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THE ANOMALY FREE CP¹ MODEL AND ITS S-MATRIX

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ABSTRACT

After showing that the CP¹ model factorizes we obtain the S-matrix for fundamental fields and bound states.

I. THE ANOMALY FREE MODEL

The classically established equivalence of the CP¹ and 0(3) non-linear sigma model⁽¹⁾, was never shown quantum mechanically. This notice aims at calling attention for a fact that, in our opinion solves the problem.

The CPⁿ⁻¹ model has a quantum anomaly preventing conservation of the non local (classically conserved) charge⁽²⁾. However, when coupled minimally to fermions, the anomaly cancels⁽³⁾, the gauge field zero mass pole disappears, and the model turns out to be factorizable⁽⁴⁾. The S-matrix can be calculated, and compared with the 1/n expansion of the model⁽⁴⁾. Accordance is obtained in lowest order. The S-matrix has no pole in the physical sheet, and the z-field interacts via a repulsive force, preventing bound states. This situation is very different from the pure CPⁿ⁻¹ model, where long range forces confine partons in mesons. This long range forces, responsible for confinement, imply also the existence of the anomaly, as shown in a explicit calculation⁽²⁾.

Briefly the CPⁿ⁻¹ model is the theory of an n-component complex z-field which Lagrangian density is given by⁽¹⁾

$$\mathcal{L} = \overline{D_\mu z} D_\mu z \quad (1)$$

where

$$D_\mu z = \partial_\mu z - A_\mu z \quad (2a)$$

$$A_\mu = -\frac{f}{n} \bar{z} \frac{\delta}{\delta z} z \quad (2b)$$

$$\text{and the constraint } \bar{z}z = \sum_i \bar{z}_i z_i = n/2f \quad (2c)$$

At classical level this model is known to possess an infinite number of conservation laws⁽⁵⁾ and the simplest classically conserved non-local charge is given by:

$$Q^{ij} = \int dy_1 dy_2 \varepsilon(y_1 - y_2) J_0^{ik}(t, y_1) J^{kj}(t, y_2) - \frac{n}{2f} \int J_1^{ij}(t, y) dy \quad (3)$$

where

$$J_\mu^{ij}(x) = z^i(x) \frac{\delta}{\delta z^\mu} \bar{z}^j(x) + 2 A_\mu z^i(x) \bar{z}^j(x) \quad (4)$$

is the classical traceless Nöther current associated to the SU(n) rotations.

The classical integrability condition

$$\partial_\mu J_\nu^{ij} - \partial_\nu J_\mu^{ij} + 2 \frac{2f}{n} [J_\mu(x), J_\nu(x)]^{ij} = 0 \quad (5)$$

is equivalent to $\frac{dQ^{ij}}{dt} = 0$.

At the quantum level we have problems because the charge (3) involves a product of two currents at the same point and so it is not well defined. To give a proper definition for the quantum non-local charge we must look at the short distance behavior of the product which appears in the commutator of equation (5). This was done⁽²⁾ and we are left with:

$$[J_\mu(x+\epsilon), J_\nu(x)]^{ij} = C_{\mu\nu}^{\rho}(\epsilon) J_\rho^{ij}(x) + D_{\mu\nu}^{\rho\sigma}(\epsilon) \partial_\sigma J_\rho^{ij}(x) + E_{\mu\nu}^{\rho\sigma}(\epsilon) z_i \bar{z}_j F_{\rho\sigma}(x) \quad (6)$$

where $C_{\mu\nu}^{\rho}$, $D_{\mu\nu}^{\rho\sigma}$ and $E_{\mu\nu}^{\rho\sigma}$ are non zero and $F_{\rho\sigma} = \partial_\rho A_\sigma - \partial_\sigma A_\rho$.

Now we are able to define the quantum non local charge $Q^{ij} = \lim_{\delta \rightarrow 0} Q_\delta^{ij}$.

$$Q_\delta^{ij} = \frac{1}{n} \left\{ \int_{|y_1 - y_2| > \delta} dy_1 dy_2 \epsilon(y_1 - y_2) J_\rho^{ik}(t, y_1) J_\rho^{kj}(t, y_2) - Z_\delta \int dy J_\rho^{ij}(t, y) \right\} \quad (7)$$

where the dependence of Z on the cutoff δ is such as to cancel the linear divergences which appear in the commutator (6). So that, in order to obtain a well-defined charge Q we must have

$$Z_\delta = \frac{n}{2\pi} \ln \left(\frac{e^{\gamma-1} m \delta}{2} \right) \quad (8)$$

The mass m is dynamically generated and given by $m^2 = \mu^2 e^{-n/2f}$ where μ is the renormalization point and γ is the Euler-Mascheroni constant.

In accordance with the confining properties of the theory one can verify using (6) and (7) that the quantum non-local charge is no longer conserved.

$$\frac{dQ^{ij}}{dt} = -\frac{2}{\pi} \int_{-\infty}^{\infty} z_i \bar{z}_j F_{10} dy \quad (9)$$

This means that the model has an anomaly in its quantum non-local charge and because of this the model is not factorizable, and consequently has no factorizable S-matrix.

However, for $n=2$ the picture changes. In this case the anomaly can be easily shown to be a total divergence, and we are able to construct a new quantum non-local conserved charge. We can then show that the CP^1 model has a quantum conserved non-local charge just redefining the old one (7).

The Nöther current (4) can be written as follows⁽⁶⁾:

$$J_\mu = -D_\mu X X^+ + X D_\mu X^+ = Y D_\mu Y^+ - D_\mu Y Y^+ \quad (10)$$

where the fields X and Y (for the CP^1 case) are the two-components fields:

$$X = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad Y = \begin{pmatrix} \bar{z}_2 \\ -\bar{z}_1 \end{pmatrix} \quad (11)$$

satisfying

$$Y^+ X = 0 \quad (12a)$$

$$X^+ X = Y^+ Y = 1 \quad (12b)$$

$$\text{and connected by } Y_i = \epsilon_{ij} X_j^+, \quad (\epsilon_{21} = -1). \quad (12c)$$

In general the following identity holds

$$X F_{\mu\nu}^X X^+ + Y F_{\mu\nu}^Y Y^+ = \frac{1}{2} (\partial_\mu J_\nu - \partial_\nu J_\mu) \quad (13)$$

Now, the anomaly is just

$$(X_i X_j^+ - \frac{1}{2} \delta_{ij}) F_{uv}^x = (Y_j^+ Y_i - \frac{1}{2} \delta_{ij}) F_{uv}^y \quad (14)$$

The above identity being a direct consequence of (11). So that we can take the mean value of both sides obtaining (13), which is a total divergence of the current (10).

At this point a standard definition for a conserved quantum non-local charge can be made and we conclude that the model is anomaly free.

This fact follows from a very simple criterium obtained in the context of group theory⁽⁷⁾, for the so called non linear sigma models defined on symmetric Riemannian spaces. There it is shown a general criterion for the presence or absence of anomalies in these quantum models. The conclusion is the following. Let a model be defined on a symmetric space $M = G/H$. One can have two possibilities:

- 1) The model is anomaly free if H is simple.
- 2) Anomalies are allowed if H contains nontrivial ideals.

II. FUNDAMENTAL z-FIELDS S-MATRIX

In this section we construct, from the knowledge of the asymptotic field, the quantum non-local charge which has no contribution of the gauge field A_μ . To justify this procedure we remember that the asymptotic part of a conserved current can be taken as the one which has the same commutation relations and vacuum expectation value as the interacting current⁽⁸⁾. The procedure as well as all the normalizations are completely

analogous to the ref. (2).

The non-local charge can be written as:

$$Q^{ij} = -\frac{1}{2} \int_{-\infty}^{\infty} d\mu(p_1) d\mu(p_2) \bar{\epsilon}(p_1 - p_2) : (a_{in}^i(p_1) a_{in}^{+k}(p_1) - b_{in}^{+i}(p_1) b_{in}^k(p_1)) (a_{in}^k(p_2) a_{in}^{+j}(p_2) - a_{in}^{+k}(p_2) a_{in}^j(p_2)) : - \frac{1}{i\pi} \int d\mu(p) \ell_n \frac{(p^0 + p)}{m} : \left\{ a_{in}^i(p) a_{in}^{+j}(p) - b_{in}^{+i}(p) b_{in}^j(p) - \frac{\delta^{ij}}{2} (a_{in}^{+k}(p) a_{in}^k(p) - b_{in}^k(p) b_{in}^{+k}(p)) \right\} : \quad (15)$$

where a and b are creation and destruction operators obeying the usual commutator rules. The out-form of the same charge Q^{ij} differs from (15) just on the sign of the first term.

The action of the non-local charge Q^{ij} on asymptotic states of two particles characterized by the rapidities θ_1, θ_2 and by the isospin indices $c_\ell, d_\ell, c'_\ell, d'_\ell$ is given by:

$$Q^{ij} |\theta_1 c_1, \theta_2 c_2\rangle_{in} = \left[M_{in}^{ij} \right]_{d_1 d_2 c_1 c_2} |\theta_1 d_1, \theta_2 d_2\rangle_{in} \quad (16a)$$

$${}_{out} \langle \theta'_1 c'_1, \theta'_2 c'_2 | Q^{ij} = {}_{out} \langle \theta'_1 d'_1, \theta'_2 d'_2 | \left[M_{out}^{ij} \right]_{c'_1 c'_2 d'_1 d'_2} \quad (16b)$$

where

$$\left[M_{in}^{ij} \right]_{d_1 d_2 c_1 c_2} = -\frac{1}{2} (\delta^{ic_1} \delta^{jd_2} \delta_{c_2 d_1} - \delta^{ic_2} \delta^{jd_1} \delta_{c_1 d_2}) - \frac{\theta_1}{i\pi} (\delta^{ic_1} \delta^{jd_1} \delta_{c_2 d_2} + \frac{\delta^{ij}}{2} \delta_{c_1 d_1} \delta_{c_2 d_2}) - \frac{\theta_2}{i\pi} (\delta^{ic_2} \delta^{jd_2} \delta_{c_1 d_1} + \frac{\delta^{ij}}{2} \delta_{c_1 d_1} \delta_{c_2 d_2}) \quad (17)$$

$$\begin{aligned} \left(M_{out}^{ij} \right)_{c_1 c_2 d_1 d_2} &= \frac{1}{2} (\delta^{id_1} \delta^{jc_2} \delta^{d_2} c_1 - \delta^{id_2} \delta^{jc_1} \delta^{d_1} c_2) - \\ &- \frac{\theta_1}{i\pi} (\delta^{id_1} \delta^{jc_1} \delta^{c_2} d_2 + \frac{\delta^{ij}}{2} \delta^{c_1 d_1} \delta^{c_2 d_2}) - \\ &- \frac{\theta_2}{i\pi} (\delta^{id_2} \delta^{jc_2} \delta^{c_1} d_1 + \frac{\delta^{ij}}{2} \delta^{c_1 d_1} \delta^{c_2 d_2}) \end{aligned} \quad (18)$$

Now the elastic scattering amplitude of two particles with rapidities θ_1 and θ_2 can be written as:

$$\begin{aligned} \text{out} \langle \theta_1^i c_1^i, \theta_2^j c_2^j | \theta_1 c_1, \theta_2 c_2 \rangle_{in} &= (4\pi)^2 \delta(\theta_1^i - \theta_1^i) \delta(\theta_2^j - \theta_2^j) \left\{ \delta^{c_1 c_1^i} \delta^{c_2 c_2^j} u_1(\theta) + \right. \\ &+ \left. \delta^{c_1 c_2^i} \delta^{c_2 c_1^j} u_2(\theta) \right\} - (4\pi)^2 \delta(\theta_1^i - \theta_2^j) \delta(\theta_2^j - \theta_1^i) \left\{ \delta^{c_1 c_1^i} \delta^{c_2 c_2^j} u_1(\theta) + \right. \\ &+ \left. \delta^{c_1 c_2^i} \delta^{c_2 c_1^j} u_2(\theta) \right\} \end{aligned} \quad (19)$$

where $\theta = \theta_1 - \theta_2$ and $u_1(\theta)$, $u_2(\theta)$ are restricted by the non-local conservation law obtained imposing that:

$$\text{out} \langle \theta_1^i c_1^i, \theta_2^j c_2^j | Q^{ij} | \theta_1 c_1, \theta_2 c_2 \rangle_{in} = \begin{cases} \left(M_{out}^{ij} \right)_{c_1 c_2 d_1 d_2} \text{out} \langle \theta_1^i d_1^i, \theta_2^j d_2^j | \theta_1 c_1, \theta_2 c_2 \rangle_{in} & (20a) \\ \text{out} \langle \theta_1^i c_1^i, \theta_2^j c_2^j | \theta_1 d_1, \theta_2 d_2 \rangle_{in} \left(M_{in}^{ij} \right)_{d_1 d_2 c_1 c_2} & (20b) \end{cases}$$

This set of linear equations for $u_1(\theta)$ and $u_2(\theta)$ can be solved giving us one of the so called "factorization equations".

$$u_2(\theta) = - \frac{i\pi}{\theta} u_1(\theta) \quad (21)$$

The other factorization equation which relates $t_1(\theta)$ with $t_2(\theta)$ can be obtained by the usual crossing symmetry or by the same way as $u_1(\theta)$ and $u_2(\theta)$ just writing the elastic scattering amplitude of one particle and its anti-particle as follows:

$$\begin{aligned} \text{out} \langle \theta_1^i c_1^i, \theta_2^j \bar{c}_2^j | \theta_1 c_1, \theta_2 \bar{c}_2 \rangle_{in} &= (4\pi)^2 \delta(\theta_1^i - \theta_1^i) \delta(\theta_2^j - \theta_2^j) \left\{ \delta^{c_1 c_1^i} \delta^{c_2 \bar{c}_2^j} t_1(\theta) + \right. \\ &+ \left. \delta^{c_1 \bar{c}_2^i} \delta^{c_2 c_1^j} t_2(\theta) \right\} - (4\pi)^2 \delta(\theta_1^i, \theta_2^j) \delta(\theta_2^j - \theta_1^i) \left\{ \delta^{c_1 c_1^i} \delta^{c_2 \bar{c}_2^j} r_1(\theta) + \right. \\ &+ \left. \delta^{c_1 \bar{c}_2^i} \delta^{c_2 c_1^j} r_2(\theta) \right\} \end{aligned} \quad (22)$$

From the above equations we obtain both relations among $t_1(\theta)$, $t_2(\theta)$ and $r_1(\theta)$, $r_2(\theta)$ which read

$$t_2(\theta) = - \frac{i\pi}{i\pi - \theta} t_1(\theta) \quad (23a)$$

$$\text{and } r_1(\theta) = r_2(\theta) = 0 \quad (23b)$$

Finally we see that the relations (21) and (23) correspond to those of class II of ref. (9).

III. BOUND STATES S-MATRIX FOR CP¹ MODEL

Now it turns out that pure CP¹ and the model coupled to fermions have the same factorization equations. We claim that the difference between both models lies on the bound state spectrum. For the model with fermions there is no bound state pole⁽⁴⁾, consequently the S-matrix for the partons is the

complete one. On the other hand the pure model should have one bound state pole, in order that quantum mechanically, the equivalence between CP¹ and O(3) non linear sigma models holds⁽¹⁰⁾

We define bound state as^{(10), (11)}:

$$|\pi_{\alpha_1\alpha_2}^a \frac{(\theta_1+\theta_2)}{2}\rangle = \frac{1}{2} \left\{ |\alpha_1(\theta_1)\bar{\alpha}_2(\theta_2)\rangle - |\bar{\alpha}_2(\theta_1)\alpha_1(\theta_2)\rangle \right\} \lambda_{\alpha_1\alpha_2}^a \quad (24)$$

where $\lambda_{\alpha_1\alpha_2}^a$ are the Pauli matrices.

We suppose that a bound-state is defined with a difference in the rapidity variables given by a constant α , which characterizes the bound-state pole⁽¹¹⁾:

$$\theta_1 - \theta_2 = i\pi\alpha$$

The bound state S-matrix is defined as

$$\begin{aligned} 4\langle \gamma_1' \bar{\gamma}_2' \delta_1' \bar{\delta}_2' | \alpha_1 \bar{\alpha}_2 \beta_1 \beta_2 \rangle &= \sigma_1 \delta_{\alpha_1 \beta_2} \delta_{\alpha_2 \beta_1} \delta_{\gamma_1' \delta_2'} \delta_{\gamma_2' \delta_1'} + \sigma_2 \delta_{\alpha_1 \gamma_1} \delta_{\alpha_2 \beta_1} \delta_{\beta_2 \delta_2} \delta_{\gamma_2' \delta_1'} \\ &+ \sigma_3 \delta_{\alpha_1 \beta_2} \delta_{\alpha_2 \gamma_2} \delta_{\beta_1 \delta_1} \delta_{\gamma_1' \delta_2'} + \sigma_4 \delta_{\alpha_1 \delta_1} \delta_{\alpha_2 \beta_1} \delta_{\beta_2 \gamma_2} \delta_{\gamma_1' \delta_2'} + \sigma_5 \delta_{\alpha_1 \beta_2} \delta_{\alpha_2 \delta_2} \delta_{\beta_1 \gamma_1} \delta_{\gamma_2' \delta_1'} + \\ &+ \sigma_6 \delta_{\alpha_1 \gamma_1} \delta_{\alpha_2 \gamma_2} \delta_{\beta_1 \delta_1} \delta_{\beta_2 \delta_2} + \sigma_7 \delta_{\alpha_1 \delta_1} \delta_{\alpha_2 \delta_2} \delta_{\beta_1 \gamma_1} \delta_{\beta_2 \gamma_2} + \sigma_8 \delta_{\alpha_1 \gamma_1} \delta_{\alpha_2 \delta_2} \delta_{\beta_1 \delta_1} \delta_{\beta_2 \gamma_2} + \\ &+ \sigma_9 \delta_{\alpha_1 \delta_1} \delta_{\alpha_2 \gamma_2} \delta_{\beta_1 \gamma_1} \delta_{\beta_2 \delta_2} \end{aligned} \quad (25)$$

and the action of the non-local charge Q^{ij} on the bound states is well known.

$$Q^{ij} |\pi_{\alpha_1\alpha_2}^a \pi_{\beta_1\beta_2}^b \rangle_{in} = \left(M_m^{ij} \right)_{\alpha_1' \alpha_2' \beta_1' \beta_2'} \pi_{\alpha_1' \alpha_2'}^a \pi_{\beta_1' \beta_2'}^b \rangle_{in} \quad (26a)$$

$$\langle out | \pi_{\gamma_1 \gamma_2}^a \pi_{\delta_1 \delta_2}^b | Q^{ij} = \langle out | \pi_{\gamma_1 \gamma_2}^a \pi_{\delta_1 \delta_2}^b \left(M_{out}^{ij} \right)_{\gamma_1 \gamma_2 \delta_1 \delta_2} \gamma_1' \gamma_2' \delta_1' \delta_2' \quad (26b)$$

where

$$\begin{aligned} \left(M_{in}^{ij} \right)_{\alpha_1' \alpha_2' \beta_1' \beta_2'} \pi_{\alpha_1 \alpha_2}^a \pi_{\beta_1 \beta_2}^b &= \frac{1}{2} (\delta^{i\alpha_2'} \delta^{j\beta_1'} \delta^{\alpha_2 \beta_1} - \delta^{i\beta_1'} \delta^{j\alpha_2'} \delta^{\alpha_2 \beta_1}) \delta^{\alpha_1 \alpha_1'} \delta^{\beta_2 \beta_2'} + \\ &+ \frac{1}{2} (\delta^{i\beta_2'} \delta^{j\alpha_2'} \delta^{\beta_2 \alpha_2'} - \delta^{i\alpha_2'} \delta^{j\beta_2'} \delta^{\alpha_2 \beta_2'}) \delta^{\alpha_1 \alpha_1'} \delta^{\beta_1 \beta_1'} + \\ &+ \frac{1}{2} (\delta^{i\beta_1'} \delta^{j\alpha_1'} \delta^{\alpha_1 \beta_1'} - \delta^{i\alpha_1'} \delta^{j\beta_1'} \delta^{\alpha_1 \beta_1'}) \delta^{\alpha_2 \alpha_2'} \delta^{\beta_2 \beta_2'} + \\ &+ \frac{1}{2} (\delta^{i\alpha_1'} \delta^{j\beta_2'} \delta^{\alpha_1 \beta_2'} - \delta^{i\beta_2'} \delta^{j\alpha_1'} \delta^{\alpha_1 \beta_2'}) \delta^{\alpha_2 \alpha_2'} \delta^{\beta_1 \beta_1'} + \\ &+ \alpha \delta^{ij} \delta^{\alpha_1 \alpha_1'} \delta^{\alpha_2 \alpha_2'} \delta^{\beta_1 \beta_1'} \delta^{\beta_2 \beta_2'} - \phi_1 \delta^{i\alpha_1'} \delta^{j\alpha_2'} \delta^{\alpha_2 \alpha_2'} \delta^{\beta_1 \beta_1'} \delta^{\beta_2 \beta_2'} + \\ &+ \phi_2 \delta^{i\alpha_2'} \delta^{j\alpha_2'} \delta^{\alpha_1 \alpha_1'} \delta^{\beta_1 \beta_1'} \delta^{\beta_2 \beta_2'} - \phi_3 \delta^{i\beta_1'} \delta^{j\beta_1'} \delta^{\alpha_1 \alpha_1'} \delta^{\alpha_2 \alpha_2'} \delta^{\beta_2 \beta_2'} + \\ &+ \phi_4 \delta^{i\beta_2'} \delta^{j\beta_2'} \delta^{\alpha_1 \alpha_1'} \delta^{\alpha_2 \alpha_2'} \delta^{\beta_1 \beta_1'} \end{aligned} \quad (27a)$$

$$\text{with } \phi_i = \frac{\theta_i}{i\pi}$$

and

$$\begin{aligned} \left(M_{out}^{ij} \right)_{\gamma_1 \gamma_2 \delta_1 \delta_2} \gamma_1' \gamma_2' \delta_1' \delta_2' &= -\frac{1}{2} (\delta^{i\delta_1'} \delta^{j\gamma_1'} \delta^{\gamma_1 \delta_1} - \delta^{i\gamma_1'} \delta^{j\delta_1'} \delta^{\gamma_1 \delta_1}) \delta^{\gamma_2 \gamma_2'} \delta^{\delta_2 \delta_2'} - \\ &- \frac{1}{2} (\delta^{i\gamma_1'} \delta^{j\delta_2'} \delta^{\gamma_1 \delta_2} - \delta^{i\delta_2'} \delta^{j\gamma_1'} \delta^{\gamma_1 \delta_2'}) \delta^{\gamma_2 \gamma_2'} \delta^{\delta_1 \delta_1'} - \\ &- \frac{1}{2} (\delta^{i\gamma_2'} \delta^{j\delta_1'} \delta^{\gamma_2 \delta_1} - \delta^{i\delta_1'} \delta^{j\gamma_2'} \delta^{\gamma_2 \delta_1}) \delta^{\gamma_1 \gamma_1'} \delta^{\delta_2 \delta_2'} - \\ &- \frac{1}{2} (\delta^{i\delta_2'} \delta^{j\gamma_2'} \delta^{\gamma_2 \delta_2} - \delta^{i\gamma_2'} \delta^{j\delta_2'} \delta^{\gamma_2 \delta_2}) \delta^{\gamma_1 \gamma_1'} \delta^{\delta_1 \delta_1'} + \\ &+ \alpha \delta^{ij} \delta^{\gamma_1 \gamma_1'} \delta^{\gamma_2 \gamma_2'} \delta^{\delta_1 \delta_1'} \delta^{\delta_2 \delta_2'} - \phi_1 \delta^{i\gamma_1'} \delta^{j\gamma_2'} \delta^{\gamma_2 \gamma_2'} \delta^{\delta_1 \delta_1'} \delta^{\delta_2 \delta_2'} + \end{aligned}$$

$$\begin{aligned}
 & + \phi_2 \delta^{i\gamma_2} \delta^{j\gamma_1} \delta^{i_1} \delta^{j_1} \delta^{i_2} \delta^{j_2} - \phi_3 \delta^{i\delta_1} \delta^{j\delta_1} \delta^{i_1} \delta^{j_1} \delta^{i_2} \delta^{j_2} + \\
 & + \phi_4 \delta^{i\delta_2} \delta^{j\delta_2} \delta^{i_1} \delta^{j_1} \delta^{i_2} \delta^{j_2} \delta^{i_1} \delta^{j_1} \quad (27b)
 \end{aligned}$$

The non-local conservation law

$$\begin{aligned}
 & \left(M_{out}^{ij} \right)_{\gamma_1 \gamma_2 \delta_1 \delta_2 \gamma_1' \gamma_2' \delta_1' \delta_2'} \langle \gamma_1' \bar{\gamma}_2' \delta_1' \bar{\delta}_2' | \alpha_1 \bar{\alpha}_2 \beta_1 \bar{\beta}_2 \rangle \equiv \\
 & \equiv \langle \gamma_1 \bar{\gamma}_2 \delta_1 \bar{\delta}_2 | \alpha_1' \bar{\alpha}_2' \beta_1' \bar{\beta}_2' \rangle \left(M_{in}^{ij} \right)_{\alpha_1' \alpha_2' \beta_1' \beta_2' \alpha_1 \alpha_2 \beta_1 \beta_2} \quad (28)
 \end{aligned}$$

implies the following equations

$$\sigma_1 = \frac{\sigma_2}{\phi + \alpha - 1} \quad (29a)$$

$$\sigma_2 = \frac{\sigma_6}{\phi - 1 - \alpha} \quad (29b)$$

$$\sigma_3 = \frac{\sigma_6}{\phi + \alpha - 1} \quad (29c)$$

$$\sigma_4 = \sigma_5 = 0 \quad (29d)$$

$$\sigma_7 = \frac{\sigma_6}{\phi^2} \quad (29e)$$

$$\sigma_8 = \sigma_9 = -\frac{\sigma_6}{\phi} \quad (29f)$$

where $\phi = \frac{\phi_1 + \phi_2}{2} - \frac{\phi_3 + \phi_4}{2}$

We have then as solution for the S-matrix

$$\begin{aligned}
 i_j S_{k\ell} &= \left[\frac{2\phi(\phi-1) + (1-\alpha^2)}{\phi(\phi-\alpha-1)(\phi+\alpha-1)} \right] \sigma_6 \delta_{ij} \delta_{k\ell} + \\
 &+ \left[\frac{\phi(\phi-1)^2 + (\phi-1)(1-\alpha^2)}{\phi(\phi-\alpha-1)(\phi+\alpha-1)} \right] \sigma_6 \delta_{ik} \delta_{j\ell} - \\
 &- \left[\frac{2\phi(\phi-1)^2 + (\phi-1)(1-\alpha^2)}{\phi^2(\phi-\alpha-1)(\phi+\alpha-1)} \right] \sigma_6 \delta_{i\ell} \delta_{jk} \quad (30)
 \end{aligned}$$

Now we see that we must fix α in order to satisfy crossing and for $\alpha=1$ the final S-matrix becomes that one from the 0(3) non-linear sigma model⁽¹⁰⁾⁽¹¹⁾.

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