry groups. It seems that most important is the mass generation and consequently Weyl invariance breaking turns out to be a fundamental feature for this kind of model. For more complicated models(19), we wonder about the consequence for other structures, such as θ-vacuum and confinement of flavor and colour degrees of freedom.

APPENDIX

Metric

$$\eta_{\mu\nu} = \mathrm{diag}\ (1,-1) .$$

γ_μ matrices representation

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi} = \psi^T \gamma_0$$

Fierz transformation:

$$\delta_{\alpha\beta}\ \delta_{\lambda\rho} = \frac{1}{2}\left[\delta_{\alpha\rho}\delta_{\lambda\beta} + \gamma_{5\alpha\rho}\gamma_{5\lambda\beta} + \gamma_{\mu\alpha\rho}\gamma^{\mu}_{\lambda\beta}\right]$$

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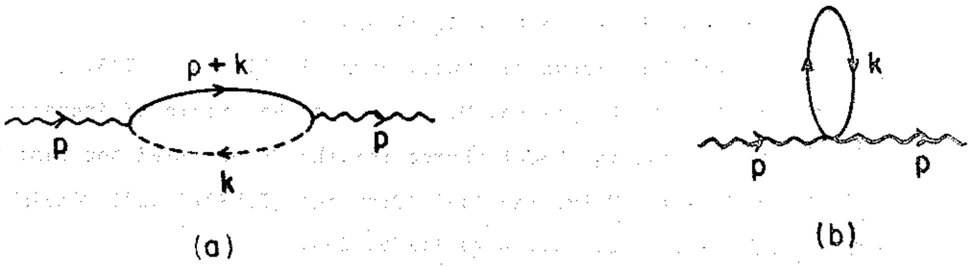


Fig 1

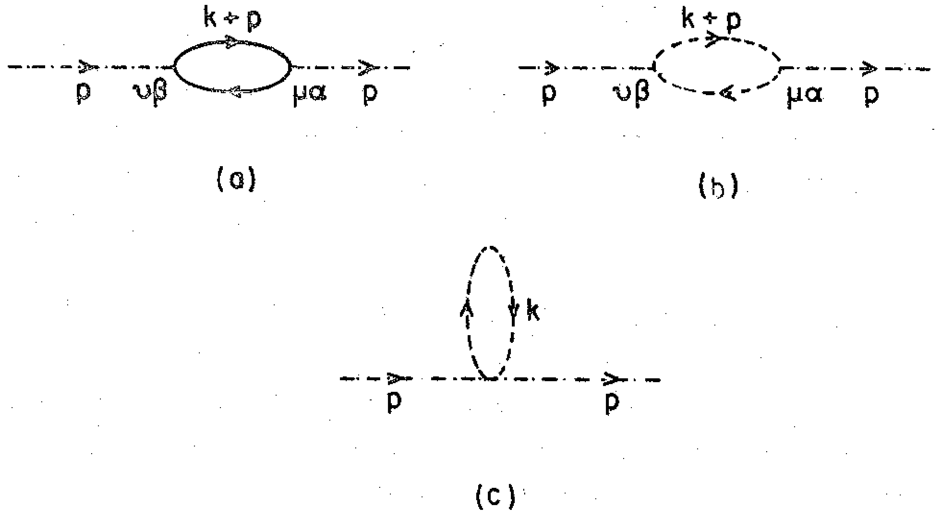
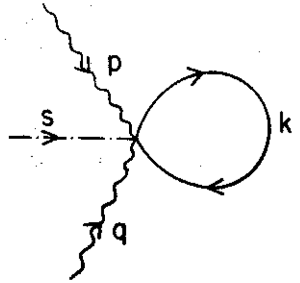
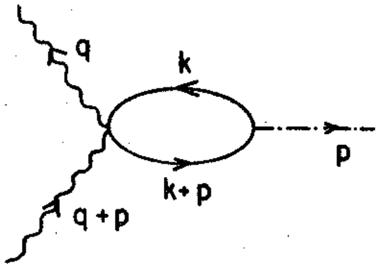


Fig 2



(a)



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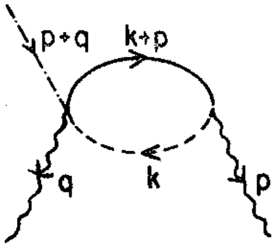

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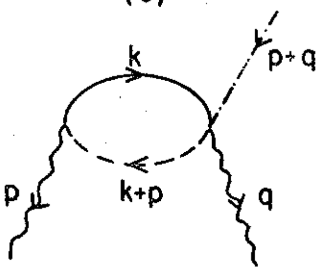
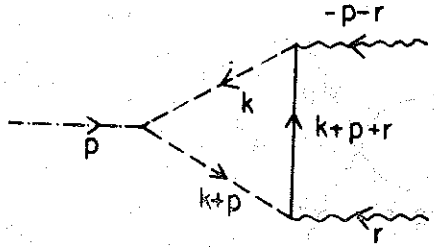


(d)

Fig 3



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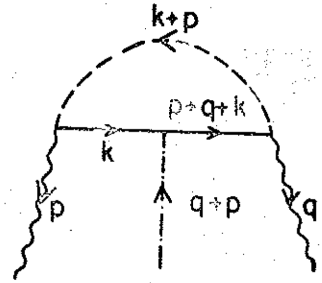

(f)

Fig 3

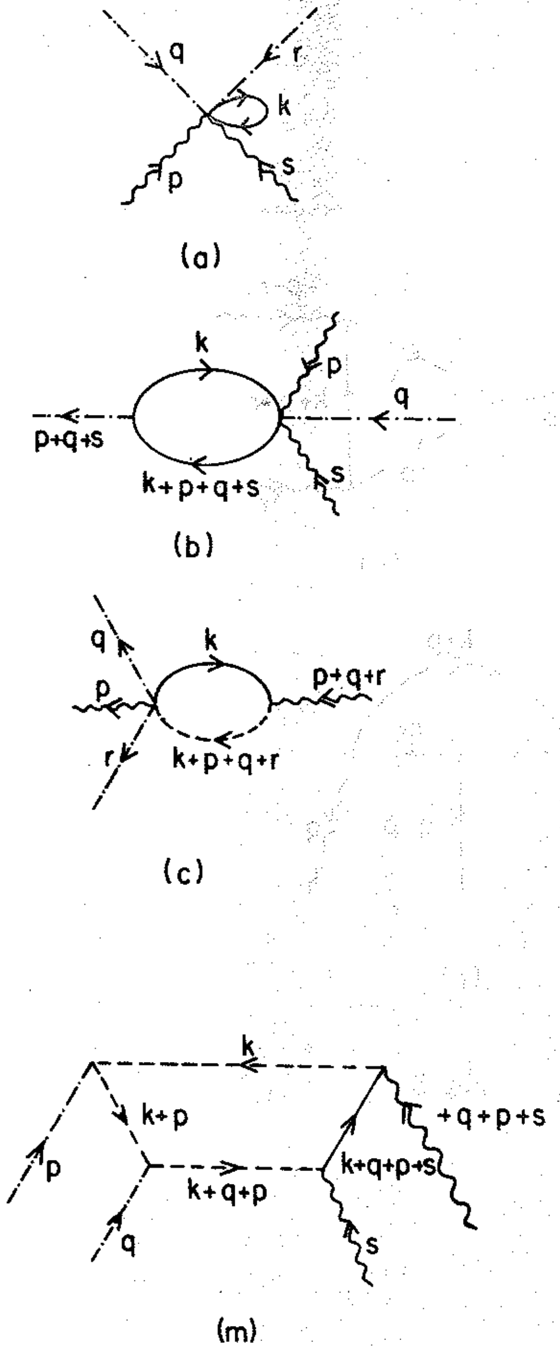

Fig 4

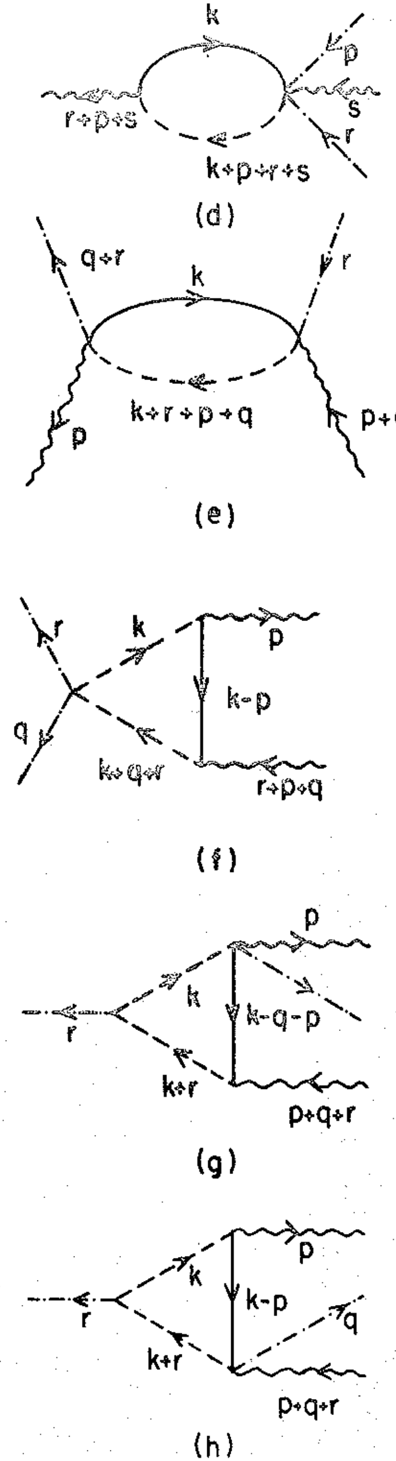

Fig 4

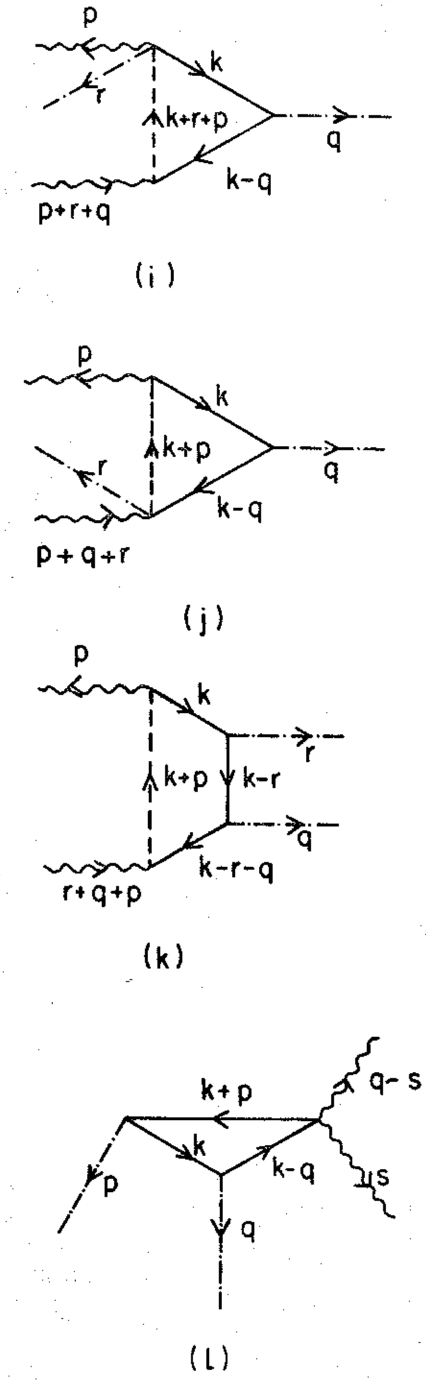

Fig 4