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Abstract

We study a field-theoretical model where the decay rate of the false vacuum can be computed up to the first quantum corrections in both the high-temperature and zero-temperature limits. We find that the dependence of the decay rate on the height and width of the potential barrier does not follow the same simple area rule as in the quantum-mechanical case. Furthermore, its behaviour is strongly model-dependent.

I. INTRODUCTION

Questions relating to phase transitions in the very early universe have attracted a great deal of attention recently. This interest arouse chiefly because of the recent afforts aiming at the unification of the theories which are believed to account for the basic forces of nature-namely, Quantum Chromodynamics [1], which describes the strong interactions, and the Weinberg-Salam-Glashow model [2] for the electroweak forces - into a single Grand Unified Theory [3,4] . Candidates for such a theory must - much as the Weinberg-Salam-Glashow model itself - undergo a spontaneous symmetry breakdown in order to reproduce the "low" energy phenomenology observed in the laboratory.

It is a general feature of theories with a symmetry which is spontaneously broken at zero temperature that, for temperatures above a critical value, this symmetry is restored [5]. The physical system under study will thus possibly exist in one of two different phases: the unbroken and the broken-symmetry phases. Since the temperature at the very early stages of the universe was extremely high one aspects that phase transitions would occur as the universe gets older and cooler. Their study should play a relevant role in understanding many aspects of our universe such as it is today. Among these we shall mention the present density of monopoles, walls and strings which are expected to have been produced in these phase transitions, [6] and the so-called flatness and horizon problems which might be understood if the conjecture that the universe has undergone a supercooling is correct [7].

The most popular way of studying such phase transitions is the analysis of the effective potential [8] at finite temperatures. Figure (1) shows the typical behaviour of the effective potential of a gauge theory as the temperature is lowered. The phase transition pictured is a first-order one. The picture we have of the evolution of the system is the following [5]: at high temperatures, the symmetry is unbroken; the corresponding effective potential is shown in fig.(1a). As the temperature is lowered, the potential acquires local minima for values of the field different from zero (fig.(1b)).There will be a critical value of the temperature for which these local minima will be degenerate with the symmetric minimum (fig.(1c)); and for still lower temperatures they will have lower energy, thus rendering the symmetric vacuum metastable (fig.(1d)).

Two mechanisms compete to drive the system from the symmetric, false vacuum, to the asymmetric real vacuum. There are, on the one hand, the classical thermal fluctuations, and, on the other, the quantum tunneling through the barrier separating the two vacua. The balance between these two mechanisms will depend upon the details of the barrier. When the system begins to overcome the barrier, a bubble of real vacuum will form inside the false vacuum [9]. Radiation will occasionally fill this bubble, if not all of the energy difference between the two vacua goes into expanding its wall.

In this work we will study a model which exhibits a phase transition having the same features of the more realistic models described by the effective potential approach. Besides finding the bounce solution (which is relevant for the computation of the decay rate of the false vacuum), we will be able to compute also the determinant associated with the fluctuations around the bounce in the two extreme situations of very high and very low temperatures. We will get in this way a closed expression for the decay rate due to thermal fluctuations and tunneling.

The paper is organized as follows. In section II, we present a brief review of the Functional Integration approach, setting the stage for the following sections. In section III we introduce the model and study its decay at zero temperature (sub section a); its subsection b gives an account of the finite-temperature decay amplitude of the false vacuum and finally a summary (Section IV) closes the main body of the paper. In an Appendix we apply the formalism of the generalized zeta function for regularizing and computing the pre-exponentail factor.

II. FUNCTIONAL INTEGRATION FORMALISM

We will briefly review in this section the functional integration formalism applied to Quantum Field Theories at finite temperature and with a single scalar field [10].

To define a field theory at finite temperature the partition function associated with its canonical ensemble must be specified. The partition function is given by

$$Z = \mathrm{Tr}\, e^{-\beta H} \tag{1}$$

where β is the inverse of the temperature and H the Hamiltonian (units are such that c=k=1). We can write this partition function in the Feynman path-integral representation [11]: it is a functional integral over all periodic (antiperiodic for fermions) field

configurations with period β, with the exponential of minus the Euclidean action as the integrand*. For models involving just one scalar field we have

$$Z = N \int \mathcal{D}\phi \, e^{-S_E[\phi]/\hbar} \tag{2}$$

Here N is a normalization factor, to be defined later, and S_E is obtained from the expression for the Euclidean action,

$$S_E[\phi] = \int d^4x \left[\frac{1}{2}(\partial_4\phi)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V[\phi] \right] \tag{3}$$

through the formal substitution $x_4 \to \hbar\beta\tau$, with the understanding that the field ϕ obeys the periodic boundary condition

$$\phi(0,\vec{x}) = \phi(\hbar\beta,\vec{x}) \tag{4}$$

and that the integral in (3) extends from $\tau=0$ to $\tau=1$. We thus write

$$\frac{S_E[\phi]}{\hbar} = \beta\int_0^1 d\tau \int d^3x \left\{ \frac{1}{2\hbar^2\beta^2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V[\phi] \right\} \tag{5}$$

---

* For the sake of convenience, we shall adopt a terminology which is quite widespread, although inaccurate: the "Euclidean action" defined in eq.(5) below is not really an "Euclidean" object. What we are in fact doing in the case of finite temperature is to work in the Matsubara representation [12], where temperature replaces time out from the start as the relevant parameter in following the evolution of physical observables.

In the semiclassical limit, where $\hbar \to 0$, the dominant contribution to the partition function in (2) will come from those field configurations which minimize the classical Euclidean action and therefore obey the Euler-Lagrange equations of motion,

$$\sum_{i=1}^{4} \frac{\partial^2}{\partial x_i^2} \phi_c = V'[\phi_c] \tag{6}$$

where $V' = \frac{\delta V}{\delta \phi}$ and ϕ_c must satisfy the boundary condition (4). Moreover, for high temperatures, the integrand in (5) is peaked around static (i.e., τ -independent) solutions, since the term $(\hbar\beta)^{-2}\left(\frac{\partial \phi}{\partial \tau}\right)^2$ will otherwise lead to much larger values for $S_E[\phi]$. So, the configurations relevant in the semiclassical limit must verify

$$\nabla^2 \phi_c = V'[\phi_c] \tag{7}$$

and the classical action will be given by

$$S_{c\ell} = \hbar\beta \int d^3x \left\{ \frac{1}{2}(\vec{\nabla}\phi_c)^2 + V[\phi_c] \right\} = \beta S^{(3)} \tag{8}$$

where $S^{(3)}$ is the three-dimensional classical action. Solutions to the classical equations of motion such that $\phi_c(x_i = \pm\infty) = \phi_F$ where ϕ_F is the field configuration which corresponds to a false or metastable vacuum as discussed in the Introduction, are called "bounces" [13].

The corrections to the partition function arising from fluctuations around the classical solutions may be evaluated

by making a functional Taylor expansion of S_E in $\eta = \phi - \phi_c$. If we keep the quadratic term in η only (there is no term linear in η because ϕ_c is a solution to the equation of motion, $\delta S_E/\delta\phi = 0$), then the resulting expression for Z involves a gaussian integral, and can be written as

$$Z = N e^{-S_{c\ell}/\hbar} \int \mathcal{D}\eta \exp\left\{ -\frac{1}{\hbar} \int d^4x\, \eta \left[-\frac{1}{2}\sum_{i=1}^{4} \partial_i^2 + \frac{1}{2} V''[\phi_c] \right] \eta \right\} =$$
$$= N' \det^{-\frac{1}{2}}\left(-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c] \right) e^{-S_{c\ell}/\hbar} \tag{9}$$

For finite temperature, $x_4 = \hbar\beta\tau$, the "time" integral goes from 0 to $\hbar\beta$ and $\eta(0,\vec{x}) = \eta(\hbar\beta,\vec{x})$.

To compute the functional integral (or, equivalently, the determinant) in (9), we choose a set of orthonormalized eigenfunctions $\eta_n(x_4,\vec{x})$ of the operator $-\sum_{i=1}^{4}\partial_i^2 + V''[\phi_c]$ such that

$$\left(-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c] \right) \eta_n(x_4,\vec{x}) = \epsilon_n \eta_n(x_4,\vec{x}) \tag{10}$$

and then make the expansion

$$\eta(x) = \sum_n a_n \eta_n(x) \tag{11}$$

The measure of the functional integral becomes then

$$\mathcal{D}\eta = \prod_n \mu\, da_n \tag{12}$$

where μ is a measure-normalizing factor. Integration over the a_n's will lead to the following expression for the partition function

$$Z = N \prod_n \int \mu\, da_n\, e^{-\frac{\epsilon_n}{2\hbar} a_n^2}\, e^{-S_{ce}/\hbar} =$$
$$= N \prod_n \left(\frac{2\pi\hbar\mu^2}{\epsilon_n} \right)^{1/2} e^{-S_{ce}/\hbar} \tag{13}$$

But not all eigenvalues of the operator $-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c]$ are positive-definite. In fact, there is a set of zero eigenmodes for ϕ_c nontrivial (that is, not constant). They are the functions $\eta_i = \partial_i \phi_c$. These zero eigenvalues reflect the space (or space-time at zero temperature) translation invariance of the solutions to the equation of motion (6). To take into account this invariance we integrate over all possible locations of the bounce, thus getting a volume factor V (or VT at zero temperature). This is of course equivalent to integrating over the a_n which corresponds to the zero mode while dropping the zero eigenvalue from the determinant, and multiplying the final result by a factor $\mu(S_{ce})^{1/2}$ for each zero mode in order to match the normalizations of $d\chi_i$ and da_n.

This is not all, however. As the zero modes will have a node at the point where $\partial_i\phi_c = 0$, they are not the lowest-eigenvalue solutions: the operator $-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c]$ has at least one negative eigenvalue. This situation signals the existence of a metastable state. The imaginary part of the functional integral (2), in the presence of this metastable state is given by [14]

$$\mathrm{Im}\, Z_{\text{one bounce}} = \frac{1}{2} N' V T \left| \det{}'\left(-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c] \right) \right|^{-1/2} \mu^4 (S_{ce})^2\, e^{-S_{ce}/\hbar} \tag{14}$$

where the prime indicates that the zero eigenvalues must be omitted form the determinant. To get the final answer, we must still sum over all possible configurations which are relevant in the decay of the false vacuum, that is to say, over all configurations containing any arbitrary number of bounces. This sum may be easily done in the dilute-gas approximation, where the bounces are supposed to be far apart and non-overlapping. The important result is that

$$Z \sim \exp\left(i\, \mathrm{Im}\, Z_{\text{one bounce}} \right) \tag{15}$$

The decay probability of the metastable vacuum per unit time per unit volume will then be given by [14,15]

$$\Gamma = N \left| \det{}'\left(-\sum_{i=1}^{4} \partial_i^2 + V''[\phi_c] \right) \right|^{-1/2} \mu^4 (S_{ce})^2\, e^{-S_{ce}/\hbar} \tag{16}$$

where a renormalization constant has been absorbed in <u>N</u>.

III. THE MODEL

Consider a model where the potential depends on the temperature in the way described in the Introduction. Such a situation might be simulated by taking

$$V[\phi, T] = \frac{1}{2} M^2 \phi^2 + B(T)\, \phi^2 \ln \frac{\phi^2}{c^2} + \frac{\lambda}{4} \phi^4 \tag{17}$$

and by choosing an appropriate function B(T). At high temperatures, the potential should look like that depicted in fig.(2a). For T lower than a critical temperature T_c, and for B(T) adequately chosen, we have the case shown in fig.(2b). For B(T)=0, the ground state of the system corresponds to the field configuration $\phi = 0$.

In order to compute the decay rate of the metastable vacuum we shall make two approximatione. First, we assume that after the system has reached the situation in which we are interested, which is that depicted in fig.(2b), the dependence of B(T) on the temperature is sufficiently weak to be neglected. This assumption is inessential but it simplifies the computations; the potential will have in this case the same form as the effective potential in some two-dimensional models [16].

Secondly, since the decay of the false vacuum is controlled, in the WKB approximation, by the potential barrier, we neglect the $\lambda\phi^4$ term in (17). A similar approximation made by Witten in ref. [17] gives results which are in good agreement with numerical computations [18].

Our model will thus be given by the Bialynicki-Birula-Micielsky potential [19],

$$V[\phi] = \frac{m^2}{2}\phi^2\left(1 - \ln\frac{\phi^2}{c^2}\right) \tag{18}$$

a) <u>ZERO TEMPERATURE</u>

The potential defined in the equation (18) has a maximum for $\phi_M = C$ with value

$$V[\phi_M] = \frac{m^2c^2}{2} \tag{19}$$

It has two zeroes, at $\phi = 0$ and at

$$\phi_0 = c\sqrt{e} \tag{20}$$

and the Euclidean equation of motion reads

$$\left(\frac{\partial^2}{\partial x_4^2} + \vec{\nabla}^2\right)\phi + m^2\phi \ln\left(\frac{\phi^2}{c^2}\right) = 0 \tag{21}$$

At zero temperature, we will look for a solution of this equation which has an O(4) symmetry, so that the field will be a function of the variable $\rho = (x_4^2 + \vec{x}^2)^{1/2}$ only. This solution will have no nodes, so that we expect it to lead to the lowest action among the possible non-trivial solutions of (20) [13]. Under this assumption, equation (21) becomes

$$\frac{d^2\phi}{d\rho^2} + \frac{n-1}{\rho}\frac{d\phi}{d\rho} + m^2\phi \ln\left(\frac{\phi^2}{c^2}\right) = 0 \tag{22}$$

where we allowed the dimension of space-time to be <u>n</u>. We are interested in a solution with the following boundary conditions:

$$\phi(\rho = \pm\infty) = 0 \tag{23}$$

which means that the particle is sitting in the false vacuum $\phi=0$ at $\rho = \pm\infty$, (this is required of the solution is to have finite action), and

$$\left.\frac{d\phi}{d\rho}\right|_{\rho=0} = 0 \tag{24}$$

The value of the field at this point is such that $\phi_F \leq \phi(0) < \phi_T$ where ϕ_T is the configuration which corresponds to the "true" vacuum, and ϕ_F that corresponding to the false or metastable one. If we interpret ϕ as position and ρ as time, then the equation (22) with the boundary conditions (23) and (24) can be viewed as the equation of motion of a classical particle under the action of the potential $V = -\frac{m^2}{2}\phi^2\left(1 - \ln\frac{\phi^2}{c^2}\right)$ and a dissipative force $\frac{n-1}{\rho}\frac{d\phi}{d\rho}$ [13]; this particle is released at the point $\phi(0)$ at the instant $\rho = 0$; then it slides down the potential and, slowing down because of the dissipative force, it reaches $\phi=0$ at $\rho=\infty$.

The function

$$\phi_c = c\exp\left(-\frac{1}{2}m^2\rho^2 + \frac{n}{2}\right) \tag{25}$$

is a solution of this problem. The action for this bounce is

$$\begin{aligned}\frac{S_{cl}^{(n)}}{\hbar} &= \frac{2\pi^{n/2}}{\Gamma(n/2)}\int_0^\infty d\rho\, \rho^{n-1}\left\{\frac{1}{2}\left(\frac{d\phi_c}{d\rho}\right)^2 + V[\phi_c]\right\} = \\ &= \frac{(e\sqrt{\pi})^n}{2}\,\frac{c^2}{m^{n-2}}\end{aligned} \tag{26}$$

At $\rho = 0$, where

$$\phi_c(0) = c\exp(n/2) \tag{27}$$

we have $V[\phi_c] = -\frac{(n-1)e^n}{2}m^2c^2$. The difference between the energy densities of the two vacua is

$$\Delta\epsilon = \frac{n-1}{2}e^n m^2c^2 \tag{28}$$

The bounce in this problem is gaussian; this situation is to be compared with the one studied by Coleman [13], where the false and the true vacua are separated by a small energy difference. In the latter case, the bounce is constant in ρ within a region of radius R and drops exponentially to zero outside it.

In order to compute the determinant of the operator $-\sum_{i=1}^{k}\partial_i^2 + V''[\phi_c]$, with the second derivative of the potential given by

$$V''[\phi_c] = m^2\left[m^2\rho^2 - (n+2)\right] \tag{29}$$

we must solve the Schrödinger equation for the harmonic oscillator in n dimensions,

$$\left[-\sum_{i=1}^{n}\frac{\partial^2}{\partial x_i^2} + m^4\sum_{i=1}^{n}x_i^2 - (n+2)m^2\right]\psi_{\{k\}} = 2m^2\epsilon_{\{k\}}\psi_{\{k\}} \tag{30}$$

The eigenvalues $\epsilon_{\{k\}}$ depend on n integer quantum numbers and are

$$\epsilon_{l_1l_2\cdots l_n} = \left(\sum_{i=1}^{n}l_i - 1\right) \tag{31}$$

There is a single negative eigenvalue, $\epsilon_{00\cdots0}$, and n zero eigenvalues, $\epsilon_{100\cdots0}$, $\epsilon_{010\cdots0}$, etc. The determinant may now be computed with the technique of the generalized zeta function. The details are given in the Appendix.

The decay rate for $n=4$ can then be read from eq. (16):

$$\Gamma = A\,(S_{cl})^2\, e^{-S_{cl}/\hbar} \qquad (32)$$

where

$$A = N \exp\left(\tfrac{1}{2}\zeta'(0)\right) \qquad (33)$$

and the measure-normalizing factor μ has been defined so as to absorb the factor $2m^2$ in the eigenvalues as well as the $2\pi\hbar$ coming from the gaussian integration.

b) FINITE TEMPERATURE

We shall now examine the behaviour of the model defined in the beginning of this section at finite temperature. We must look for solutions of the equation of motion (21) which are periodic in x_4 with period $\hbar\beta$. Let us try an ansatz exhibiting an O(3) symmetry:

$$\phi(x_4, r) = c\, f(x_4)\, e^{-m^2 r^2/2} \qquad (34)$$

with $r^2 = \sum_{i=1}^{3} x_i^2$. The function $f(x_4)$ is periodic in x_4 and satisfies

$$\frac{d^2 f}{d x_4^2} = m^2 f\left(3 - \ln f^2\right) \qquad (35)$$

The problem is then reduced to the classical-mechanical problem of a particle of unit mass moving in the potential

$$U(f) = -\frac{m^2}{2} f^2 \left(4 - \ln f^2\right) \qquad (36)$$

with f its position.

The action is thus given by

$$S_{cl} = \pi^{3/2}\frac{c^2}{m^3}\int_0^{\hbar\beta} dx_4 \left[\frac{1}{2}\left(\frac{df}{dx_4}\right)^2 - U(f)\right] \qquad (37)$$

The solutions to (35) are characterized by a number E, which is a "constant of motion": it is in fact the energy of the classical particle in the mechanical analogy,

$$E = \frac{1}{2}\left(\frac{df}{dx_4}\right)^2 + U(f) \qquad (38)$$

and it is bounded:

$$-\frac{m^2 e^3}{2} \leqslant E \leqslant 0 \qquad (39)$$

The upper bound corresponds to the solution with infinite period (zero-temperature case), while $E = -\frac{m^2 e^3}{2}$ for the solution with period zero (infinite temperature). We can invert the equation (38) and integrate to have the period as a function of energy. One finds

$$\hbar\beta(E) = 2\int_{f_1}^{f_0} \frac{df}{[2(E-U)]^{1/2}} =$$

$$= \int_{f_1}^{f_0} \frac{df}{\left[m^2 f^2\left(1-\frac{1}{4}\ln f^2\right)+\frac{E}{2}\right]^{1/2}} \tag{40}$$

where f_0 and f_1 are the turning points for a particle with energy E moving in the potential (36); they are the positive roots of the equation

$$U(f) = E \tag{41}$$

To have the period, put $f_1=0$, $f_0=f(\beta)$ and then just invert the equation (40).

In the limit of very high temperature the decay will be controlled by the static bounces. The classical analogy corresponds to a particle sitting at the bottom of the potential (36). The static solution to eq. (35) is

$$f(x_4) = e^{3/2} \tag{42}$$

and the field $\phi(x)$ has the form

$$\phi(x) = c\, e^{3/2}\, e^{-m^2 r^2/2} \tag{43}$$

The classical action is given in this limit by the equations (8) and (25) with n=3 or equivalently by (37) with $\frac{df}{dx_4}=0$ and $U=-\frac{m^2e^3}{2}$. We have

$$S_{ce} = \frac{S^{(3)}}{T} = \frac{\pi^{3/2}}{2}\, e^3\, \frac{c^2}{m}\, \hbar\beta \tag{44}$$

Now to fix the domain of validity of this approximation, we must precise what "high temperature" means. To do so let us investigate the behaviour of the system when the function f oscillates very near its minimum $f=e^{3/2}$. Expanding the potential around this point, we find

$$U(f) \simeq \frac{m^2}{2}\left(-e^3 + 2\tilde{f}^2\right) \tag{45}$$

where $\tilde{f}=f-e^{3/2}$. We then parametrize the energy E as

$$E = -\frac{m^2e^3}{2}(1-\epsilon) \tag{46}$$

with ϵ very small for small oscillations around the minimum of the potential. The period $\hbar\beta(\epsilon)$ will be approximately independent of ϵ (because the oscillations are almost harmonic) and

$$\beta(\epsilon) \simeq \frac{\pi\sqrt{2}}{\hbar m} \tag{47}$$

This expression, with $\beta=1/T$, fixes the meaning of "high temperature". The tunneling mechanism dominates the decay amplitude when $T \ll \frac{\hbar m}{\pi\sqrt{2}}$. For $T \gtrsim \frac{\hbar m}{\pi\sqrt{2}}$ only the thermal fluctuations given by the static solution are relevant.

For temperatures near $\frac{\hbar m}{\pi\sqrt{2}}$, the function f is given by

$$f(x_4) = e^{3/2}\left[1 + \frac{\epsilon^{1/2}}{\sqrt{2}} \sin\left(\omega x_4 + \frac{\pi}{2}\right)\right] \tag{48}$$

with $\omega = m\sqrt{2}$.

Now to have the pre-exponential factor, we must compute

$$\int \mathcal{D}\eta \, e^{-\frac{1}{2\hbar}\int_0^{\hbar\beta} dx_4 \int d^3x \, \eta \hat{O} \eta} \tag{49}$$

where $\hat{O}$ is the operator

$$-\sum_{i=1}^{4} \partial_i^2 + m^2(m^2r^2 - 5) - \sqrt{2}\, m\, \epsilon^{1/2} \sin\left(\omega x_4 + \frac{\pi}{2}\right) \tag{50}$$

The x_4-dependent part of this operator is related to Mathieu's equation . In the static case where $\epsilon \simeq 0$ and the time derivative is dropped in the operator, its eigenvalues are given by

$$\epsilon_{klm} = 2m^2(k + l + m - 1) \tag{51}$$

The spectrum has three zero and one negative eigenvalues, as should be expected. The imaginary part of the free-energy density is

$$\operatorname{Im} F_0 = -\frac{1}{2} A T \left(\frac{S^{(3)}}{\hbar T}\right)^{3/2} e^{-S^{(3)}/\hbar T} \tag{52}$$

The $S^{(3)}/T$ in the pre-exponential factor comes about because of the normalization of the zero-mode eingenfunctions. The factor A is given by

$$A = N\left(2\pi\mu^2 T\right)^{\zeta(0)/2} \exp\left(\frac{1}{2}\zeta'(0)\right) \tag{53}$$

and $\zeta(0)$, here taken for the three-dimensional oscillator, has been calculated in the Appendix. We thus finally end up with the decay probability [10,14,15] :

$$\Gamma = A \frac{T}{\hbar} \left(\frac{S^{(3)}}{\hbar T}\right)^{3/2} e^{-S^{(3)}/\hbar T} \tag{54}$$

The exponential factor in (54) above can be written in terms of the height H and the width W of the potential, defined by eqs. (19) and (20) respectively:

$$H = V[\phi_M] = \frac{m^2c^2}{2}$$

$$W = \phi_0 = c\sqrt{\epsilon} \tag{55}$$

One gets

$$\Gamma \sim \exp\left[-(\pi/e^2)^{3/4} \frac{W^3}{H^{1/2}} \beta\right] \tag{56}$$

IV. <u>CONCLUSIONS.</u>

We have studied in this paper the tunneling properties of a system for which it is possible to find exact solutions. Explicit expressions are given for the static bounce which exhibits an O(3) symmetry and is relevant in the high-temperature limit, as well as for the O(4)-symmetric solution which controls the zero-temperature behaviour of the decay rate. At nigh tempera ture, the decay is purely due to thermal fluctuations as should be expected.

One very interesting feature of the model is that it allows an exact calculation of the determinant in both the zero-temperature, four-dimensional case, and the high-temperature limit. In this limit the determinant goes as

$$\sim \exp(-\mathcal{E}'(0))\, T^{-\mathcal{E}(0)}$$

Another noticeable feature of the model studied in this work is the strong dependence of the decay rate on the height and specially the width of the potential (eq.(56)).This feature is not shared by other models which seem to imply that the height and the widht do not play a significant role in the decay process. Also, an important lesson to be learned from the model is that the intuition gained in Non-relativistic Quantum Mechanics, where the decay is controlled by the area under the potential barrier, cannot be generalized to Quantum Field Theory.

We believe that the model discussed in this paper might play a relevant role in understanding some important aspects of realistic theories and of cosmological phase transitions.

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APPENDIX

In this appendix we apply the technique of generalized zeta functions [20] to the computation of the functional determinant for the n-dimensional (in Euclidean space-time) harmonic oscillator.

The determinant of an operator is the product of its eigenvalues, which in general diverges. The generalized-zeta-function procedure for this determinant consists in defining a function $\zeta(s)$ which is the sum of the inverse powers of the eigenvalues,

$$\zeta(s) = \sum_n \epsilon_n^{-s} \tag{A1}$$

and then analytically continuing it down to $s=0$, which is in general possible.

The inverse of the square root of the determinant is easily shown to be formally identical to $\exp\left(\frac{1}{2}\zeta'(0)\right)$, where $\zeta'(0)$ is the derivative of $\zeta(s)$ at the point $s=0$. In the ζ-function method, we take this formal identity as a definition. The series (A1) must not contain, of course, negative or zero eigenvalues.

With the aid of a table of integral transforms, we find that the inverse Mellin transform of $\Gamma(s)\,\epsilon_n^{-s}$ is

$$Z_n(\alpha) = e^{-\alpha\epsilon_n} \tag{A2}$$

We thus have

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty d\alpha\, \alpha^{s-1} \sum_n e^{-\alpha\epsilon_n} \tag{A3}$$

The eigenvalues for the harmonic oscillator are those given by the formula (31) (remember the factor $2m^2$ has been absorbed in the measure of the functional integral). In this case the sum under the integral sign in (A3) above is

$$\sum{}' e^{-\alpha(\ell_1+\ell_2+\cdots+\ell_n-1)} \tag{A4}$$

where the prime means that the negative and zero eigenvalues are not included; this sum reduces to a product of n identic geometric progressions. We have then:

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty d\alpha\, \alpha^{s-1}\left[\frac{e^{\alpha}}{(1-e^{-\alpha})^n} - e^{\alpha} - n\right] \tag{A5}$$

This integral defines the generalized zeta function for $\mathcal{R}e(s)>n$, but this function is meromorphic and may be continued for $\mathcal{R}e(s)<n$.

To express $\zeta(s)$ as a single infinite series of inverse powers, develop $(1-e^{-\alpha})^{-n}$ in a Taylor series of $e^{-\alpha}$ then integrate term by term. The result is:

$$\zeta(s) = \sum_{k=1}^{\infty}\binom{k+n}{n-1} k^{-s} \tag{A6}$$

This sum converges for $\Re e(s)>n$.

We now see that the generalized zeta function for the harmonic oscillator in n dimensions may be given in terms of Riemann's zeta function, $\zeta_R(s)$. For n=3, for example, we find:

$$\zeta(s) = \frac{1}{2}\left[\zeta_R(s-2) + 5\zeta_R(s-1) + 6\zeta_R(s)\right] \tag{A7}$$

We also have

$$\zeta'(0) = \frac{1}{2}\left[\zeta_R'(-2) + 5\zeta_R'(-1) + 6\zeta_R'(0)\right] \tag{A8}$$

for the three-dimensional harmonic oscillator and

$$\zeta(0) = \frac{1}{6}\left[\zeta_R(-3) + 6\zeta_R(-2) + 11\zeta_R(-1) + 6\zeta_R(0)\right] \tag{A9}$$

in the four-dimensional case.

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FIGURE CAPTIONS

Fig.1 - Behaviour of the effective potential as a function of temperature. At very high temperatures, the ground state corresponds to $\phi = 0$ (a). Symmetry is unbroken. For low temperatures, the true vacuum will be at $\phi \neq 0$ and symmetry will be spontaneously broken (d,e) (see text).

Fig.2 - Potential for the model discussed in this paper (see text) (a) is the high-temperature case, (b) shows the potential at low temperatures.

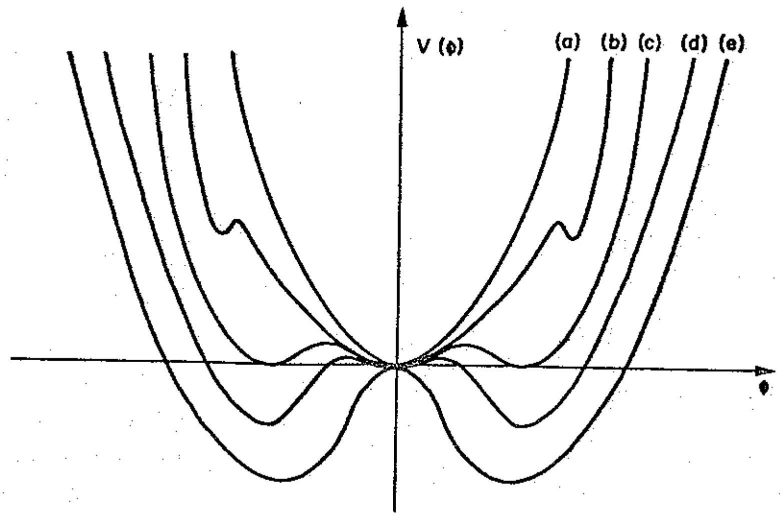
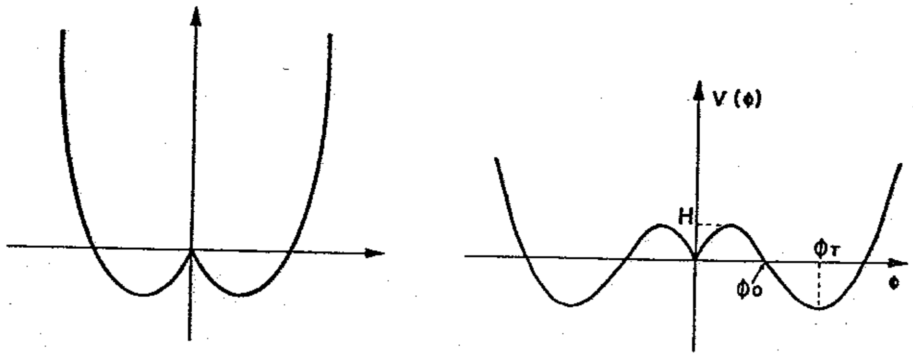


FIG. 1



(a) (b)

FIG. 2