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FINITE PROTON-NEUTRON MASS DIFFERENCE

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ABSTRACT

With the only assumption that the up-down quark mass splitting is a weak-electromagnetic radiative effect, the proton-neutron mass difference is shown to be finite. This happens in a new representation where the mass difference can be completely calculated (no unknown terms) in very good agreement with experiment.

The main purpose of the present latter is to announce (and show) that a consistent picture is emmerging in which the proton-neutron mass difference $\Delta = m_p - m_n$ is finite, calculable and in agreement with the experimental value. It is finite in a new representation where the coefficient of the divergent term can be shown to vanish. And it is completely calculable under the (mild?) assumption that the up-down quark mass splitting is due to the electroweak interaction radiative corrections. After that, a very good result follows.

The electromagnetic contribution to Δ is given by

$$\Delta_{e\ell} = i\alpha \int \frac{d^4k}{(2\pi)^3} \frac{T(|\vec{k}|,k_o)}{k^2 + i\varepsilon} , \quad (1)$$

in term of $T = g^{\mu\nu}(T^p_{\mu\nu} - T^n_{\mu\nu})$, where $T^{p(n)}_{\mu\nu}$ is the forward Compton amplitude for an off-shell photon with momentum k_μ scattering off a proton (neutron) with a momentum p_μ[1].

The standard procedure is, following Cottingham[3], to Wick-rotate Eq. (1) and obtain

$$\Delta_{e\ell} = \frac{\alpha}{(2\pi)^3} \int \frac{d^4k_E}{Q^2} T(-Q^2, i\nu) , \quad (2)$$

where d^4k_E is the volume element in Euclidean momentum space arrived at by the rotation $pk/M = \nu \to \nu \exp(i\pi/2) = i\nu$, and $Q^2 = \nu^2 + K^2$ with $K = |\vec{k}|$. The last relations hold, of course, in the nucleon rest frame.

In the framework of a unified electroweak theory we will also have contributions resembling Eq. (2) but containing the amplitudes for nucleon-intermediate weak bosons scattering (Δ_{weak}). As will be shown later on $\Delta_{e\ell}$ and Δ_{weak} are separately convergent and since $\Delta_{weak} << \Delta_{e\ell}$, $\Delta \simeq \Delta_{e\ell}$ is a very good approximation.

For T in Eq. (2) one usually writes a dispersion relation

$$T(-Q^2, i\nu) = T^{Su} + \frac{6\nu^2}{Q^2}\int_{Q^2/2M}^{\infty} \frac{d\nu'}{\nu'^2+\nu^2}\left(\frac{Q^2}{\nu'}W_1 - \nu' W_2\right) + \frac{2(Q^2+2\nu^2)}{Q^2}\int_{Q^2/2M}^{\infty}\frac{d\nu'\nu' W_2}{\nu'^2+\nu^2}, \quad (3)$$

where T^{Su} is an unknown subtraction term and the $W_i = W_i^{e-p} - W_i^{e-n}$ are the customary structure functions which can be divided according to

$$W_i = W_i^{Co} + W_i^{In} , \quad (4)$$

with W_i^{Co} containing the contribution from coherent scattering (essentially Born terms) while the W_i^{In} are related to the incoherent scattering of the components of the nucleon (essentially deep inelastic scattering). Thus, T can also be written as

$$T = T^{Su} + T^{Co} + T^{In} . \quad (5)$$

I know that more than a few readers are impaciently asking themselves: what about the up-down quark mass difference contribution to Δ? I am assuming that the up-down quark splitting originates from electroweak radiative corrections and as such its contribution to Δ is contained in the T^{In} amplitude (and part also possibly in T^{Su}).

One of the problems with the Cottingham representation is that the T^{In} contribution leads to the logarithmically divergent part

$$\Delta_{\infty}^{In} = \frac{3\alpha M}{4\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2}\int_0^1 dx\, F_2(Q^2,x) , \quad (6)$$

where F_2 is the deep inelastic non-singlet structure function $F_2 = F_2^{e-p} - F_2^{e-n}$. In order to arrive at Eq. (5) the Callan-Gross[4] relation $2xF_1 = F_2$ has been used[2].

One of mine main tasks in the present paper consists in showing that there are other representations for Δ in which the coefficient of the divergent part in Δ^{In} vanishes. In order to derive new representations for Δ let us rotate the time like integration path in Eq. (1) according to $\nu \to \nu \exp i[(\pi/2)-\beta]$ with $\beta << 1$ obtaining

$$\Delta = \frac{\alpha}{(2\pi)^3}\int \frac{d^3k\, d\nu(1-i\beta)T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2 + K^2]} , \quad (7)$$

which has to be independent of β. The vanishing of the derivative of this expression with respect to β leads, in the $\beta \to 0$ limit, to

$$\Delta = \frac{2\alpha}{(2\pi)^3}\int \frac{d^3k\, d\nu}{Q^2}\nu^2\frac{\partial T}{\partial\nu^2} + \frac{2\alpha}{(2\pi)^3}\int\frac{d^3k\, d\nu\, K^2 T(K,i\nu)}{Q^4} . \quad (8)$$

Here we see that the ν-independent components of T (as in the subtraction function T^{Su} for instance) can only contribute to the second term in the right hand side of Eq. (8).

For integrals of the type

$$I_n = i\int\frac{d^3k\, d\nu(K^2)^n T(K,\nu)}{[\nu^2-K^2+i\kappa]^{n+1}} = (-1)^n\int\frac{d^3k\, d\nu(1-i\beta)(K^2)^n\, T[(K,(i+\beta)]}{[(1-2i\beta)\nu^2 + K^2]^{n+1}} \quad (9)$$

a similar procedure yields

$$\int\frac{d^3k\, d\nu(K^2)^n\, T(K,i\nu)}{(Q^2)^{n+1}} = \frac{2}{(2n+1)}\int\frac{d^3k\, d\nu(K^2)^n}{(Q^2)^{n+1}}\nu^2\frac{\partial T}{\partial\nu^2} +$$

$$+\frac{2(n+1)}{(2n+1)}\int\frac{d^3k\,d\nu\,(K^2)^{n+1}\,T(K,i\nu)}{(Q^2)^{n+2}}\quad. \tag{10}$$

Applying Eq. (10) N times to Eq. (9) we obtain

$$\Delta=\frac{2\alpha}{(2\pi)^3}\int\frac{d^3k\,d\nu}{Q^2}\left\{\nu^2\frac{\partial T}{\partial\nu^2}\sum_{n=0}^{N}\frac{n!}{(2n+1)!!}\left(\frac{2K^2}{Q^2}\right)^n+\right.$$

$$\left.+\frac{(2N+2)!!}{2(2N+1)!!}\left(\frac{K^2}{Q^2}\right)^{N+1}T(K,i\nu)\right\}\quad. \tag{11}$$

Now, the ν-independent components in T have been pushed to the last (N+1) term in Eq. (11). After a trivial angular integration this equation can be rewritten as

$$\Delta=\frac{2\alpha}{\pi^2}\int_0^\infty\frac{dQ^2}{Q^2}\int_0^1 dy\,\sqrt{1-y^2}\left\{y^2\left[\frac{\partial T(-Q^2,i\nu)}{\partial\nu^2}\right]_{\nu=yQ}\sum_{n=0}^{N}\frac{(2n)!!(1-y^2)^n}{(2n+1)!!}+\right.$$

$$\left.+\frac{(2N+2)!!}{2(2N+1)!!}(1-y^2)^{N+1}\,T(-Q^2,iyQ)\right\}\quad, \tag{12}$$

where the variable $y=\nu/Q$ was introduced.

If the last term in Eq. (11) were to drop out in the $N\to\infty$ limit, we would obtain

$$\Delta=\frac{\alpha}{\pi^2}\int_0^\infty\frac{dQ^2}{Q^2}\int_0^Q\sqrt{Q^2-\nu^2}\,\nu^2 d\nu\,\frac{\partial T(-Q^2,i\nu)}{\partial\nu^2}\sum_{n=0}^{\infty}\frac{(2n)!!}{(2n+1)!!}(1-\nu^2/Q^2)^n. \tag{13}$$

This representation for the mass shift was already presented in Ref. (2). Since it does contain a logarithmically divergent part of the type of Eq. (6), it was assumed that something like the Weinberg-'t Hooft[5] mechanism was in operation introducing a regulating factor. Another problem [which I did not notice in Ref. (2)] is that the last term in Eq. (11) [or Eq. (12)] does

not vanish in the $N\to\infty$ limit. In fact, after the integration in y this term is given by

$$\frac{\alpha}{4\pi}\frac{(N+3/2)}{(N+2)}\int_0^\infty\frac{dQ^2}{Q^2}\,T(-Q^2,i\,y_N\,Q)\quad, \tag{14}$$

with $0\leq y_N\leq 1$ and stubbornly staying alive in the $N\to\infty$ limit.

The problems mentioned above: the non-vanishing of the term containing T (instead of $\partial T/\partial\nu^2$) and the logarithmic divergence, can both be solved at the same time in the following way. Let us subtract Eq. (8) from twice Eq. (7) with $\beta=0$ obtaining

$$\Delta=\frac{2\alpha}{(2\pi)^3}\int\frac{d^3k\,d\nu\,\nu^2}{Q^4}\,(1-Q^2\,\partial/\partial\nu^2)\,T(K,i\nu)\quad. \tag{15}$$

The logarithmic divergence comes from the incoherent (deep inelastic) part of T in Eq. (3) when inserted into Eq. (15). As long as the Callan-Gross relation holds (as it seems to be) the first integral in Eq. (3) is zero. Thus, we are left with the incoherent amplitude

$$T^{In}(-Q^2,i\nu)=\frac{2(Q^2+2\nu^2)}{Q^2}\int_{Q^2/2M}^\infty\frac{d\nu'\,\nu'\,W_2^{In}}{\nu'^2+\nu^2}\quad. \tag{16}$$

Using $\nu^2=Q^2-K^2$ and $\nu'=Q^2/2Mx$ it is easy to see that $\nu'^2>\nu^2$ as long as $Q>2M$. So, in order to study the high Q behavior we are allowed to expand Eq.(16) in powers of $(\nu/\nu')^2$ as

$$\left[T^{In}(-Q^2,i\nu)\right]_{Q>2M}=\frac{4M(Q^2+2\nu^2)}{Q^4}\int_0^1 dx\,F_2(Q^2,x)\left[1-\frac{4M^2x^2\nu^2}{Q^4}+\right.$$

$$\left.+\left(\frac{2Mx\nu}{Q^2}\right)^4+\ldots\right]=T_\infty^{In}+T_F^{In}\quad, \tag{17}$$

with

$$T_\infty^{In} = \frac{4M(Q^2+2\nu^2)}{Q^4}\int_0^1 dx\; F_2(Q^2,X) \quad ; \quad T_F^{In} = T^{In} - T_\infty^{In} \quad , \qquad (18)$$

i.e., the first term in the squared bracket expansion of Eq.(17) is contained in T_∞^{In} while the rest of the expansion gives T_F^{In}.

When T^{In} is inserted in the Cottingham representation [Eq.(2)], T_∞^{In} is the culprit for the logarithmic divergence, while T_F^{In} gives a finite contribution. With Eq.(15) the situation is much better. It is easy to see that when inserted into Eq.(15), T_∞^{In} yields the logarithmic divergence $\int dQ^2/Q^2$ multiplied by the coefficient

$$\int_0^1 dy\, \sqrt{1-y^2}\,\,(2y^2-1)y^2 = 0 \quad , \qquad (19)$$

where, again $y = \nu/Q$. Then, instead of Eq.(15) we are effectively left with

$$\Delta = \frac{2\alpha}{(2\pi)^3}\int \frac{d^3k\, d\nu\, \nu^2}{Q^4}\,(1-Q^2\partial/\partial\nu^2)\left[T^{Su}+T^{Co}+T_F^{In}\right] \quad , \qquad (20)$$

which is finite.

Since we still do not know what $T^{Su}(-Q^2)$ is, we have to transform Eq.(20) into a more convenient expression. In order to do that let us consider the integrals

$$J_n = (-1)^n i\int \frac{d^3k d\nu\nu^2 (K^2)^n}{(\nu^2-K^2)^{2+n}}\,T(K,\nu) = -i\int \frac{d^3k(3\beta+i)\nu^2 d\nu (K^2)^n\, T[K,(i+\beta)\nu]}{[(1-2i\beta)\nu^2+K^2]^{n+2}} \quad , \qquad (21)$$

where, as before, the second form follows from the first after a rotation $\nu \to \nu \exp i[(\pi/2)-\beta]$ with $\beta \ll 1$. Also as before, the vanishing of the β derivative leads to a useful relation, i.e.,

$$\int \frac{d^3k d\nu (K^2)^n T}{(Q^2)^{n+2}} = \frac{2}{(2n+1)}\int \frac{d^3k d\nu\nu^2 (K^2)^n}{(Q^2)^{n+2}}\,\nu^2\frac{\partial T}{\partial\nu^2} + \frac{2(n+2)}{(2n+1)}\int \frac{d^3k d\nu\nu^2 (K^2)^{n+1}}{(Q^2)^{n+3}} \quad . \qquad (22)$$

Applying Eq. (22) (N+1) times to Eq. (20) we obtain

$$\Delta = \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q d\nu\,\sqrt{Q^2-\nu^2}\,\left\{\nu^2\,\frac{\partial(T^{Co}+T_F^{In})}{\partial\nu^2}\left[\frac{\nu^2}{Q^2}\sum_{n=0}^{N}\frac{(2n+2)!!}{(2n+1)!!}(1-\nu^2/Q^2)^n - 1\right]\right.$$
$$\left. + \frac{(2N+4)!!}{2(2N+1)!!}\,\frac{\nu^2}{Q^2}\left(1-\frac{\nu^2}{Q^2}\right)^{N+1}\left[T^{Su}+T^{Co}+T_F^{In}\right]\right\} \quad . \qquad (23)$$

Now the last term, the only one that might contain ν-independent parts as in T^{Su}, is

$$\frac{\alpha}{8\pi}\,\frac{(N+3/2)}{(N+2)(N+3)}\int_0^\infty \frac{dQ^2}{Q^2}\,T(-Q^2, i\, y_N\, Q) \quad , \qquad (24)$$

vanishing in the $N\to\infty$ limit and living us with the representation

$$\Delta = \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q \sqrt{Q^2-\nu^2}\,\nu^2 d\nu\,\frac{\partial(T^{Co}+T_F^{In})}{\partial\nu^2}\left\{\frac{\nu^2}{Q^2}\sum_{n=0}^{\infty}\frac{(2n+2)!!}{(2n+1)!!}\left(1-\frac{\nu^2}{Q^2}\right)^n - 1\right\} \qquad (25)$$

or, rearranging term,

$$\Delta = \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q \sqrt{Q^2-\nu^2}\,\nu^2\, d\nu\,\frac{\partial(T^{Co}+T_F^{In})}{\partial\nu^2}\times$$
$$\times\left\{1 - 2\left(1-\frac{\nu^2}{Q^2}\right) + \frac{\nu^2}{Q^2}\sum_{n=1}^{\infty}\frac{(2n+2)!!}{(2n+1)!!}\left(1-\frac{\nu^2}{Q^2}\right)^n\right\} \quad . \qquad (26)$$

The summation term in this equation can still be rearranged according to

$$\frac{\nu^2}{Q^2}\sum_{n=1}^{\infty}\frac{(2n+2)!!}{(2n+1)!!}\left(1-\frac{\nu^2}{Q^2}\right) = \frac{8}{3}\left(1-\frac{\nu^2}{Q^2}\right) + \sum_{n=2}^{\infty}\frac{(2n)!!}{(2n+1)!!}\left(1-\frac{\nu^2}{Q^2}\right)^n \quad . \qquad (27)$$

This allows us to rewrite Eq. (26) as

$$\Delta = \frac{\alpha}{\pi^2}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^Q \sqrt{Q^2-\nu^2}\,\nu^2\, d\nu\,\frac{\partial(T^{Co}+T_F^{In})}{\partial\nu^2}\sum_{n=0}^{\infty}\frac{(2n)!!}{(2n+1)!!}\left(1-\frac{\nu^2}{Q^2}\right)^n \quad , \qquad (28)$$

which is exactly the same as Eq.(13) already presented in Ref.(2), except for the fact that the infinity producing T_{∞}^{In} does not appear now. Thus, the representation (27) is finite, does not depend on the unknown T^{Su} and can be calculated completely. The contribution to Δ from T^{Co} ($\Delta^{(1)}$) and the leading contribution from t_F^{In} ($\Delta^{(2)}$) were already given in Ref.(2) as[6]

$$\Delta^{(1)} = -1.36 \text{ MeV} \quad , \quad \Delta_4^{(2)} = -0.04 \text{ MeV} \qquad (29)$$

adding up to a theoretical proton-neutron mass difference of $\Delta = -1.40$ MeV to be compared with the experimental $\Delta = -1.29$ MeV.

A comparison of this work with that of Ref.(2) might be in order. There, I presented the representation (13) for Δ which does not depend on the unknown T^{Su} but still contains the logarithmic divergence. In models exhibiting the Weinberg-'t Hooft mechanism a regulating factor tames that divergence and, in particular, the T_{∞}^{In} term contributes to Δ with 0.09 MeV. Thus, in Ref.(2) the much better value $\Delta = -1.27$ MeV was obtained.

The results of the present paper are much more general. It is shown that in representations like (28) obtained from Eq.(15), Δ is finite since the coefficient of the logarithmically divergent term vanishes[7]. Thus, there is no need to invoke special features like the Weinberg-'t Hooft mechanism which only holds in certain models. Except for the assumption that the up-down quark mass splitting originates from electroweak radiative corrections, the results of the present work are model independent.

FOOTNOTES AND REFERENCES

1 - The parametrizations, normalizations etc. used for $T_{\mu\nu}$ as well as other conventions are as in Ref. (2) where citations to previous work are also given.

2 - I. Kimel, Phys. Rev. D27, 2129(1983).

3 - W.N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

4 - C. Callan and D.J. Gross, Phys. Rev. Letters 22, 156 (1969).

5 - S. Weinberg, Phys. Rev. Letters 29, 388 (1972); G. 't Hooft, Nucl. Phys. B35, 167 (1971).

6 - For details of the calculation see Ref. (2).

7 - This is to be contrasted with calculations based on the Cottingham formula where the logarithmic divergence is even immune to the possible softening expected from asymptotic freedom. See G.B. West, invited talk in Recent Developments in High Energy Physics, Orbis Scientiae 1980, eds. A. Perlmutter and L.F. Scott (Plenum Press, N.Y. 1980); and J. Kiskis and G.B. West, Phys. Rev. Lett. 45, 773 (1980).