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publicações

IFUSP/P-368

NEW FORMULA FOR PROTON-NEUTRON MASS DIFFERENCE

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Outubro/1982

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ABSTRACT

A new formula is presented, allowing the computation of the proton-neutron mass difference without the knowledge of the substraction function in the dispersion integral. The Born terms contribute with the correct sign. The integral over the deep-inelastic region is finite whenever the Weinberg-'t Hooft mechanism is operative and gives a small value. The net result is in very good agreement with the experimental value.

I. INTRODUCTION

The proton-neutron mass difference, $\Delta = M_p - M_n$, is one of those subjects that remain latent most of the time while surfacing to the literature every so often whenever a development in a related area revives the hope that the problem might, finally, be solved[1] .

The simplest way to tackle the problem is to assume that the mass difference is a low energy effect and try to compute it in a Born approximation. Feynman and Speisman[2] were the first to notice that the Born approximation (in a Feynman diagramatic sense) could, in principle, even yield the right (negative) sign.

Later, it was realized that the Born approximation leads to a situation where the proton is about 1 MeV heavier than the neutron[3,1]. When calculating Feynman diagrams, however, different parametrizations of the nucleon electromagnetic vertex, lead to different results when taken off-mass-shell. Thus, even when some possible ways of writing the Born approximation might lead to the right sign[4] , the result is at best ambiguous.

A new formulation of the Δ problem started with Cottingham's work[5] . Also in this formulation the simplest thing to do is to keep the Born term (now in a dispersion relation sense) only. But here again, there are ambiguities. One can talk of an unsubstracted Born term as in Cottingham's original work or a substracted Born term as in, for example, Elitzur and Harati's paper[6] . And both things are quite different. In any case, it is clear after Harari's work[7] that there must be a substraction function in the dispersion relations and in spite of some interesting speculations[6] there is no reliable way to estimate what that substraction should be.

The main motivation for Cottingham's work was to relate the high energy contribution to Δ, to the deep inelastic phenomenology. Unfortunately, that contribution seems to be divergent. This divergence discouraged many people from trying to solve the Δ puzzle. Besides, with the advent of the quark model, it became less clear that Δ should be calculable as an electromagnetic effect since (intrinsic) mass differences between the quarks might be responsible for Δ.

A justification for believing that mass differences within a multiplet should be calculable as radiative effects was provided by the work of 't Hooft[8] and Weinberg[9]. As these authors pointed out, in a unified renormalizable theory the symmetry might be broken spontaneously in such a way as to keep the masses equal in zero order. They went on to show that then, in second order radiative correction the exchanges of the various gauge vectors are related in such a way as to make the whole contribution finite. Unfortunately, elaborations on Weinberg's ideas in the framework of pure weak-electromagnetic models[10,11] have shown that, more likely than not, Δ would come out with the wrong sign. Thus, more realistic schemes incorporating strong interactions have to be considered.

In the present work I incorporate the 't Hooft-Weinberg mechanism in its simplest form into a phenomenological framework where the strong interaction effects enter in the guise of form factors and structure functions. The calculations are based on a new representation for Δ that makes unnecessary the knowledge of the (unknown) substraction function in the dispersion relations for the off-shell Compton amplitudes[12]. Thus, Δ is completely calculable and a quite satisfactory result is obtained.

In section II, I comment on the ambiguity in defining what might be called the Born approximation to the Cottingham formula for Δ. This ambiguity is one of the unpleasant features of the usual formalism. Another unpleasant feature, which is discussed in section III, is the divergence of the deep-inelastic contribution. This divergence does not appear in models exhibiting the Weinberg-'t Hooft mechanism which is discussed in section IV.

The new formula for Δ is presented in section V where the (now unambiguous) contribution of the Born term is also calculated. The contribution from the deep-inelastic region to the new formula is studied in section VI while section VII contains some last remarks.

II. COTTINGHAM FORMULA AND BORN APPROXIMATION

To second order in the coupling e, the nucleon electromagnetic self mass can be written as[13]

$$\delta M = \frac{ie^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{T(|\vec{k}|,k^0)}{k^2+i\varepsilon} , \qquad (2.1)$$

where $T = g^{\mu\nu}T_{\mu\nu}$ and $T_{\mu\nu}$ is the forward Compton amplitude (for an off-mass-shell photon) defined by

$$T_{\mu\nu}(|\vec{k}|,k^0) = i \int d^4x \; e^{ikx} \langle p|T\{j_\mu(x),j_\nu(0)\}|p\rangle$$

$$= -\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) T_1 + \frac{1}{M^2}\left(p_\mu - \frac{M\nu}{k^2} k_\mu\right)\left(p_\nu - \frac{M\nu}{k^2} k_\nu\right) T_2$$

$$= \frac{\pi}{\alpha}\left\{(k^2 g_{\mu\nu} - k_\mu k_\nu)t_1 + \left[-\nu^2 g_{\mu\nu} - \frac{k^2}{M^2} p_\mu p_\nu + \frac{\nu}{M}(p_\mu k_\nu + p_\nu k_\mu)\right] t_2\right\} , \qquad (2.2)$$

where $p_\mu(k_\mu)$ is the nucleon (photon) momentum, M the nucleon mass and $\nu = pk/M$. An average over nucleon spins is understood in Eq. (2.2) and I have explicitly written the two most usual parametrizations of $T_{\mu\nu}$: in terms of the T_i which are simply related to electron-nucleon scattering structure functions by $\mathrm{Im}\, T_i = \pi W_i$; and in terms of the t_i which are free of kinematical singularities.

As we can see from Eq. (2.1) the self mass depends on the combination

$$T(|\vec{k}|,k^O) = (1-\nu^2/k^2)T_2(|\vec{k}|,k^O) - 3\ T_1(|\vec{k}|,k^O)$$

$$= \frac{\pi}{\alpha}\left\{3k^2\ t_1(|\vec{k}|,k^O) - (2\nu^2+k^2)t_2(|\vec{k}|,k^O)\right\} \quad . \qquad (2.3)$$

From Eq. (2.1) we can get to the Cottingham representation by performing a Wick rotation in the k_o plane $(k_o \to ik_4)$, which in the rest frame of the in (or out) going nucleon is equivalent to $\nu \to i\nu_I$. The rotated counterpart of Eq. (2.1) is

$$\delta M = \frac{\alpha}{(2\pi)^3}\int \frac{d^4k_E}{Q^2}\ T(-Q^2,i\nu_I) \quad , \qquad (2.4)$$

with $Q = \sqrt{-k^2}$. This equation leads, after an angular integration, to

$$\delta M = \frac{-1}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q d\nu_I\sqrt{Q^2-\nu_I^2}\left\{3Q^2\ t_1(-Q^2,i\nu_I) - (Q^2+2\nu_I^2)t_2(-Q^2,i\nu_I)\right\} \quad . \qquad (2.5)$$

The next step taken by Cottingham was to write (unsubstracted) dispersion relations for the t_i

$$t_1(-Q^2,i\nu_I) = \frac{2\alpha}{\pi Q^2}\int_{Q^2/2M}^{\infty} \frac{\nu\ d\nu}{\nu^2+\nu_I^2}\left(W_1 - \frac{\nu^2}{Q^2}W_2\right) \quad , \qquad (2.6)$$

$$t_2(-Q^2,i\nu_I) = \frac{2\alpha}{\pi Q^2}\int_{Q^2/2M}^{\infty} \frac{d\nu\ \nu\ W_2(-Q^2,\nu)}{\nu^2+\nu_I^2} \quad . \qquad (2.7)$$

For elastic scattering, the structure functions are

$$W_1^{e\ell}(-Q^2,\nu) = (Q^2/4M^2)G_M^2(Q^2)\,\delta(\nu-Q^2/2M) \quad , \qquad (2.8)$$

$$W_2^{e\ell}(-Q^2,\nu) = \frac{G_E^2(Q^2) + (Q^2/4M^2)G_M(Q^2)}{1+Q^2/4M^2}\,\delta(\nu-Q^2/2M) \quad , \qquad (2.9)$$

leading to the Born terms

$$t_1^B(-Q^2,i\nu_I) = \frac{4M\alpha Q^2\left[G_M^2(Q^2) - G_E^2(Q^2)\right]}{\pi(Q^4+4M^2\nu_I^2)\ (Q^2+4M^2)} \quad , \qquad (2.10)$$

$$t_2^B(-Q^2,i\nu_I) = \frac{4M\alpha\left[4M^2G_E^2(Q^2) + Q^2G_M^2(Q^2)\right]}{\pi(Q^4+4M^2\nu^2)(Q^2+4M^2)} \quad . \qquad (2.11)$$

With these, the elastic contribution to the contracted Compton amplitude turns out to be

$$T^B(-Q^2,i\nu_I) = 4M\left\{\frac{(2\nu_I^2+Q^2)\left[4M^2G_E^2(Q^2) + Q^2G_M^2(Q^2)\right] - 3Q^4(G_M^2 - G_E^2)}{(Q^4+4M^2\nu_I^2)\ (Q^2+4M^2)}\right\} \quad , \qquad (2.12)$$

which, when inserted into Eq. (2.5) yields the Born approximation to the mass shift[1]

$$\delta M^B = -\frac{2\alpha M}{\pi^2}\int_0^\infty dQ^2 \int_0^Q \frac{\sqrt{Q^2-\nu_I^2}\; d\nu_I}{(Q^4+4M^2\nu_I^2)(Q^2+4M^2)}\left\{3Q^2(G_M^2-G_E^2)\right.$$

$$\left. - (1+2\nu_I^2/Q^2)(4M^2G_E^2+Q^2G_M^2)\right\} \quad . \qquad (2.13)$$

Actually, the situation is not that clear cut and other "Born approximations" can be found in the literature. This freedom (or ambiguity) in choosing a "Born approximation" is mainly due to two facts: first, one can either write dispersion relations for the ti , or for the Ti , or still for the contracted T . And second, as originally pointed out by Harari[7] , most probably Eq. (2.6) does not converge and a substracted dispersion relation has to be used instead,

$$\bar{t}_1(-Q^2,i\nu_I) = t_1(-Q^2,0) - \frac{2\alpha\,\nu_I^2}{\pi Q^2}\int_{Q^2/2M}^\infty \frac{d\nu}{(\nu^2+\nu_I^2)}\left[\frac{W_1}{\nu} - \frac{\nu\, W_2}{Q^2}\right] \quad , \qquad (2.14)$$

which contains the substracted Born term

$$\bar{t}_1^B = -\frac{8\alpha M^3\nu_I^2\left[G_M^2(Q^2) - G_E^2(Q^2)\right]}{\pi Q^2(Q^4+4M^2\nu_I^2)(Q^2+4M^2)} \quad . \qquad (2.15)$$

This, together with Eq. (2.11), leads to a T^B different from Eq. (2.12), namely to the substracted

$$\bar{T}^B = 4M\left\{\frac{(2\nu_I^2+Q^2)\left[4M^2G_E^2(Q^2) + Q^2G_M^2(Q^2)\right] + 12M^2\nu_I^2(G_M^2-G_E^2)}{(Q^4+4M^2\nu_I^2)(Q^2+4M^2)}\right\} \quad , \qquad (2.16)$$

whose contribution to Eq. (2.5) defines the substracted Born

approximation

$$\delta\bar{M}^B = \frac{2\alpha M}{\pi^2}\int_0^\infty dQ^2 \int_0^Q \frac{d\nu_I\,\sqrt{Q^2-\nu_I^2}}{(Q^4+4M^2\nu_I^2)(Q^2+4M^2)}\left\{\frac{12M^2\nu_I}{Q^2}\left[G_M^2(Q^2) - G_E^2(Q^2)\right]\right.$$

$$\left. + (1+2\nu_I^2/Q^2)\left[4M^2G_E^2(Q^2) + Q^2G_M^2(Q^2)\right]\right\} \quad . \qquad (2.17)$$

It should be obvious that the "Born approximations" (2.13) and (2.17) will give very different results and it is not clear which one, if either, should be favored. Still other possibilities were considered like, for instance, the use of an unsubstracted dispersion relation for T[15] leading to

$$T^{B'}(-Q^2,i\nu_I) = \frac{4MQ^2\left[G_E^2(Q^2) - (Q^2/2M^2)G_M^2(Q^2)\right]}{Q^4 + 4M^2\nu_I^2} \quad , \qquad (2.18)$$

and to a third "Born approximation"

$$\delta M^{B'} = \frac{2\alpha M}{\pi^2}\int_0^\infty dQ^2 \int_0^Q \frac{d\nu_I\,\sqrt{Q^2-\nu_I^2}}{Q^4 + 4M^2\nu_I^2}\left[G_E^2(Q^2) - \frac{Q^2}{2M^2}G_M^2(Q^2)\right] \qquad (2.19)$$

The main purpose of the present section was to emphasize that there is no way of knowing if the "Born term" contribution provides by itself a good approximation to Cottingham formula since there is an arbitrariness in the choice of that Born term.

Whatever its form the Born term is, of course, the contribution from the nucleon pole. It can be shown that the contribution from the resonances, besides being much smaller[1] than the Born term is also subject to the same kind of arbitrariness.

III. HIGH ENERGY REGION AND DIVERGENCES

The main ingredient in the calculation of mass shifts is, as we saw, the off-mass-shell forward Compton scattering amplitude. This amplitude can be naturally divided into two parts: the first part describing the coherent scattering of the nucleon and the second part the incoherent scattering of the constituents (partons, quarks, etc.) inside the nucleon. The coherent scattering contains the contributions from the resonances and the nucleon pole and was discussed in the last section.

The incoherent scattering starts to be important at some Q_o and is directly related to the deep inelastic phenomenology according to

$$\text{Im } T_i^{\text{Incoherent}} = \pi\, W_i^{\text{deep Inelastic}} \quad . \tag{3.1}$$

In the present section we will always have $Q \geq Q_o$ in which case the whole Compton amplitude is practically incoherent since the coherent part is negligible above a suitable Q_o. The incoherent scattering contribution to the mass shift will be called δM^{In}.

In order to obtain the dominant part of δM^{In} it is convenient to write the t_i's with one more substraction each according to

$$t_1(-Q^2,i\nu_I) = t_1(-Q^2,0) + \nu_I^2\left[\frac{\partial t_1(-Q^2,i\nu_I)}{\partial \nu_I^2}\right]_{\nu_I^2=0}$$

$$- \frac{2\alpha\ \nu_I^2}{\pi Q^2}\int_{Q^2/2M}^{\infty}\frac{d\nu}{(\nu^2+\nu_I^2)}\left[\frac{W_1}{\nu} - \frac{\nu\ W_2}{Q^2}\right] \quad , \tag{3.2}$$

$$t_2(-Q^2,i\nu_I) = t_2(-Q^2,0) - \frac{2\alpha\ \nu_I^2}{\pi Q^2}\int_{Q^2/2M}^{\infty}\frac{d\nu\ \nu\ W_2}{\nu^2(\nu^2+\nu_I^2)} \quad . \tag{3.3}$$

The new substraction term in Eq. (3.2) can be easily obtained by taking the derivative of Eq. (2.14) with respect to ν_I^2,

$$\left(\frac{\partial t_1}{\partial \nu_I^2}\right)_{\nu_I^2=0} = \frac{2\alpha}{\pi Q^2}\int_{Q^2/2M}^{\infty}\frac{d\nu}{\nu^2}\left[\frac{W_1}{\nu} - \frac{\nu\ W_2}{Q^2}\right]$$

$$= \frac{4M\alpha}{\pi Q^6}\int_0^1 dx\ \left[2x\ F_1(Q^2,x) - F_2(Q^2,x)\right]$$

$$= \frac{4M\alpha}{\pi Q^6}\left[2\ \mathcal{M}_1(3,Q^2) - \mathcal{M}_2(2,Q^2)\right] \quad , \tag{3.4}$$

where, as usual, $x = Q^2/2M\nu$, $F_1 = MW_1$, $F_2 = \nu W_2$ and $\mathcal{M}_i(j,Q^2)$ is the j Cornwall-Norton moment of F_i[16].

The substraction term in Eq. (3.3) is obtained from Eq. (2.7) as

$$t_2(-Q^2,0) = \frac{4M\alpha}{\pi Q^4}\int_0^1 dx\ F_2(Q^2,x) = \frac{4M\alpha}{\pi Q^4}\mathcal{M}_2(2,Q^2) \quad . \tag{3.5}$$

Thus, the incoherent contribution to T can be written, in its full glory, as

$$T^{In} = (2\nu_I^2+Q^2)\left[\frac{4M}{Q^4}\ \mathcal{M}_2(2,Q^2) - \frac{2\nu_I^2}{Q^2}\int_{Q^2/2M}^{\infty}\frac{d\nu\ \nu\ W_2}{\nu^2(\nu^2+\nu_I^2)}\right] -$$

$$- \frac{3\pi Q^2}{\alpha}\ t_1(-Q^2,0) - \frac{12M\nu_I^2}{Q^4}\left[2\ \mathcal{M}_1(3,Q^2) - \mathcal{M}_2(2,Q^2)\right] +$$

$$+ \frac{6\nu_I^2}{\pi}\int_{Q^2/2M}^{\infty} \frac{d\nu}{(\nu^2+\nu_I^2)\,\nu^2}\left(\frac{W_1}{\nu} - \frac{\nu\, W_2}{Q^2}\right) \tag{3.6}$$

δM^{In} is obtained by inserting Eq. (3.6) into Eq. (2.5). It is easy to see that the substraction terms in Eq. (3.6) give the dominant contribution which is

$$\delta M^{In} = \frac{2\alpha M}{\pi^2}\int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^4}\int_0^Q d\nu_I \sqrt{Q^2-\nu_I^2}\,\Big\{3(\nu_I^2/Q^2)\left[\mathcal{M}_2(2,Q^2) - 2\,\mathcal{M}_1(3,Q^2)\right]$$

$$+ (1+2\nu_I^2/Q^2)\,\mathcal{M}_2(2,Q^2) - 3Q^4 t_1(-Q^2,0)\Big\} \quad . \tag{3.7}$$

Once the ν_I integration is performed this expression reduces to

$$\delta M^{In} = \frac{3\alpha M}{8\pi}\int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2}\left\{3\,\mathcal{M}_2(2,Q^2) - 2\,\mathcal{M}_1(3,Q^2) - \frac{16}{\pi}\,Q^4 t_1(-Q^2,0)\right\} . \tag{3.8}$$

In spite of some interesting work[6] , no reliable way for estimating $t_1(-Q^2,0)$ is known. Besides that, the terms in Eq. (3.8) containing the moments are logarithmically divergent. The divergent contribution to δM can be found in the literature[17] in a somewhat different form obtained from dispersion relations for the T_i instead of the t_i . Both forms though, reduce to the same expression when the Callan-Gross[18] relation $2xF_1 = F_2$ is used; namely to

$$\delta M^{In} = \frac{3\alpha M}{4\pi}\int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2}\left[\mathcal{M}_2(2,Q^2) - \frac{8}{\pi}\,Q^4 t_1(-Q^2,0)\right] \quad . \tag{3.9}$$

A possible solution to the divergence problem will be considered in the next section.

IV. WEINBERG-'T HOOFT MECHANISM

From what we have seen up to now the situation with the mass shift can be summarized as follows: i) There is no unambiguous way of defining the "Born approximation" which contains most of the coherent contribution to δM . ii) There is a completely unknown contribution depending on the substraction function $t_1(-Q^2,0)$. iii) The part of the incoherent scattering that is known seems to give divergent contributions to individual mass shifts, and to the nucleon mass difference Δ .

In order to overcome the divergence just mentioned I will invoke the interesting Weinberg-'t Hooft mechanism for generating finite mass differences within a isotopic multiplet, in the framework of renormalizable unified gauge theories. In the type of models proposed by these authors[8,9] , the spontaneous symmetry breaking is such that it leaves the masses equal in zero order radiative correction. This can be acomplished, for instance, by the absence of Yukawa coupling to the scalars that break the symmetry. In such a case mass differences are computable as second order effects in the gauge couplings.

Since the models are renormalizable due to their group representation content, the couplings of the various gauge bosons in the radiative corrections arrange themselves in such a way as to make these corrections finite. In the simplest model considered, that of $SU(2) \times U(1)$ for example, instead of having just the photon propagator in δM as in Eq. (2.1), the heavy neutral gauge bozon z radiative contribution combines with the photon contribution in such a way as to produce, inside the self mass expressions, the change

$$\frac{1}{k^2+i\epsilon} \longrightarrow \frac{1}{k^2+i\epsilon} - \frac{1}{k^2-m_z^2+i\epsilon} \quad , \tag{4.1}$$

where m_z is the mass of the heavy neutral gauge boson z that will be taken as m_z = 90 GeV. In more complicated (or realistic) models more propagators appear[10,11] but we can assume that the net effect is as in Eq. (4.1) where m_z is certain effective mass.

In the following I will assume that something like the Weinberg-'t Hooft mechanism is operative in nature. Also from now on, all mass shifts I will write down contain radiative effects only (no zero order contributions which drop out in mass differences anyway). In these mass shift expressions I will accordingly introduce the change (4.1) which, for instance, transforms Eq. (2.1) into

$$\delta M = \frac{\alpha}{(2\pi)^3} \int \frac{d^4k_E \; T(-Q^2,\, i\nu_I)}{Q^2(1+Q^2/m_z^2)} \quad . \tag{4.2}$$

Eq. (4.2) leads to expressions which are no longer divergent at high momentum. The problem of the unknown substraction function, however, still remains to be solved and will be tackled next.

V. NEW REPRESENTATION FOR MASS SHIFTS

In this section I will derive a formula for the mass shift in which the substraction function $t_1(-Q^2,0)$ of Eq. (2.14) does not enter. The derivation will be based on a generalization of Cottingham's rotation. I will consider first an expression like Eq. (2.1), obtain from it the new representation and afterwards, discuss the required modifications whenever the Weinberg-'t Hooft mechanism is in operation.

For the mass shift formula (2.1) rewritten as

$$\delta M = \frac{i\alpha}{(2\pi)^3} \int \frac{d^3k \; d\nu \; T(K,\nu)}{\nu^2-K^2+i\epsilon} \tag{5.1}$$

where $K = |\vec{k}|$, Cottingham's original proposal was to rotate the ν integration path from the real to the imaginary axis in the ν complex plane in accordance to $\nu \to \nu\, e^{i\pi/2}$. Given the necessary conditions for the validity of that rotation [6] , other choices are possible. Let us, for instance, consider the integration along a straight line at an angle $(\pi/2)-\beta$ with the real axis. In the rotation involved in going from the real axis to that path, the integration variable changes according to $\nu \to \nu\, e^{i(\pi/2-\beta)}$ and, whenever $\beta \ll 1$, Eq. (5.1) changes into

$$\delta M = \frac{\alpha}{(2\pi)^3} \int \frac{d^3k(1-i\beta)d\nu \; T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2+K^2]} \quad . \tag{5.2}$$

Since the result has to be independent of β (as long as $\beta \ll 1$), we set the derivative with respect to β equal to zero and obtain

$$\frac{\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu\ T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2+K^2]} - \frac{2\alpha}{(2\pi)^3}\int \frac{d^3k(1-i\beta)d\nu\ \nu^2\ T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2+K^2]^2}$$

$$+\frac{\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu\ \nu}{[(1-2i\beta)\nu^2+K^2]}\ \frac{\partial\ T[K,(i+\beta)\nu]}{\partial\nu} = 0 \qquad (5.3)$$

The $\beta\to 0$ limit of this expression yields for the mass shift

$$\delta M = \frac{\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu_I\ T(K,i\nu_I)}{(\nu_I^2+K^2)} = \frac{\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu_I\ \nu_I}{(\nu_I^2+K^2)}\ \frac{\partial\ T(K,i\nu_I)}{\partial\nu_I}$$

$$+\frac{2\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu_I\ K^2\ T(K,i\nu_I)}{(\nu_I^2+K^2)^2}\ , \qquad (5.4)$$

where, as in Sect. II-VI, $i\nu_I$ is used whenever ν is taken on the imaginary axis.

The procedure just described can be applied to the integrals

$$I_n = i\int \frac{d^3k\ d\nu(K^2)^n\ T(K,\nu)}{[\nu^2-K^2+i\epsilon]^{n+1}}$$

$$= (-1)^n\int \frac{d^3k(1-i\beta)d\nu\ (K^2)^n\ T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2+K^2]^{n+1}}\ , \qquad (5.5)$$

which should also be independent of β . The vanishing of the derivative of Eq. (5.5) with respect to β followed by the $\beta\to 0$ limit leads to the relation

$$\int \frac{d^3k\ d\nu_I(K^2)^n\ T(K,i\nu_I)}{(\nu_I^2+K^2)^{n+1}} = \frac{1}{(2n+1)}\int \frac{d^3k\ d\nu_I(K^2)^n\ \nu_I}{[\nu_I^2+K^2]^{n+1}}\ \frac{\partial T(K,i\nu_I)}{\partial\ \nu_I} +$$

$$+\frac{2(n+1)}{(2n+1)}\int \frac{d^3k\ d\nu_I(K^2)^{n+1}\ T(K,i\nu_I)}{(\nu_I^2+K^2)^{n+2}}\ . \qquad (5.6)$$

From Eq. (5.4) and repeated use of Eq. (5.6) it is a simple matter to obtain the expansion

$$\delta M = \frac{2\alpha}{(2\pi)^3}\int \frac{d^3k\ d\nu_I\ \nu_I^2}{(\nu_I^2+K^2)}\ \frac{\partial T(K,i\nu_I)}{\partial\nu_I^2}\Big[1+\frac{2}{3}\ \frac{K^2}{\nu_I^2+K^2}+$$

$$+\frac{2.4}{3.5}\left(\frac{K^2}{\nu_I^2+K^2}\right)^2+\frac{2.4.6}{3.5.7}\left(\frac{K^2}{\nu_I^2+K^2}\right)^2+\ldots\Big]\ , \qquad (5.7)$$

where knowing that T is an even function of ν_I , I have used $\nu_I(\partial T/\partial\nu_I) = 2\nu_I^2(\partial T/\partial\nu_I^2)$. In terms of the Euclidean 4-momentum squared $Q^2 = \nu_I^2+K^2$, Eq. (5.7) can be rewritten, after a trivial angular integration, in the form

$$\delta M = \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q \sqrt{Q^2-\nu_I^2}\ \ \nu_I^2\ d\nu_I\ \frac{\partial T(-Q^2,i\nu_I)}{\partial(\nu_I^2)}$$

$$\times\left\{\sum_{n=0}^{\infty}\frac{n!}{(2n+1)!!}\left[2\left(1-\frac{\nu_I^2}{Q^2}\right)\right]^n\right\}\ . \qquad (5.8)$$

This expression for the mass shift, depending as it is on $\partial T/\partial(\nu_I^2)$ will not contain contributions from unknown, ν_I^2 independent substraction functions like $t_1(-Q^2,0)$ in Eq.(2.14).

What happens now if we start from an expression for δM incorporating the Weinberg-'t Hooft mechanism? In such a case we will have to deal with derivatives like

$$\frac{\partial}{\partial\beta}\frac{1}{\left[(1-2i\beta)\nu^2+K^2\right]\left[1+\frac{(1-2i\beta)\nu^2+K^2}{m_z^2}\right]}\xrightarrow[\beta\to 0]{}$$

$$=\frac{2i\nu^2}{Q^2(1+Q^2/m_z^2)}+\frac{2i\nu^2}{m_z^2(1+Q^2/m_z^2)^2} \quad . \tag{5.9}$$

Since m_z is a very large mass (compared to the nucleon mass, for instance), we can adopt the criterion of neglecting, in the second line of Eq. (5.9), the second term in comparison with the first. In such an approximation, the Weinberg-'t Hooft mechanism modifies Eq. (5.8) into

$$\delta M \approx \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2(1+Q^2/m_z^2)}\int_0^Q \sqrt{Q^2-\nu_I^2}\ \nu_I^2\ d\nu_I\ \frac{\partial T(-Q^2,i\nu_I)}{\partial(\nu_I^2)}$$

$$\times\left\{\sum_{n=0}^{\infty}\frac{n!}{(2n+1)!!}\left[2\left(1-\frac{\nu_I^2}{Q^2}\right)\right]^n\right\} \quad . \tag{5.10}$$

For the Born term as well as other possible resonant contributions the factor $(1+Q^2/m_z^2)$ in the denominator of Eq. (5.10) makes very little difference. On the other hand this factor is necessary for the convergence of the high energy incoherent scattering contribution to δM. The approximations leading to Eq. (5.10) are reasonable but are not essential for the obtention of a convergent formula for δM. Starting from

$$\delta M = \frac{i\alpha}{(2\pi)^3}\int\frac{d^3k\ d\nu\ T(K,\nu)}{(\nu^2-K^2)\left[1-(\nu^2-K^2)/m_z^2\right]}$$

$$=\frac{i\alpha}{(2\pi)^3}\int d^3k\ d\nu\ T(K,\nu)\left[\frac{1}{\nu^2-K^2}-\frac{1}{\nu^2-K^2-m_z^2}\right] \quad ,$$

and following the procedure leading to Eq. (5.8) we arrive now at

$$\delta M = \frac{\alpha}{\pi^2}\int_0^\infty dQ^2\int_0^Q\sqrt{Q^2-\nu_I^2}\ \ \nu_I^2\ d\nu_I\ \frac{\partial T(-Q^2,i\nu_I)}{\partial(\nu_I^2)}$$

$$\left\{\sum_{n=0}^{\infty}\frac{n!2^n}{(2n+1)!!}\left[Q^{-2}(1-\nu_I^2/Q^2)^n-(Q^2+m_z^2)^{-1}(1-\nu_I^2/Q^2+m_z^2)^n\right]\right\} , \tag{5.11}$$

rather more cumbersome than the approximate expression (5.10).

It is now convenient to separate δM according to $\delta M = \delta M^{(1)}+\delta M^{(2)}$ with $\delta M^{(1)}$ ($\delta M^{(2)}$) containing the contribution from coherent (incoherent) scattering. By far, the largest contribution to $\delta M^{(1)}$ comes from the Born term. A nice feature of our new formula is that the unsubstracted Born terms (2.12) and (2.18) as well as the substracted (2.16), when inserted into Eq. (5.10), all yield exactly the same $\delta M^{(1)}$, namely

$$\delta M^{(1)} = -\frac{8\alpha M}{\pi^2}\int_0^\infty dQ^2\int_0^Q\sqrt{Q^2-\nu_I^2}\ \nu_I^2 d\nu_I\ \frac{\left[2M^2G_E^2(Q^2)-Q^2G_M^2(Q^2)\right]}{(1+Q^2/m_z^2)(Q^4+4M^2\nu_I^2)^2} \quad .$$

$$\left\{1+\frac{2}{3}(1-\nu_I^2/Q^2)+\frac{2.4}{3.5}(1-\nu_I^2/Q^2)^2+\frac{2.4.6}{3.5.7}(1-\nu_I^2/Q^2)^3+\dots\right\} \tag{5.12}$$

This, of course, is not what is commonly called the Born approximation to the mass shift; it must also contain part of the contribution of the substraction term to the customary Cottingham representation.

The usual dipole fit for the form factors,

$$G_E(Q^2)/q = G_M(Q^2)/\mu = (1+Q^2/m_o^2)^{-2} \quad , \qquad (5.13)$$

in terms of the nucleon charge q in units of e, its magnetic moment μ and $m_o^2 \approx 0.72\ \mathrm{GeV}^2$, will be employed. With these form factors and calling $\nu_I = yQ$, $Q^2 = 4M^2 t$, Eq. (5.12) turns into

$$\delta M^{(1)} = -\frac{4\alpha M}{\pi^2}\int_0^\infty dt \int_0^1 \frac{dy\ y^2\sqrt{1-y^2}\,[q^2-2\mu^2 t]}{(1+\frac{4M^2}{m_Z^2}t)(t+y^2)^2(1+Rt)^4}$$

$$\times \left\{1+\frac{2}{3}(1-y^2)+\frac{2.4}{3.5}(1-y^2)^2+\frac{2.4.6}{3.5.7}(1-y^2)^3+\dots\right\}, \qquad (5.14)$$

where $R = 4M^2/m_o^2 = 4.90$. Since the integral in Eq. (5.14) decreases quite fast with t, the factor $(1+4M^2t/m_Z^2)$ makes little difference and can be safely supressed.

Keeping only the first term in the expansion of Eq. (5.14) (the 1 in the braces) I was able to perform the integration analytically. So, the result to first order in that expansion is

$$\delta M^{(1)}(1) = -\frac{\alpha M}{12\pi(R-1)^{7/2}}\left\{q^2\left[\frac{3}{2}R(5R^2-6R+16)\left(\frac{\pi}{2}-\tan^{-1}\sqrt{(R-1)^{-1}}\right)\right.\right.$$

$$\left.\left.+\frac{1}{4}(36R^2-8R-33)\right]-\mu^2\left[3(R^2-2R-4)\left(\frac{\pi}{2}-\tan^{-1}\sqrt{(R-1)^{-1}}\right)+(26-3R)\right]\right\}. \qquad (5.15)$$

This part of the radiative shifts contributes to the proton-

neutron mass difference with

$$\Delta^{(1)}(1) = \delta M_p^{(1)}(1) - \delta M_n^{(1)}(1) = -1.35\ \mathrm{MeV} \quad . \qquad (5.16)$$

This is to be compared with a numerical integration with the four terms written explicitly inside the braces of Eq. (5.14) and leading to the result

$$\Delta^{(1)} \approx \Delta^{(1)}(4) = -1.36\ \mathrm{MeV} \quad . \qquad (5.17)$$

This is the value I will take for the Born contribution to the mass difference in the framework of the formalism presented in this paper. This $\Delta^{(1)}$ is to be added to $\Delta^{(2)}$ calculated in the next section.

VI. INCOHERENT SCATTERING CONTRIBUTION

What remains to be computed is $\Delta^{(2)}$ the contribution from incoherent scattering to the mass difference Δ. In order to make this section a little more self contained let me repeat a few things already stated in Sect. II and III where for the incoherent scattering I have used the usual notation $x = Q^2/2M\nu$, $F_1 = MW_1$, $F_2 = \nu W_2$ and the Callan-Gross relation $2xF_1 = F_2$. With all that, the contribution from incoherent scattering to the contracted Compton amplitude is

$$T^{In}(-Q^2, i\nu_I) = (1-\nu_I^2/Q^2)T_2 - 3T_1 =$$

$$= -\frac{3\pi Q^2}{\alpha}\,t_1(-Q^2,0) + 2(1+2\nu_I^2/Q^2)\int_{Q^2/2M}^{\infty}\frac{d\nu\ \nu\ W_2}{\nu^2+\nu_I^2} =$$

$$= -\frac{3\pi Q^2}{\alpha} t_1(-Q^2,0) + 4M(Q^2+2\nu_I^2)\int_0^1 \frac{dx\, F_2(Q^2,x)}{(Q^4+4M^2x^2\nu_I^2)} \quad . \qquad (6.1)$$

What enters into the mass shift equation (5.10), as we can see, is the derivative of Eq. (6.1) with respect to ν_I^2

$$\frac{\partial T^{In}(-Q^2,i\nu_I)}{\partial(\nu_I^2)} = 8MQ^2\int_0^1 \frac{dx\ (Q^2-2M^2x^2)F_2}{(Q^4+4M^2x^2\nu_I^2)^2} \quad . \qquad (6.2)$$

The lower limit of the integration in Q^2 in Eq. (5.10) should be a Q_o^2 where the incoherent scattering starts to be important. Let us make the simplifying choice $Q_o^2 = 4M^2 \simeq 3.5\ GeV^2$ and use the variables $t = Q^2/4M^2$ and $y = \nu_I/Q$ keeping in mind that $1 \leq t \leq \infty$ and $0 \leq y \leq 1$. Then, Eq. (6.2) can be expanded according to

$$\frac{\partial T^{In}(-Q^2,i\nu_I)}{\partial(\nu_I^2)} = \frac{1}{4M^3t^3}\int_0^1 \frac{dx(2t-x^2)F_2}{(1+x^2y^2/t)^2}$$

$$= \frac{1}{4M^3t^3}\int_0^1 dx\ F_2(2t-x^2)\left\{1-(2x^2y^2/t) + 3(x^2y^2/t)^2 + \ldots\right\}$$

$$= \frac{1}{2M^3t^2} M_2(2,Q^2=4M^2t) - \frac{(1+4y^2)}{4M^3t^3} M_2(4,Q^2=4M^2t) + \ldots \quad , \qquad (6.3)$$

where the Cornwall-Norton moments [16] naturally appear.

The incoherent scattering contribution to the mass difference, $\Delta^{(2)} = \delta M_p^{(2)} - \delta M_n^{(2)}$, will be a function of the nonsinglet moments $M_2^{NS}(j,Q^2)$ which, to leading order, are

given by [19]

$$M_2^{NS}(j,Q^2) = \frac{M_2^{NS}(j,Q_o^2)\left[\ell n(Q_o^2/\Lambda^2)\right]^{dj}}{\left[\ell n(Q^2/\Lambda^2)\right]^{dj}} \quad , \qquad (6.4)$$

where for simplicity Q_o^2 here will be taken the same as the lower limit of integration in Q^2 ($Q_o^2 = 3.5\ GeV^2$). Inserting Eq. (6.3) into Eq. (5.10) we get

$$\Delta^{(2)} = \Delta_2^{(2)} + \Delta_4^{(2)} + \ldots = \frac{8\alpha M}{\pi^2}\int_1^\infty \frac{dt}{t(1+\frac{4M^2}{m_Z^2}t)}$$

$$\int_0^1 \sqrt{1-y^2}\ y^2dy\left[M_2(2,4M^2t) - \frac{(1+4y^2)}{2t} M_2(4,4M^2t) + \ldots\right]$$

$$\times\left\{1 + \frac{2}{3}(1-y^2) + \frac{2.4}{3.5}(1-y^2)^2 + \frac{2.4.6}{3.5.7}(1-y^2)^3 + \ldots\right\} \quad , \qquad (6.5)$$

where $\Delta_j^{(2)}$ contains the contribution from the $M_2(j,4M^2t)$ moment.

Other parameters used in the computation were $\Lambda = 0.2$ GeV and, for six flavours, $d_2 \simeq 1/2$ and $d_4 \simeq 1$. $M_2(2,\ 3.5\ GeV^2)$ and $M_2(4,\ 3.5\ GeV^2)$ were obtained from integrations of

$$F_2^{ep} - F_2^{en} = 0.4\ x^{1.2}\left[(1-x)^3 - 0.4(1-x)^2/Q^2 + 1.7(1-x)/Q^4\right] \quad , \qquad (6.6)$$

a parametrization of experimental data given in Ref. 20. The results were $M_2(2,\ 3.5\ GeV^2) = 0.018$ and $M_2(4,\ 3.5\ GeV^2)=0.0043$.

Numerical integrations of Eq. (6.5) keeping the four terms explicitly written inside the braces yielded

$$\Delta_1^{(2)} = 0.13 \text{ MeV} \quad , \quad \Delta_2^{(2)} = -\ 0.04 \text{ MeV} \quad , \quad \Delta^{(2)} \simeq 0.09 \text{ MeV} \ . \tag{6.7}$$

This, together with the Born contribution of Eq. (5.17), gives for the proton-neutron mass difference the value

$$\Delta = \Delta^{(1)} + \Delta^{(2)} = -\ 1.27 \text{ MeV} \quad , \tag{6.8}$$

to be compared with the experimental $\Delta_{exp} = -\ 1.29$ MeV . One could hardly ask for a closer agreement.

A question that can be asked at this point is the following: how come that on the basis of the same type of contributions that in Cottingham formalism give a positive sign, here using the new representation (5.10) the sign comes out negative? The difference is that in Cottingham formula besides the Born and inelastic contributions we have the contribution from the substraction function. One can imagine that it should be possible to find ways to allow us to relate the substraction function to the Born and inelastic terms. Then, the whole Δ could be expressed in terms of elastic and inelastic data which, however, will enter in Δ in a different way than in Cottingham's formula. Elitzur and Harari[6] for example, tried to do precisely that through finite energy sum rules that, unfortunately, involve unproved assumptions of superconvergence of certain amplitudes. With the use of Eq. (5.10) we short circuit that procedure. If Eq. (5.10) is correct it automatically takes into account the contribution of $t_1(-Q^2,0)$ to Cottingham formula, but written

in terms of $\partial T/\partial(\nu_I^2)$ of the new representation.

VII. CONCLUDING REMARKS

A new expression for the mass shift, Eq. (5.8) or (5.10), was presented. With this new expression, mass shifts or mass differences within a multiplet can be calculated even without a knowledge of substraction functions like $t_1(-Q^2,0)$. A nice feature of formula (5.10) is that substracted and unsubstracted Born terms lead to exactly the same result: a contribution to the proton-neutron mass difference with the right (negative) sign.

It is well known that the contribution from the deep-inelastic region leads to a divergent integral in the usual Cottingham formalism. Here, we have resorted to the Weinberg-'t Hooft mechanism which, while providing a justification for the idea that mass differences within a multiplet should be calculable as radiative effects, also introduces a convenient convergence factor into the integrals. The resulting deep inelastic contribution is positive.

On the whole, the theoretical mass difference obtained from the formalism presented in this paper, Eq. (6.8), is amazingly close to the experimental value.

FOOTNOTES AND REFERENCES

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