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ABSTRACT

The description of nuclear scattering using classical trajectory concepts is discussed. Analysis of the condition of validity of this description within the N. Bohr uncertainty arguments, is made. Applications to the scattering systems, $p+^{28}Si$ and $^{16}O+^{28}Si$ at E_{CM} = 8.0 MeV and 55.6 MeV, respectively are made.

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I. INTRODUCTION

The usual conditions cited in connection with the applicability of classical mechanics in the description of atomic and nuclear scattering, are related, in one way or another, to the underlying WKB approximation. This approximation is valid when the de-Broglie wave length of the scattered particle is much smaller than the range over which the interaction potential varies appreciably. Though this condition results in a systematic and controlable method of approximation, one nevertheless, ends up, in scattering situations, dealing with trajectories. The final aim of this is the utilization of the classical deflection function in the construction of the semi-classical amplitude[1).

It would seem, though, very instructive to express the validity condition referred to above, directly in terms of the properties of the classical deflection function. In fact N. Bohr back in 1948[2) did exactly this for the case of Coulomb scattering, in his discussion of the energy-loss suffered by a particle travessing matter.

In the present paper we discuss Bohr's criterion in a more general context of a joint Coulomb (long-ranged) and nuclear (short-ranged) forces. We then turn to the consequences of applying this criterion for a heavy-ion and an "equivalent" light-ion scattering.

The paper is organized as follows. In Section II a detailed presentation of Bohr's semiclassical criterion is given. In Section III we present our application of the criterion to nuclear scattering, and, finally, in Section IV we present several concluding remarks.

II. BOHR'S SEMI-CLASSICAL CRITERION

Consider the "experimental" arrangement shown in fig. (1). A small aperture, is inserted in the path of particles. The quantal diffraction of these particles will create an angular dispersion, $\delta\theta_D$, which together with two angular uncertainties to be discussed below, will decide upon the total angular dispersion that measures the deviation from a perfect classical behaviour. We now calculate this dispersion in details.

The quantal diffraction arising from the passage of particles through the aperture, creates a dispersion proportional to the de-Broglie wave length, $\lambdabar$, of the particle,

$$\delta\theta_D = \lambdabar / d \tag{1}$$

where d is the diameter of the opening. Further, the impact parameter (unobserved quantity!) is undefined within a distance $\frac{d}{2}$, thus giving rise to another uncertainty in θ , namely

$$\delta\theta_b = \frac{1}{2}\left(\frac{\partial\theta}{\partial b}\right) d \tag{2}$$

Finally, owing to the uncertainty in the incident energy, ΔE, one has

$$\delta\theta_E = \left(\frac{\partial\theta}{\partial E}\right)\Delta E \tag{3}$$

Combining these three factors, one obtains finally the total angle uncertainty

$$(\delta\theta)^2 = (\delta\theta_D)^2 + (\delta\theta_b)^2 + (\delta\theta_E)^2 \tag{4}$$

The optimal angular spread is found by minimizing Eq. (4) with respect to the radius of the hole $\frac{d}{2}$. The following values of d_{opt} is then obtained $\left(\left(\frac{\partial\theta}{\partial d}\right)_{d_{op}} = 0\right)$

$$d_{opt} = \left(\frac{2\lambdabar}{|\partial\theta/\partial b|}\right)^{1/2} \tag{5}$$

With d_{opt} above one then finds for $\delta\theta_{opt}$ the following simple expression

$$(\delta\theta_{opt.})^2 = \left|\frac{\partial\theta}{\partial b}\right| \lambdabar + \left(\frac{\partial\theta}{\partial E}\right)^2 (\Delta E)^2 \tag{6}$$

Classical description of atomic and nuclear scattering is applicable whenever the above values of the angle-spread is much smaller than the deflection function $\theta(b,E)$, i.e.

$$\frac{\delta\theta_{opt.}}{\theta(b,E)} \ll 1 \qquad \text{For all } b \text{ and } E . \tag{7}$$

To give a specific example, we apply Eq. (6) to the pure Rutherford scattering of two point charges, Z_1e and Z_2e . The deflection function is then given by

$$\theta(b,E) = 2\tan^{-1}(a/b) \tag{8}$$

with a being half the distance of closest approach for head-on collision, $a = \frac{Z_1 Z_2 e^2}{2E_{CM}}$. We then obtain for Eq. (7), the following

$$\frac{\left[(2\lambda\!\!\!^{-}/a)\sin^2\theta/2 + \left(\frac{\Delta E}{E}\right)^2 \sin^2\theta\right]^{1/2}}{\theta(b,E)} \ll 1 \qquad (9)$$

Eq. (9) is clearly very well satisfied in the full angular range ($0^\circ \leq \theta \leq 180^\circ$), as long as

$$2a/\lambda\!\!\!^{-} \gg 1 \quad \text{and} \quad \frac{\Delta E}{E} \ll 1 \qquad (10)$$

The first condition, $2\frac{a}{\lambda\!\!\!^{-}} >> 1$, could be easily satisfied in a given scattering system, if the de-Broglie wave length, $\lambda\!\!\!^{-}$, is very small, or equivalently if the energy, E, is very high. For a given energy (given $\lambda\!\!\!^{-}$) the above condition may also be satisfied if the charges, Z_1e and Z_2e are large. Note that the ratio $\frac{a}{\lambda\!\!\!^{-}}$ is nothing but the Sommerfeld parameter η ($\eta \equiv \frac{Z_2 Z_1 e^2}{\hbar v}$ where v is the asymptotic wave number). The second condition, $\frac{\Delta E}{E} << 1$ is almost always valid in present-day nuclear experiments, and may be dropped altogether.

In figure 2 we show the results of applying Eq. (9) to two nuclear systems that are energetically equivalent. It is quite clear that for the heavier system $^{16}O+^{12}C$, $\delta\theta_{op}$ is much smaller than θ over the full angular range. For the light system, $p+^{12}C$, the scattering in the angular range $\leq \theta \leq 120^\circ$ is clearly <u>not</u> classical in nature.

The point Coulomb interaction discussed above is a valid approximation for the description of nuclear scattering

at very low energies. As the energy is increased to values above the height of the Coulomb barrier, both the nuclear interaction and the modified Coulomb interaction (for an extended charge distribution) have to included in the discussion concerning $\delta\theta_{opt}$. We first consider the changes in $\theta(b,E)$ and $\delta\theta_{opt}$ owing to the finite size of the nucleus. The Coulomb interaction to be used has the form

$$\begin{aligned} V(r) &= Z_1 Z_2 e^2/r \,, & r \geq R_c \\ V(r) &= \frac{Z_1 Z_2 e^2}{2R_c}\left[3 - \left(\frac{r}{R_c}\right)^2\right] \,, & r \leq R_c \end{aligned} \qquad (11)$$

where R_c is a radius related to the size of the system. Typically one has $R_c \approx 1.4\,(A_1^{1/3} + A_2^{1/3})$ [fm] where A_i is the mass number of nucleus i. The deflection function, defined by

$$\theta(b,E) = \pi - 2\int_{r_{min}(b,E)}^{\infty} \frac{b\,dr/r^2}{\left(1 - \frac{V(r)}{E} - \frac{b^2}{r^2}\right)^{1/2}} \qquad (12)$$

with the distance of closest approach $r_{min}(b,E)$ identified with the largest root of the equation $1 - \frac{V(r_{min})}{E} - \frac{b^2}{r_{min}^2} = 0$, can be obtained in closed form for the $V(r)$ of Eq. (11). For $E < E_B \equiv \frac{Z_1 Z_2 e^e}{R_c}$, $\theta(b,E)$ is just the Rutherford function, Eq. (8). For center-of-mass energies, E, within the range $E_B \leq E \leq \frac{3}{2}E_B = V(0)$, the deflection function attains the following form

$$\theta(b,E) = \frac{\pi}{2} + \sin^{-1}\left[\frac{\left(\frac{3}{2}\frac{E_B}{E}-1\right)+2b^2/R_c^2}{\left[\left(\frac{3}{2}\frac{E_B}{E}-1\right)^2+2\frac{b^2}{R_c^2}\frac{E_B}{E}\right]^{1/2}}\right]$$

$$+2\sin^{-1}\left[\frac{E_B/E}{\left[4\frac{b^2}{R_c^2}+\left(\frac{E_B}{E}\right)^2\right]^{1/2}}\right]$$

$$-2\sin^{-1}\left[\frac{2b^2/R_c^2+E_B/E}{\left[4\frac{b^2}{R_c^2}+(E_B/E)^2\right]^{1/2}}\right]$$

$$; \quad b \le R_c\,(1-E_B/E)^{1/2}, \tag{13}$$

$$\theta(b,E) = 2\tan^{-1}(a/b) \quad ; b \ge R_c\,(1-E_B/E)^{1/2} .$$

For $E > \frac{3}{2}E_B$, a similar form of $\theta(b,E)$ as in Eq. (13) is obtained except for the replacement of the second term of the RHS by $-\sin^{-1}\left[\frac{\left(1-\frac{3}{2}\frac{E_B}{E}\right)-2b^2/R_c^2}{\left[\left(\frac{3}{2}\frac{E_B}{E}-1\right)^2+\frac{2b^2}{R_c^2}\frac{E_B}{E}\right]^{1/2}}\right]$. It is interesting to note that in this last case the deflection function attains a maximum value at a finite value of b ; a rainbow!

The equation that expresses the validity of the semiclassical description of scattering, Eq. (7), becomes rather involved for $\theta(b,E)$ of Eq. (13). However we found it to hold very well for heavy systems as in the case with the pure point-Coulomb scattering.

A more realistic discussion of Eq. (7) in the

context of nuclear scattering at $E > E_B$ should clearly involve the short-ranged nuclear interaction. It is a common practice to take for this interaction a Saxon-Woods potential of the form $V_N(r) = -V_o[1+(r-R)/d]^{-1}$ with R slightly smaller than R_c , and d of the order of 0.6 fm. Unfortunately the deflection function for this general case cannot be evaluated in closed form and one has to resort to a numerical integration of Eq. (12). Before turning to this discussion (see Section III), it would be interesting to exhibit qualitatively the changes in $\theta(b,E)$ that would occur due to the introduction of the short-rangel attractive nuclear potential. This may be done with the aid of classical perturbation theory[3].

Considering the form of $\theta(b,E)$ at not too small values of b , a region in the impact parameter space where the nuclear potential is small and may be approximated by $V_N(r) \simeq -V_o e^{-(r-R)/d}$. The deflection function, for these values of b , may be approximated by

$$\theta(b,E) \cong 2\tan^{-1}\frac{a}{b} + \Delta\theta_N \tag{14}$$

with $\Delta\theta_N$ representing the perturbation on the Rutherford angle due to the nuclear interaction. This perturbation, (negative), may be easily evaluated by considering the slight change in the linear momentum that will occur upon reaching a distance close to the turning point. This change is basically due to the component of the nuclear force perpendicular to the trajectory. This latter may be approximated by a straight line tangent to the Rutherford trajectory at the Coulomb distance of closest approach $r_{min} = a + \sqrt{a^2+b^2}$. We thus find

$$\Delta\theta_N \simeq -\frac{1}{mv}\int_{-\infty}^{\infty}\frac{dV_N}{dr}\,\frac{r_{min}}{r}\,dt \tag{15}$$

with $r^2 = r_{min}^2 + v^2t^2$.

Inserting the exponential form for $V_N(r)$ referred to earlier and owing to the fact that $\frac{r_{min}}{d} >> 1$, we finally find[3]

$$\Delta\theta_N \simeq -\frac{V_0}{E}\left(\frac{r_{min}\,\pi}{2d}\right)\exp\left(\frac{R-r_{min}}{d}\right) \tag{16}$$

which is clearly negative and decreases rapidly with increasing b (r_{min}(b)). The Bohr's criterion, Eq. (7) becomes, in this case (neglecting $\frac{\Delta E}{E}$, for simplicity)

$$\frac{2\lambda\!\!\!^{-}}{d}\left|\sin^2\theta/2 + \frac{V_0}{E}\,a\left(\frac{r_{min}(\theta)\pi}{2d}\right)^{1/2}\cos\theta/2\,.\right.$$
$$\left.\cdot\exp\left[\frac{R-r_{min}(\theta)}{d}\right]\left(\frac{1}{2r_{min}(\theta)}-\frac{1}{d}\right)\right| \tag{17}$$
$$<< \theta(b,E)$$

where $\quad r_{min}(\theta) = a\left[1+\frac{1}{\sin\theta/2}\right]$

Eq. (17) clearly shows, at least in cases where it is valid, that, as long as $d < 2r_{min}(\theta)$, the validity of the classical description is guaranteed for V_o/E small. The condition, $d < 2r_{min}(\theta)$, is clearly well satisfied over the full angular range since d is almost always smaller than a fermi. In the next section we present the exact evaluation of $\frac{\delta\theta_{op}}{\theta}$ for two realistic nuclear scattering cases.

III. APPLICATION TO NUCLEAR SCATTERING

In this section we discuss the validity of Eq. (7) in two nuclear scattering situations; a light-ion system, $p+{}^{28}Si$ and a heavy-ion system, ${}^{16}O+{}^{28}Si$. The two systems are energetically equivalent, namely the center-of-mass energy per nucleon* is the same for both systems. To make the comparison more consistent we have to introduce interaction potentials that are simply related through the single-folding formula to be discussed below.

The nuclear potential for the ${}^{16}O+{}^{28}Si$ system has been fixed to be the real part of the E-18 optical potential[4]. This potential has been found to reproduce rather well the average behaviour of the ratio $\frac{\sigma_{e\ell}}{\sigma_{Ruth}}(\theta)$, where $\theta_{e\ell}(\theta)$ is the elastic scattering differential cross section and $\sigma_{Ruth}(\theta)$ is the Rutherford cross section, $\sigma_{Ruth}(\theta) = \frac{a^2}{4\sin^4\theta/2}$. The parameters of the E-18 interaction are, $V_o = 10.0$ MeV , $R = 1.35\sum_{i=1}^{2}A_i^{1/3}$ fm and $d = 0.618$ fm.

The interaction of p with ${}^{28}Si$ is described by a potential that we define to be such that when folded with the ${}^{16}O$ matter density, ρ_{16_O} , we recove the E-18 potential referred to above. Thus[5]

$$V_{E-18}(\vec{r}) = \int d\vec{r}'\,\rho_{16_O}(\vec{r}')\,V_{p+{}^{28}Si}(\vec{r}'-\vec{r}) \tag{18}$$

We have inverted Eq. (18) numerically, assuming for simplicity a Saxon-Woods form for ρ_{16_O} . The obtained numerical values of $V_{p+{}^{28}Si}$ were then adjusted to a Saxon-Woods potential with the following parameters $V_o = 85$ MeV , $d = 0.6115$ fm ,

*Measured from the Coulomb barrier.

$R = 1.048\ A^{1/3}$ fm. The value of V_o is quite large and should be contrasted with the usual estimate[6] based on the independent-particle picture of the nucleus, which gives $V_o \approx 40$ MeV. Actually the inversion procedure does not supply a unique $V_{p+^{28}Si}(F)$ in view of the fact that several combinations of V_o and d could give rise to the same V_{E-18}. However, we shall use the parameters alluded to above for definiteness purpose.

We have evaluated the deflection function $\theta(b,E)$, Eq. (12), and its derivation, $\frac{\delta\theta}{\delta b}$, numerically. The results are shown in Figs. (3) and (4). In the region of b near and farther out from the Coulomb rainbow, the deviation $\delta\theta_{opt}$ is seen to be smaller than θ. However the $^{16}O+^{28}Si$ system shows a ratio $\frac{\delta\theta_{opt}}{\theta}$ much smaller than that of the $p+^{28}Si$, indicating clearly that the former system is more fit for a semiclassical description than the latter. This is, of course, an expected result. On the other hand one sees that in the b-region inside of $b_{g\ell}$, the glory impact parameter, and owing to the more complicated nature of $\theta(b,E)$ (presence of caustics), $\frac{\delta\theta_{opt}}{\theta}$ varies appreciably in value from point to point, and in particular attains an indefinite value (∞/∞) at $b_{or.}$, the orbitting impact parameter. The above feature of $\frac{\delta\theta_{opt}}{\theta}$ in the "internal" region seems to be common to both systems. We should mention, though, that in realistic considerations of heavy-ion scattering, this internal region is irrelevant, because of strong absorption. The absorption in the light-ion system is weaker, rendering the details of the deflection function in the internal region for e.g. $p+^{28}Si$ more important and pointing again for what has already bean concluded before, namely that the heavier system ($^{16}O+^{28}Si$) behaves more classically than the lighter system.

IV. DISCUSSION AND CONCLUSION

The condition of validity of the classical description of nuclear scattering, originally formulated by N. Bohr for point Coulomb scattering, has been generalized and applied to several systems. As expected the heavy-ion system, is found to behave more classically than the light-ion one. The above considerations, basically based on simple uncertainty arguments, are quite general with regards to the nature of the interactio responsible for the scattering. Clearly the final aim of such a discussion is to supply a simple check on the widely used semi-classical description of nuclear scattering and reactions.

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4) J.C. Cramer, R.M. De Vries, D.A. Goldberg, M.S. Zisman and C.F. Maguire, Phys. Rev. C14, 2158 (1976).

5) See, e.g., D.M. Brink and N. Rowley, Nucl. Phys. A219, 79 (1974).

6) A. Bohr and B.R. Mottelson, "Nuclear Structure", Vol. 1, North-Holland, (1969).

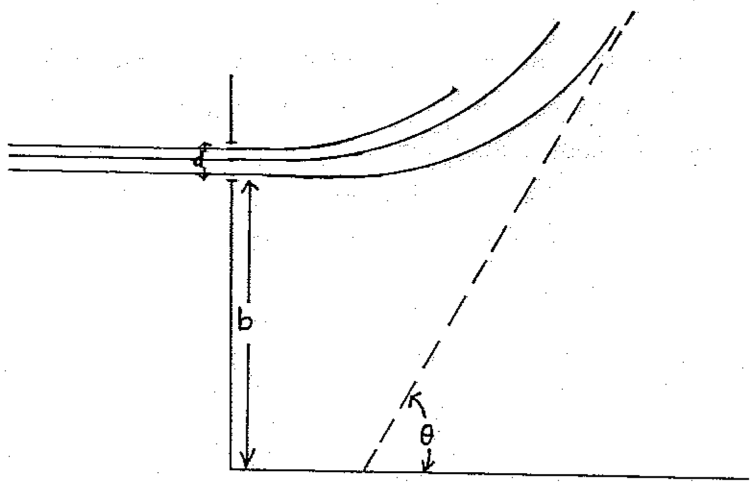
FIGURE CAPTIONS

FIGURE 1 - The experimental set up discussed in the text.

FIGURE 2 - The ratio $\frac{\delta\theta_{opt}}{\theta}$ for the Coulomb scattering $^{16}O+^{12}C$ and $p+^{12}C$ at E_{Lab} = 100.0 MeV and 8.0 MeV, respectively.

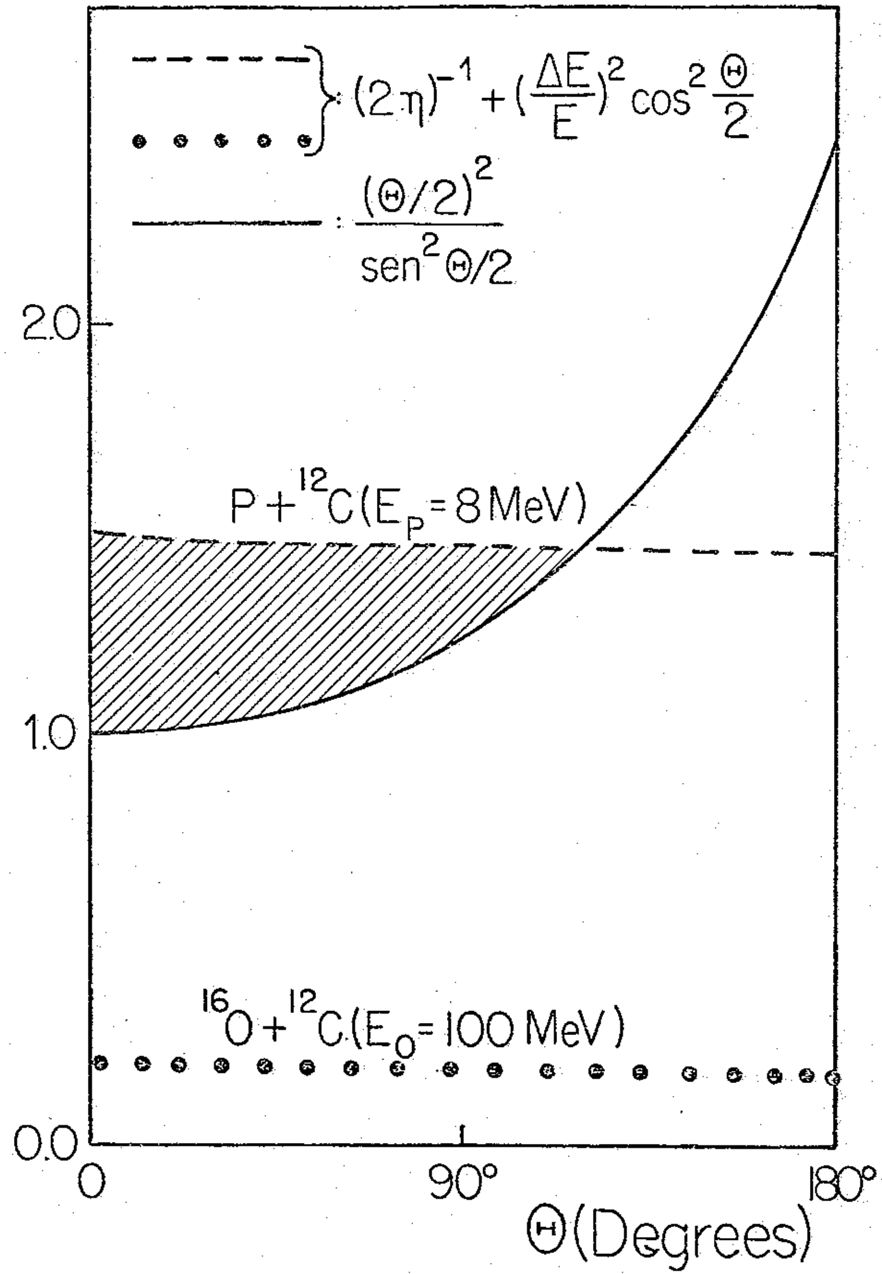
FIGURE 3 - The deflection function, $\theta(\ell,E)$ and the quantity $\frac{\partial\theta}{\partial\ell}$ for the system $^{16}O+^{28}Si$ at E_{CM} = 55.61 MeV. Notice that $\theta(0,E) = 0$ owing to the fact that for $b=0$, the turning point $r_{min}=0$ for $E > 3/2\, E_B$. The particle passes undeflected through the interaction center.

FIGURE 4 - Same as Fig. 3 for $p+^{28}Si$ at E_{CM} = 8 MeV.



d
b
θ

Fig. 1



$(2\eta)^{-1} + (\frac{\Delta E}{E})^2 \cos^2 \frac{\Theta}{2}$
$\frac{(\Theta/2)^2}{\mathrm{sen}^2 \Theta/2}$
$P + {}^{12}C (E_P = 8 \text{ MeV})$
${}^{16}O + {}^{12}C (E_O = 100 \text{ MeV})$
2.0
1.0
0.0
0
90°
180°
Θ(Degrees)

Fig. 2

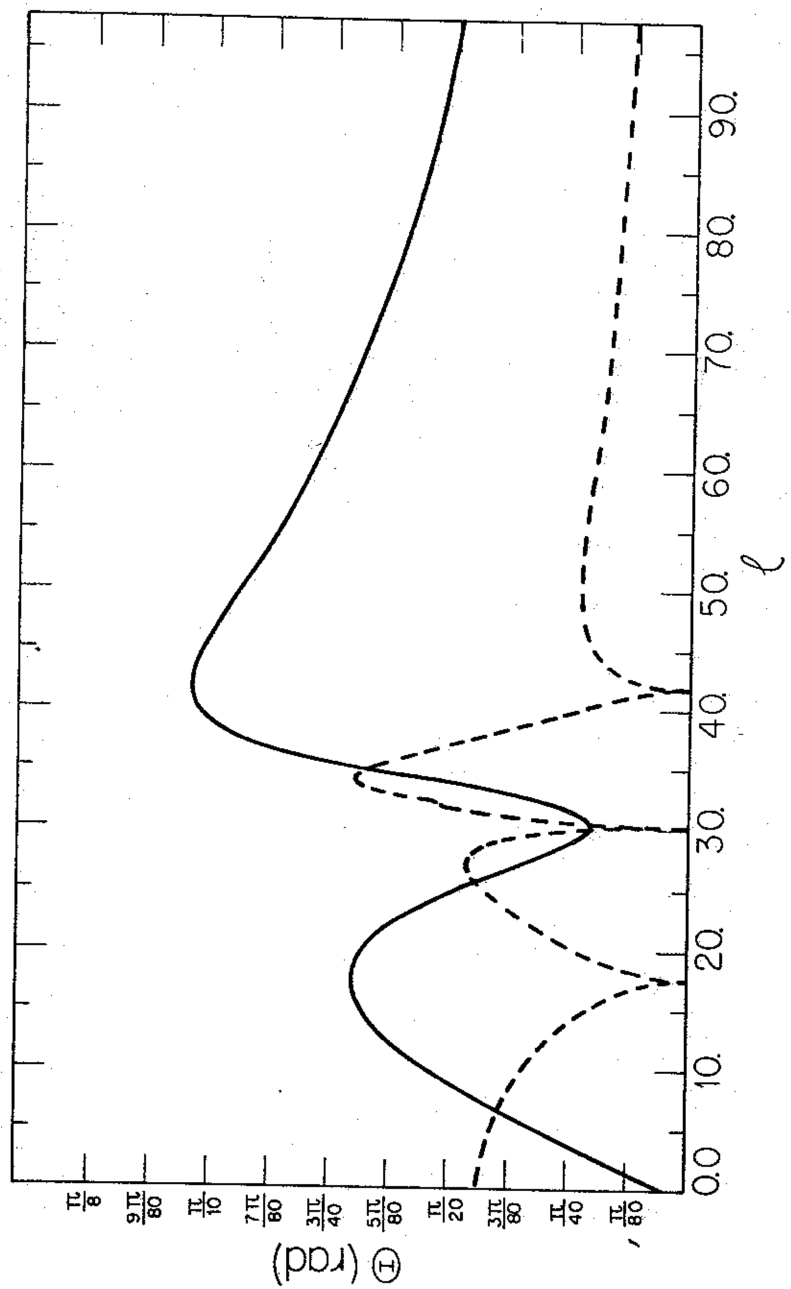

Θ (rad)
π/8
9π/80
π/10
7π/80
3π/40
5π/80
π/20
3π/80
π/40
π/80
0.0
10.
20.
30.
40.
50.
60.
70.
80.
90.
ℓ

Fig. 3

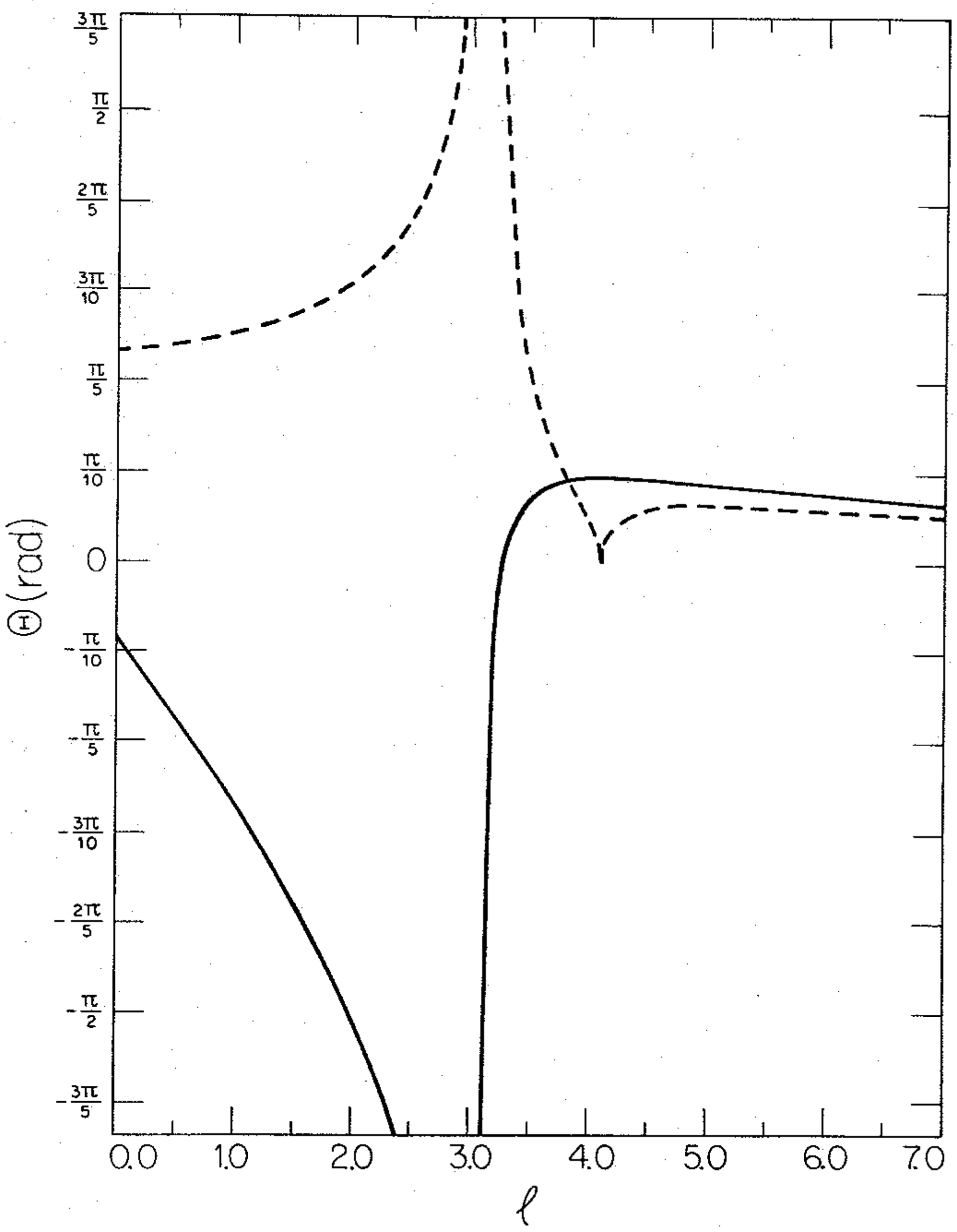

Θ (rad)
3π/5
π/2
2π/5
3π/10
π/5
π/10
0
-π/10
-π/5
-3π/10
-2π/5
-π/2
-3π/5
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
ℓ

Fig. 4