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 $\beta$ -DECAY IN AN  $SU(2) \times U(1)$  MODEL

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ABSTRACT

We consider an  $SU(2) \times U(1)$  model having both Dirac and Majorana mass terms for the neutrinos, with an extended Higgs sector without natural flavor conservation. Under these conditions, we show that for a certain range of the mass parameters of the model, some new contributions become important for the neutrinoless double  $\beta$ -decay  $[(\beta\beta)_{0\nu}]$ .

Recently it has been noticed by different authors<sup>1</sup> that in models where the neutrino acquires a Majorana mass from a Higgs boson triplet, new contributions arise to  $(\beta\beta)_{0\nu}$  which are of the six-fermion form and are not included in the usual 4-fermion parametrization. Schechter and Valle<sup>1</sup> have shown that in  $SU(2) \times U(1)$  models these contributions are negligible even when the Higgs sector contains, in addition to the triplet and usual doublet, another doublet. On the other hand in left-right symmetric models<sup>2</sup>, the new contributions can be as large as the usual one, involving left-handed currents and a virtual Majorana neutrino (Fig. 1a).

In this paper we show that it is possible to enhance the new contributions in an  $SU(2) \times U(1)$  model, provided the neutrino is given Dirac and Majorana mass terms<sup>3</sup>. Furthermore, in the model to be considered, natural flavor conservation (NFC)<sup>4</sup> is relaxed and two Higgs doublets are introduced. As a consequence, another contribution to  $(\beta\beta)_{0\nu}$  becomes important, still mediated by a virtual massive neutrino, which however simulates a right-handed current admixture, thus allowing for transitions of the type  $0^+ + 2^+$ <sup>5</sup>.

We use, in the following, a model previously considered by us, in a study of CP violation in the leptonic sector<sup>6</sup>. The essential features of the model are:

- i) it has two Higgs doublets and one triplet;
- ii) the neutrinos are given Majorana and Dirac masses;
- iii) NFC is relaxed.

We can represent the Higgs triplet by a  $2 \times 2$  matrix,

$$\vec{\tau} \cdot \vec{\eta} = \begin{bmatrix} \eta^+ & \sqrt{2} \eta^{++} \\ \sqrt{2} \eta^0 & -\eta^+ \end{bmatrix} \quad (1)$$

For the applications of this paper we need to consider the following trilinear and Yukawa couplings,

$$\begin{aligned} & \left[ \alpha_1 \phi_1^+ \phi_1^+ + \alpha_2 \phi_2^+ \phi_2^+ + \alpha_3 \phi_1^+ \phi_2^+ \right] \eta^{--} + \text{h.c.} \\ & \left[ \beta_1 \lambda \eta^+ \eta^+ + \beta_2 \lambda_1 \phi_1^+ \eta^+ + \beta_3 \lambda_2 \phi_2^+ \eta^+ + \beta_4 \lambda_1 \phi_2^+ \phi_2^+ \right] \eta^{--} + \text{h.c.} \end{aligned} \quad (2)$$

and

$$\begin{aligned} & \left( \frac{\lambda_1}{\sqrt{2}} \right)^{-1} \phi_1^- \bar{e}_R \left[ (m_e \cos \theta - D_1) \nu_{eL} + m_e \nu_{\mu L} \right] \\ & + \left( \frac{\lambda_1}{\sqrt{2}} \right)^{-1} \phi_2^- (D_1 \bar{e}_R \nu_{eL} - D_1 \cos \theta \bar{e}_L \nu_{eR} - D_2 \sin \theta \bar{e}_L \nu_{\mu R}) \\ & - \left( \frac{\lambda}{\sqrt{2}} \right)^{-1} A \eta^+ (\bar{\nu}_{eL}^c e_L + \bar{e}_L^c \nu_{eL}) - \sqrt{2} \left( \frac{\lambda}{\sqrt{2}} \right)^{-1} A \eta^{++} \bar{e}_L^c e_L \end{aligned} \quad (3)$$

for two lepton generations. Had we assumed an  $U(1)$  symmetry<sup>7</sup>, the  $\alpha_i$  terms in expression (2) would not be allowed. The coupling constants  $\alpha_i$  have dimension of mass and their order of magnitude will be important for what follows.  $\lambda_i$  and  $\lambda$  are the vacuum expectation values of the two doublets ( $i = 1, 2$ ) and the triplet, respectively,  $\lambda_i/\sqrt{2} = \langle \phi_i^0 \rangle$ ,  $\lambda/\sqrt{2} = \langle \eta^0 \rangle$ . Now, for the Yukawa couplings, we notice that due to the relaxation of NFC, the  $\phi_2$  doublet is coupled to the right-handed projection of both the charged and neutral sector. The right-handed neutrino in expression (3) appears as a consequence of having both Dirac and Majorana mass terms (in the model of Ref. 6 the  $\nu_R$  is a singlet so that there are no right-handed currents). In (3),  $\theta$  is a Cabibbo-like angle for the leptonic

sector and  $D_1$  and  $A$  are mass parameters related to the neutrino's mass eigenvalues,

$$m_i^\pm = \frac{B+A}{2} \pm \frac{1}{2} \left[ D_i^2 + (B-A)^2 \right]^{1/2} \quad (4)$$

where  $B$  is a bare mass term for the right-handed, sterile neutrino.

Defining the Majorana fields  $\chi$  and  $\omega$  as,

$$\chi = \nu_L + \nu_L^c, \quad \omega = \nu_R + \nu_R^c, \quad (5)$$

the phenomenological neutrinos are expressed in terms of the mass eigenstates,  $\chi'$  and  $\omega'$ , as follows,

$$\begin{bmatrix} \chi_e \\ \chi_\mu \\ \omega_e \\ \omega_\mu \end{bmatrix} = \begin{bmatrix} c c_1 & s c_2 & c s_1 & s s_2 \\ -s c_1 & c c_2 & -s s_1 & c s_2 \\ -s_1 & 0 & c_1 & 0 \\ 0 & -s_2 & 0 & c_2 \end{bmatrix} \begin{bmatrix} \chi'_e \\ \chi'_\mu \\ \omega'_e \\ \omega'_\mu \end{bmatrix} \quad (6)$$

$$(c_i \equiv \cos \theta_i, \quad s_i \equiv \sin \theta_i, \quad c = \cos \theta, \quad s = \sin \theta)$$

where the mixing matrix has been parametrized as in Ref. 8.

The Majorana mixing angles are given by,

$$\tan 2\theta_i = -D_i/(A-B) \quad (i = 1, 2)$$

It is important for our considerations that the parameters  $A$ ,  $B$  and  $D_i$  can assume quite large values, pushing the rate for processes like  $\mu \rightarrow e \gamma$  close to the experimental

upper limits<sup>8</sup>. As a matter of fact, the following hierarchy can be chosen,  $A \approx 0.1 \text{ GeV}$ ,  $D_i \approx 10 \text{ GeV}$ ,  $B \approx 10^3 \text{ GeV}$ . In such a case it can be seen from eq. (4) that  $m_i^-$  is the mass of a very light neutrino, of the order of a few electron-volts, while  $m_i^+$  is a heavy mass of the order of  $B$ .

We show in Fig. 1, at the quark level, the diagrams contributing to  $(\beta\beta)_{0\nu}$  decay in this model. Fig. 1a is the standard left-handed contribution parametrized by two effective four-fermion interactions. Its magnitude will be used as a reference value against which the other diagrams will be compared.

Let us now analyse the order of magnitude and helicity structure of each diagram. In the standard diagram (Fig. 1a), the two electrons are left-handed and its strength is given by

$$\frac{g^4 m_i^-}{m_W^4 \langle p^2 \rangle} \quad (7a)$$

when a  $\chi^+$  is exchanged or by

$$\frac{g^4}{m_W^4 m_i^+} \quad (7b)$$

when a  $\omega^+$  is exchanged. In the above  $\langle p^2 \rangle$  is an average four-momentum transfer squared, of the order  $(20\text{MeV})^2$  (see Nishiura, Ref. 5). Notice that in (7b), the neutrino mass appears in the denominator since,  $m_i^{+2} \gg \langle p^2 \rangle$ . In writing (7), we employed the neutrino propagator  $\langle \chi_{iL}^{\prime T} \chi_{iL}' \rangle = \frac{m_i^-}{p^2 - m_i^-}$  since

the vertices are of the same chirality. For vertices of different chiralities, as occurs in other diagrams, we use  $\langle \chi_R^{\prime T} \chi_L' \rangle = \frac{\not{p}}{p^2 - m_i^-}$  and analogously for  $\omega_i^+$ . Notice that we have omitted, for simplicity, the mixing parameters coming from eq. (6). As for the other diagrams, we show their respective strength in Table 1. In all these diagrams, the electrons are produced with the same chirality, namely, left-handedly. However, the diagram 1c allows electrons of different helicities in the final state (Fig. 2). Notice that, had we considered in Fig. 2 the triplet  $\eta^-$  instead of one of the doublets  $\phi_2^-$ , the contribution would vanish, as can be seen from helicity arguments (this term would be proportional to the neutrino mass but with opposite helicities for the neutrinos). The strength of the diagram in Fig. 2 is

$$\frac{g_{qh}^2 g_{lh}^2 \not{p}}{m_H^4 \langle p^2 \rangle} \quad (8a)$$

for a  $\chi_i^+$  exchange or

$$\frac{g_{qh}^2 g_{lh}^2 \not{p}}{m_H^4 m_i^{+2}} \quad (8b)$$

for a  $\omega_i^+$  exchange.

In Table 1 and expressions (8),  $g_{qh}$  and  $g_{lh}$  denote, respectively, the couplings of quarks and leptons to the Higgs bosons. For  $g_{qh}$  we may assume the usual couplings,  $g_{qh} \approx m_q G_F^{1/2}$ , while for  $g_{lh}$  we use the Yukawa couplings from expression (3). The trilinear coupling  $f$  in Fig. 1e is read from expression (2).

It must be stressed that the contribution shown in Fig. 2 is typical of our model and is a direct consequence of having relaxed NFC, thus allowing the coupling of  $\phi_2$  with the two charge sectors, as mentioned before. We see from Fig. 2 that this new contribution, permits transitions of the type  $0^+ \rightarrow 2^+$ , which have been regarded as typical of right-handed currents. Our model with explicit flavour violation simulates a right-handed current admixture.

Coming back to the standard contribution (Fig. 1a), we notice that it is dominated by  $\omega'$  exchange, since we have been assuming  $m_i^- \approx 0$  (10 eV),  $m_i^+ \approx 0$  (1 TeV) and  $\langle p^2 \rangle \approx 0$  (20 MeV)<sup>2</sup>. In this case,

$$\frac{\text{Amp. (1a, } \chi')}{\text{Amp. (1a, } \omega')} = \frac{1}{40}$$

and we compare the contributions from Table 1 and (8), with Amp. (1a,  $\omega'$ ). Diagrams 1b and 1c are also dominated by  $\omega'$  exchange and we have,

$$\frac{\text{Amp. (1.b)}}{\text{Amp. (1.a)}} = \frac{g_{qh} g_{lh} m_w^2}{m_H^2 g^2} \quad (9)$$

and

$$\frac{\text{Amp. (1.c)}}{\text{Amp. (1.a)}} = \frac{g_{qh}^2 g_{lh}^2 m_w^4}{g^4 m_H^4} \quad (10)$$

It is straightforward to check that both Fig. 1b and Fig. 1c give a small contribution,

$$\frac{\text{Amp. (1b,c)}}{\text{Amp. (1a)}} \sim 10^{-(5-6)}$$

For Fig. 1d, we have,

$$\frac{\text{Amp. (1.d)}}{\text{Amp. (1.a)}} \approx \frac{A m_i^+}{m_\eta^2} \approx 10^{-2} \quad (11)$$

while for Fig. 1e, the following result applies,

$$\frac{\text{Amp. (1.e)}}{\text{Amp. (1.a)}} \approx 10^{-4} (f/\text{GeV}) \quad (12)$$

The trilinear coupling  $f$ , comes from two different contributions as shown in expression (2). The first is induced by a tadpole and can never be made as large as  $10^4$  GeV, so as to make (12) of order one. However, the other trilinear couplings that are present in the Lagrangian, even before spontaneous breaking, are arbitrary, with dimension of mass and can be made as large as we please. In particular, differently from Schechter and Valle<sup>1</sup>, we do not need  $f$  to be of the order of the grand unification scale, for Fig. 1e to give a contribution of the same order as the standard mechanism. If  $f$  is about  $10^4$  GeV, a neighbouring mass scale, Fig. 1e is comparable to the usual contribution.

We now turn to the estimate of Fig. 2. In this case the dominant contribution comes from  $\chi'$  exchange (eq.8a) and,

$$\frac{\text{Amp. (2)}}{\text{Amp. (1a)}} = \frac{m_q^2 G_F^{-1}}{m_H^4} \left( \frac{D}{\lambda} \right)^2 \frac{m_i^+}{\langle p \rangle} \quad (13)$$

With the values for the mass parameters, as discussed before, we come to the interesting result that,

$$\frac{\text{Amp. (2)}}{\text{Amp. (1a)}} = 0 \quad (1) \quad (14)$$

thus enhancing transitions of the type  $0^+ \rightarrow 2^+$ .

Recently, Haxton et al.<sup>(9)</sup> have argued that nuclear physics effects suppress the contribution from the trilinear term of Fig. 1e. These authors estimate that this contribution is about three orders of magnitude smaller than the previous evaluation by Mohapatra and Vergados<sup>(1)</sup>. This argument does not invalidate our result since, as shown in Eq. (12), the magnitude of such a contribution is controlled by the trilinear coupling  $f$ , which is arbitrary. There is no a priori reason for  $f$  to be small, it could well be of the order  $10^7$  GeV, in which case even taking into account the suppression factor pointed out by Haxton et al., Fig. 1e would still contribute significantly to  $(\beta\beta)_{0\nu}$ . On the other hand for the diagram shown in Fig. 2 the arguments in Ref. 9 do not apply (Fig. 2 is of the 4-fermion type).

In conclusion, we have seen that in a model with both Dirac and Majorana masses and without NFC, it is possible to enhance a 6-fermion effective interaction as shown in Fig. 1e, provided there is an intermediate or neighbouring mass scale, wherein lepton number is explicitly broken. A consequence of abandoning NFC is to have a contribution to the transition  $0^+ \rightarrow 2^+$  which is of the same order of magnitude as the standard contribution, simulating a right-handed current effect.

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FIGURE CAPTIONS

FIG. 1 - Diagram for  $(\beta\beta)_{0\nu}$ -decay.

- 1.a - The usual diagram via massive Majorana neutrinos,  $\chi_i^1$   
being the light neutrino ( $m_1^-$ ) and  $\omega_i^1$  the heavy one  
( $m_1^+$ ).
- 1.b - As in 1.a with one of the W-boson replaced by a single  
charged Higgs. Our notation implies a transition  
propagators  $\langle \phi_i^- \eta^- \rangle$  where  $\phi_i^-$  ( $i = 1, 2$ ) and  $\eta^-$  are  
not the mass eigenstate.
- 1.c - Two-Higgs contributions, notation as in 1.b.
- 1.d - A contribution from the trilinear  $W^- W^- \eta^{++}$  term.
- 1.e - A contribution from the pure Higgs trilinear (eq.(2)).

FIG. 2 - Contributions from the  $\phi_2^-$  doublet to  $(\beta\beta)_{0\nu}$ .  
This diagram gives a  $0^+ \rightarrow 2^+$  transition.

Diagram Neutrino Exchange	1.a	1.b	1.c	1.d	1.e
$\chi'_i$	$\frac{g^4 m_i^-}{m_W^4 \langle p^2 \rangle}$	$\frac{g_{gh} g_{\ell h} g^2 m_i^-}{m_W^4 m_H^2 \langle p^2 \rangle}$	$\frac{g_{gh}^2 g_{\ell h}^2 m_i^-}{m_H^4 \langle p^2 \rangle}$	-	-
$\omega'_i$	$\frac{g^4}{m_W^4 m_i^+}$	$\frac{g_{gh} g_{\ell h} g^2}{m_W^2 m_H^2 m_i^+}$	$\frac{g_{gk}^2 g_{\ell h}^2}{m_H^4 m_i^+}$	-	-
no neutrino	-	-	-	$\frac{g^4 A}{m_\eta^2 m_H^4}$	$\frac{g_{gh}^2 f}{m_\eta^2 m_H^4}$

TABLE 1





