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by

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ABSTRACT

A new method for the analysis of heavy-ion elastic scattering is suggested. The method is based on a decomposition of the cross-section into a Rutherford, incoherent and coherent components. The incoherent component is directly related to the total reaction cross-section, and may be calculated once the latter is given. Through a careful study of the remaining, coherent, component of the cross-section, one may extract several useful properties of the system. Applications to the scattering of ^{12}C on ^{24}Mg and ^{16}O on ^{28}Si at several energies are made.

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Elastic scattering of heavy ions at above-barrier energies is characterized by two dominating effects; strong Coulomb interaction (large values of the Sommerfeld parameter, $\eta = \frac{z_1 z_2 e^2}{\hbar v}$), and strong absorption. Several seemingly different optical interpretations of the nature of $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta)$ (where $\sigma_{e\ell}(\theta) \equiv \frac{d\sigma_{e\ell}}{d\Omega}$, is the elastic scattering differential cross-section, and $\sigma_{Ruth}(\theta)$ is the corresponding Rutherford cross-section) have been advanced, e.g. Fresnel diffraction[1)], rainbow scattering[2)], Fraunhofer diffraction, etc.. The success of these models in describing the behaviour of $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta)$ in the forward-angle regime suggests strongly that the heavy-ion system behaves basically optically (i.e. geometrically). Deviation from the optical behaviour is then attributed to specific nuclear structure effects (e.g. particle transfer).

The analysis of the data has always been done through the ratio $\sigma_{e\ell}(\theta)/\sigma_{Ruth}(\theta)$. An interesting optical characteristic of the system that emerges naturally in the above ratio is the quarter-point property, namely $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta_{1/4})=0.25$, with $\theta_{1/4}$ related to the grazing trajectory[3)], in close analogy with similar features seen in the Fresnel diffraction of light from a sharp edge[4)]. With the above association of $\theta_{1/4}$ with the grazing trajectory, one obtains a very efficient and direct method for the determination of the total reaction cross-section, σ_R.

However, the nature of the angle-oscillations at small angles and the almost exponential drop at $\theta \gtrsim \theta_{1/4}$ of $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta)$, has so far escaped a clear identification with one of two effects; Fresnel diffraction (the exponential drop of $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta)$ at $\theta \geq \theta_{1/4}$ is then attributed to the creeping of waves into the shadow region) and rainbow scattering (scattering into the classically forbidden region defined by $\theta > \theta_r$, the rainbow angle). So far the question has not been settled[5].

In this letter we suggest an alternative method of analysis that may lead eventually to the solution of the above mentioned problem. Our method is based on an appropriate decomposition of $\sigma_{e\ell}(\theta)$ into three well defined pieces, attached to Coulomb scattering, incoherence (absorption) and coherence, as we shall clarify below.

We start with the usual partial wave decomposition of the elastic scattering amplitude

$$f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)\left(S_\ell^n S_\ell^c - 1\right) P_\ell(\cos\theta) \tag{1}$$

where k denotes the asymptotic wave number in the elastic channel, related to the center of mass energy E and reduced mass μ by $k = \left(\frac{2\mu E}{\hbar^2}\right)^{1/2}$. In Eq. (1), S_ℓ^c is the point-Coulomb S-function and $S_\ell^n \equiv |S_\ell^n| e^{2i\delta_\ell}$ represents the effect of the short-range nuclear force. The modulus of S_ℓ^n, $|S_\ell^n|$, is smaller than unity for low partial waves, as a consequence of short-range absorption. The elastic scattering cross-section $\sigma_{e\ell}(\theta) \equiv |f(\theta)|^2$, may be decomposed into the following, as was shown in Ref. 6)

$$\sigma_{e\ell}(\theta) = \sigma_{Ruth}(\theta) - \sigma_{INC}(\theta) + \sigma_{COH}(\theta) \tag{2}$$

where $\sigma_{Ruth}(\theta) = \frac{\eta^2}{4k^2\sin^4(\theta/2)}$, $\sigma_{INC}(\theta)$ is defined through

$$\sigma_{INC}(\theta) = \frac{1}{4k^2}\sum_{\ell=0}^{\infty}(2\ell+1)^2\left(1-|S_\ell^n|^2\right)\left[P_\ell(\cos\theta)\right]^2 \tag{3}$$

and $\sigma_{COH}(\theta)$ is given by a rather complicated expression discussed in Ref. 6). Two important properties of $\sigma_{INC}(\theta)$ and $\sigma_{COH}(\theta)$ are worth mentioning at the present moment; $\int\sigma_{INC}(\theta)d\Omega = \sigma_R \equiv \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)(1-|S^n|)^2$ and $\int\sigma_{COH}(\theta)d\Omega = 0$. We further note that $\sigma_{INC}(\theta)$ is symmetrical about $\theta = 90^\circ$. This explains our choice for the subscripts INCOHERENT and COHERENT.

Equation (2) is valid throughout the full angular range $(0^\circ \leq \theta \leq 180^\circ)$, except in cases where forward glory is present in the system in which case a fourth term, proportional to $\delta(1-\cos\theta)$, appears on the RHS of Eq. (2). For a rather detailed discussion on this point see Ref. 6). In the present paper we disregard forward glory effects completely and consider Eq. (2) as an exact relation. Since $\sigma_{Ruth}(\theta)$ is completely specified and, as we show below, $\sigma_{INC}(\theta)$ is determined by the total reaction cross-section, σ_R, one may use Eq. (2) to extract from the data, i.e. $\sigma_{e\ell}(\theta)$, the piece, $\sigma_{COH}(\theta)$, that contains the information about the nuclear phases and accordingly the ion-ion potential.

In order to set Eq. (2) into practical use, we first calculate $\sigma_{INC}(\theta)$. For this purpose we use the well known asymptotic form of the Legendre polynomial $P_\ell(\cos\theta) \simeq \left[\frac{2}{\pi(\ell+1/2)\sin\theta}\right]^{1/2}\cos\left[(\ell+1/2)\theta-\frac{\pi}{4}\right]$ valid in the angular range $\frac{1}{\ell} \leq \theta \leq \pi - \frac{1}{\ell}$, and the relation $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$, to obtain immediately the following simple form for $\sigma_{INC}(\theta)$

$$\sigma_{INC}(\theta) \cong \frac{\sigma_R}{2\pi^2\sin\theta}\left[1 + \langle \sin 2\lambda\theta \rangle_T\right] \tag{4}$$

where σ_R is the total reaction cross-section, $\sigma_\ell = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell$ with the ℓ-th transmission coefficient T_ℓ defined as usual

$T_\ell = 1-|S_\ell^n|^2$, and the T_ℓ-averaged sine function is defined by

$$\langle \sin 2\lambda\theta \rangle_T \equiv \int_0^\infty d\lambda\, \lambda\, T(\lambda) \sin 2\lambda\theta \Big/ \int_0^\infty d\lambda\, \lambda\, T(\lambda) \qquad (5)$$

with $\lambda \equiv \ell + 1/2$.

For large values of the grazing angular momentum, ℓg , $\langle\sin 2\lambda\theta\rangle_T$ gives rise to small oscillations that become more damped as θ approaches 90^o[7]. We may, therefore, in our application to heavy-ion scattering at above-barrier energies (large ℓ_g), drop the second term on the RHS of Eq. (4). With $\sigma_{INC}(\theta)$ constructed simply as $\sigma_R/2\pi^2\sin\theta$, and $\sigma_{Ruth}(\theta)$ for the point-charge scattering given by $\sigma_{Ruth}(\theta) = \eta^2/4k^2\sin^4(\theta/2)$, we may obtain $\sigma_{COH}(\theta)$ straightforwardly from the data, vis.

$$\sigma_{COH.}(\theta) = \sigma_{e\ell.}(\theta) - \frac{\eta^2}{4k^2\sin^4\theta/2} - \frac{\sigma_R}{2\pi^2\sin\theta} \qquad (6)$$

For the application of Eq. (6) in the full angular range, one has to first remove the singularity at $\theta=0^o$ and 180^o arising from the approximate form of $\sigma_{INC}(\theta)$ (the third term on the RHS of Eq. (6)). For this purpose we suggest the use of the simple formula $\sigma_{INC}(\theta) = \frac{\sigma_R}{2\pi^2\sin\theta}$ in its own angle-range of validity, $\frac{1}{\ell_g} \leq \theta \leq \pi - \frac{1}{\ell_g}$, and then extrapolate the results obtained at the extremes, to the exact value of σ_{INC} at 0^o and 180^o. It is a very simple matter to calculate $\sigma_{INC}(0^o) = \sigma_{INC}(180^o)$ from the defining Eq. (3), in the sharp cut-off limit ($T_\ell = \theta(\ell_g-\ell)$ with θ being the step function), which we expect to be a very reasonable approximation. We obtain

$$\sigma_{INC.}(0^o) = \sigma_{INC.}(180^o) = \frac{1}{3k^2}\left[(\ell_g+1)^3 - \frac{1}{4}(\ell_g+1)\right] \approx \frac{1}{3k^2}(\ell_g+1)^3 = \frac{1}{3\pi}(\ell_g+1)\,\sigma_R \qquad (7)$$

The last form of $\sigma_{INC}(0^o)$ being valid when $\ell_g >> 1$.

Eq. (6) is the principal result of the present letter. It shows clearly that the coherent nuclear component of the elastic scattering cross-section, is easily extractable from the data, once the total reaction cross-section is given (from, e.g., the quarter-point angle). It is interesting to observe that upon integrating Eq. (6) over the solid angle, and owing to the fundamental property of $\sigma_{COH}(\theta)$, namely $\int\sigma_{COH}(\theta)d\Omega = 0$, one obtains the sum-of-differences equation, $\sigma_R = \int d\Omega[\sigma_{Ruth}-\sigma_{e\ell}]$.

In order to give a theoretical background to the data analysis given below, we first present our result for $\sigma_{COH}(\theta)$ obtained from an optical model calculation for the scattering of the system $^{16}O+^{28}Si$ using the E-18 potential[8]. Figure 1) summarizes our results for $\sigma_{INC}(\theta)$ and $\sigma_{COH}(\theta)$ for this case. To be consistent with the procedure we suggest in our data analysis, we have used the quarter-point recipe to extract σ_R , even in this theoretical model calculation. We notice first strong oscillations in $\sigma_{COH}(\theta)$ at small angles that persist up to an angle close to, but smaller than $\theta_{1/4}$, after which $\sigma_{COH}(\theta)$ attains over-all negative values that reache a maximum (negative) at $\theta \simeq \theta_{1/4}$, then turns around and gradually, after reaching and crossing the θ = 0-axis, joins in value the $\sigma_{INC}(\theta)$, which is clearly always positive. The important point we would like to emphasize is that $\sigma_{COH}(\theta)$ attains a zero value at several angles. In particular, we suggest that at the two largest angles, θ_1 and θ_2, at which σ_{COH} becomes zero, being rather well-defined, may be easily used to obtain an independent value of the total reaction cross-section (see Eq. (6)), vis

$$\sigma_R = 2\pi^2 \sin\theta_{1,2}\left[\sigma_{Ruth.}(\theta_{1,2}) - \sigma_{el.}(\theta_{1,2})\right] \tag{8}$$

The values of σ_R extracted from Eq. (8) are very close to the one used in the construction of σ_{INC}, double checking our contention. It would be interesting to obtain an independent solution of the equation $\sigma_{COH}(\theta_{1,2}) = 0$, in order to obtain a completely independent (from the quarter-point recipe) method for the obtention of σ_R. We should emphasize that the quarter-point angle comes out very close to the angle at which $\sigma_{COH}(\theta)$ attains its maximum (negative) value. The above calculation with the E-18 potential was performed at $E_{CM} = 24.3$ MeV. Increasing the energies merely shifts the angles θ_1, θ_2 and $\theta_{1/4}$ to smaller values, without changing the overall qualitative features of $\sigma_{COH}(\theta)$ shown in Fig. (1).

We have calculated $\sigma_{COH}(\theta)$ for two heavy-ion systems studied extensively in the literature, $^{16}O + ^{28}Si$ [9] and $^{12}C + ^{24}Mg$ [10], at center of mass energies $E_{CM} = 24.3$, 26.2, 31.6 and 34.8 MeV and $E_{CM} = 16.5$, 19.1, 20.0, 23.2 and 26.66 MeV, respectively. All these systems exhibit a $\sigma_{COH}(\theta)$ of exactly the same qualitative behaviour as that of Fig. (1). In Fig. (2) we show the behaviour of the angles θ_1, θ_2 and $\theta_{1/4}$ as functions of the center-of-mass energies considered. Since the E-18 potential that resulted in $\sigma_{COH}(\theta)$ of Fig. (1) is a proto-type strong absorbing potential that should result in a purely "Fresnel diffraction" elastic cross-section in the forward-angle regime, we reach the conclusion that at least the above two systems show similar features (Fresnel diffraction and <u>not</u> rainbow).

To check the sensitivity of $\sigma_{COH}(\theta)$ to the nature of the elastic scattering, we have performed another calculation for the system $^{16}O + ^{58}Ni$ at $E_{Lab} = 200$ MeV, using two different optical potentials, one being strongly absorbing ($V = W = 100.0$ MeV, $R_V = R_W = 5.87$ fm, $a_V = a_W = 0.64$ fm) and the other purely real ($V = 100.0$ MeV, $W = 0.0$, $R_V = 5.87$ fm, $a_V = 0.64$ fm). In the latter case one finds a very strong nuclear rainbow effect in $\sigma_{e\ell}(\theta)$, besides the usual Coulomb rainbow. In the strong absorption case, however, no rainbow effect was seen. The $\sigma_{INC}(\theta)$ for the two cases was found to be completely different. Whereas in the strong absorption case the behaviour of $\sigma_{INC}(\theta)$ is similar to that of Fig. (1), in the no-absorption case, however, the "over-all" negative branch of $\sigma_{INC}(\theta)$ becomes positive and reaches a positive maximum at an angle intermediate between θ_1 and θ_2.

In conclusion, we have presented a new method of analysis for the heavy-ion elastic scattering data. Through a careful study of $\sigma_{COH}(\theta)$, one may obtain information about the heavy-ion system that would otherwise be hard to find through the usual analysis of $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}$. In a way, our method should be applied together with the usual one in order to obtain a more complete picture of the mechanism involved. Application of our method for the analysis of heavy-ion quasi-elastic cross-sections is certainly worthwhile and is being attempted.

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7) As an example of the behaviour of $\langle \sin 2\lambda\theta \rangle_T$, we have used a simple Ericson form for $|S_\ell^n| = \dfrac{1}{1+\exp\left(\frac{\ell_g-\ell}{\Delta}\right)}$ and evaluated the integrals appearing in Eq. (5). The resulting expression for $\langle \sin 2\lambda\theta \rangle_T$ is

$$\langle \sin 2\lambda\theta \rangle_T \simeq \frac{2\pi}{k^2}\left\{ \frac{1}{(2\theta)^2}\left[1-\theta\frac{d}{d\theta}\right]\left(\sin[2\lambda_g\theta]\cdot\frac{2\pi\theta\Delta}{\sinh 2\pi\theta\Delta}\right) + \frac{1}{2(\pi-\theta)^2}\left[1-(\pi-\theta)\frac{d}{d(\pi-\theta)}\right]\left(\sin[2\lambda_g(\pi-\theta)]\cdot\frac{2\pi(\pi-\theta)\Delta}{\sinh 2\pi(\pi-\theta)\Delta}\right)\right\}$$

clearly the higher the energy is, the larger ℓ_g would be and similarly k^2 , rendering $\langle \sin 2\lambda\theta \rangle_T$ very small.

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FIGURE CAPTIONS

FIG. 1 - The coherent (solid line) and incoherent (dashed line) components of the elastic scattering cross-section for the $^{16}O + ^{28}Si$ system at $E_{CM} = 24.3$ MeV, calculated with the E-18 potential of Ref. 8). θ_1 : ● , θ_2 : ○

FIG. 2 - The angles θ_1, θ_2 and $\theta_{1/4}$ as function of the center of mass energies for the systems $^{16}O + ^{28}Si$ (a) and $^{12}C + ^{24}Mg$. The results were obtained using the data obtained in Refs. 9) and 10). The value of θ_2 at $E_{CM} = 26.6$ MeV (Ref. 10b)) was not available.
▲ : θ_1 , ● : $\theta_{1/4}$, ■ : θ_2

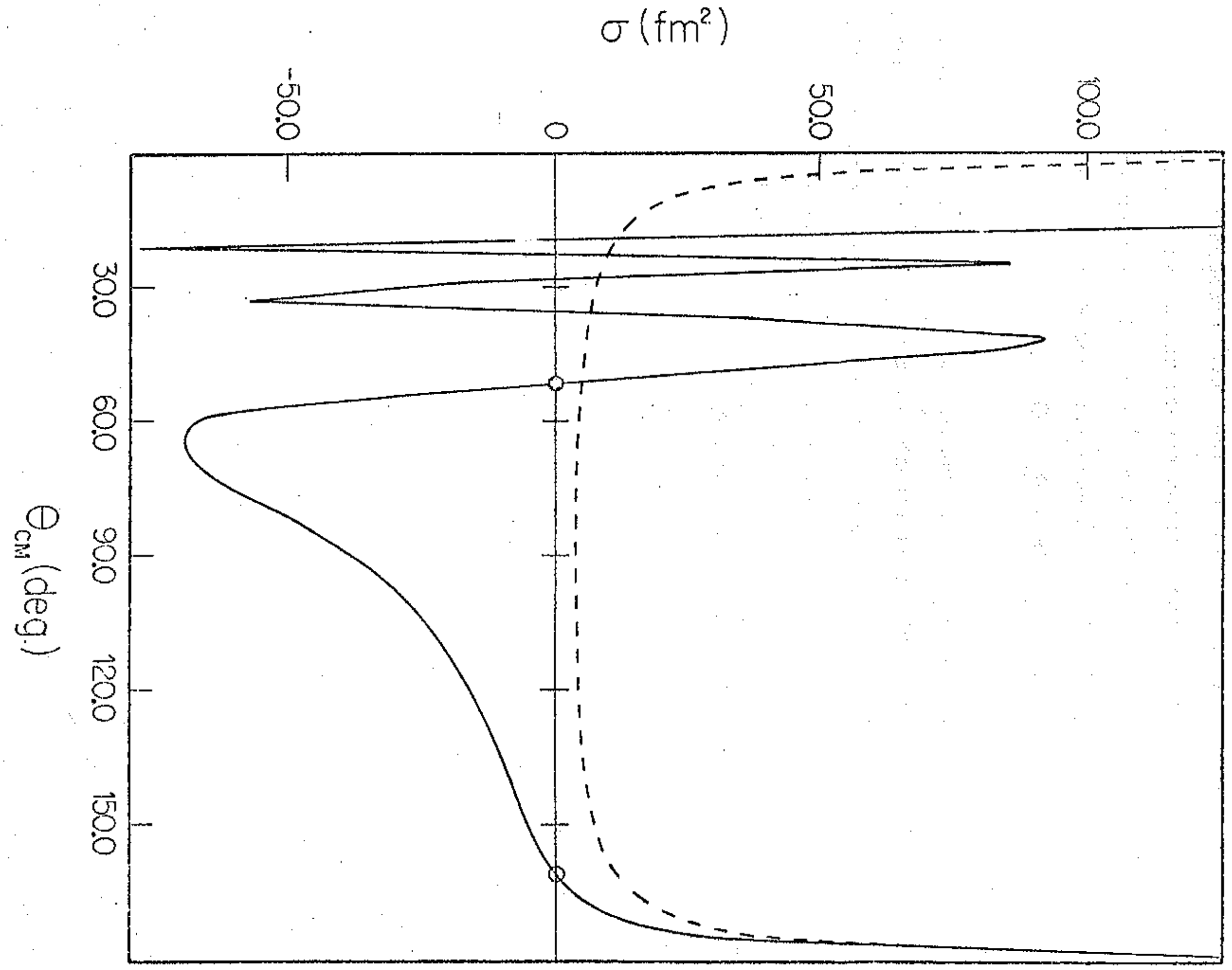
σ (fm²)
-50.0
0
50.0
100.0
30.0
60.0
90.0
120.0
150.0
Θ_CM (deg.)

Fig. 1

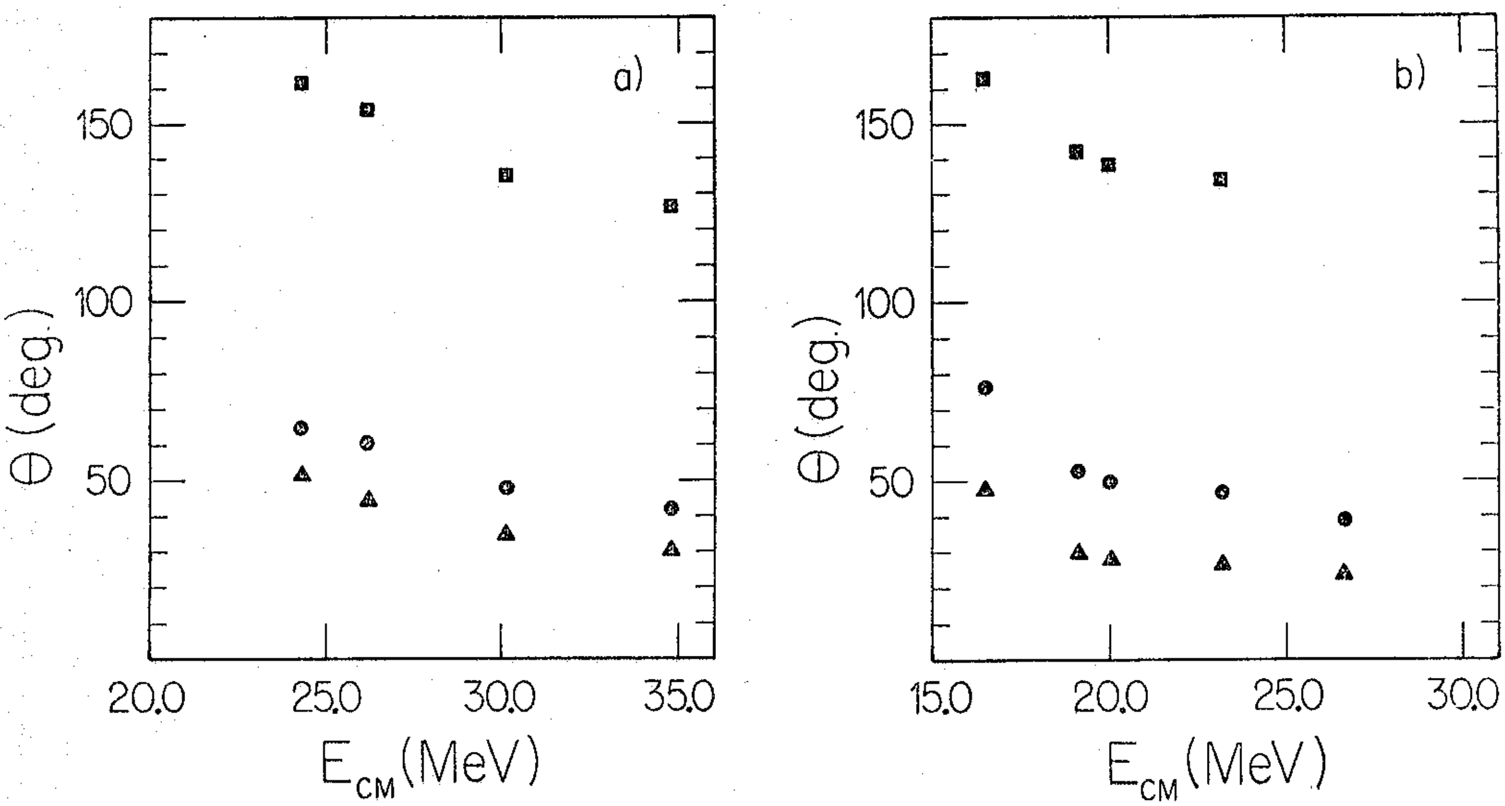
a)
b)
150
100
50
Θ (deg.)
20.0
25.0
30.0
35.0
15.0
E_CM (MeV)

Fig. 2