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ABSTRACT

The Feshbach projection formalism is used to calculate the form factors for the (d,n) stripping process to isolated analog resonances. These are used in a standard DWBA stripping calculation in which the radial integration over all space is accomplished by including outerspace contributions evaluated along the complex contours of Vincent and Fortune. It turns out that the shape and magnitude of the predicted cross section is quite insensitive to the continuum proton wave emanating from the resonant residual state.

1. INTRODUCTION

A great deal of interest has been devoted to the DWBA treatment of the stripping process to unbound residual states[1,2,3,4]. Although in a variety of cases these states are analog resonances, no special attempts at exploiting this property have appeared in the literature.

In this paper we present an analysis of stripping to isolated analog resonances for which the DWBA form factors are calculated from the analog state projection formalism[5,6,7]. The result permits an explicit separation of a resonant part from the total breakup amplitude. The resonant part of the form factor is expressed as the sum of the wavefunction of a bound-like state $|A\rangle$ in the residual nucleus plus a virtual proton wave which appears as the result of the coupling of $|A\rangle$ to the non-resonant continuum.

Even though all the wavefunctions appearing in the DWBA radial integral are regular and the integral is finite, the usual numerical convergence difficulties were encountered. These were resolved using the procedure of Vincent and Fortune which employs a deformed integration contour in the complex r-plane[4].

The method was tested for an angular distribution at E_d = 13.0 MeV of the neutrons from the reaction $^{12}C(d,n_1)^{13}N$ associated with the sharp proton resonance at 2.365 MeV excitation energy in ^{13}N, the unbound mirror of the first excited state of ^{13}C. The results show that the dominant contribution to the resonant form factor comes from the wave function associated with $|A\rangle$ and that the general shape and magnitude of the calculated angular distribution turn out to be quite insensitive to the term involving the proton wave. This allows an immense simplification

to be introduced into the analysis.

Application of this analysis to an energy averaged angular distribution at E_d = 12.4 MeV for the reaction $^{13}C(d,n_1)^{13}N$ used by H. Schelin et al.[8] is included as an illustration.

2. THE RESONANCE FORM FACTOR

The phase space used to describe the residual resonant state may be analysed in terms of a) the normalized single proton state associated with an orbital u_n identical to the corresponding single-neutron parent state and b) a smooth (non-resonant) continuum, orthogonal to this single particle state. The proton resonance is evidenced by taking into account the coupling of the normalized orbital to the non-resonant continuum.

Let H be the hamiltonian for the residual system. Then

$$H\psi^{\pm}_{E_R}(\xi,r) = E_R\psi^{\pm}_{E_R}(\xi,r) \tag{1}$$

where ξ represents the internal coordinates, r the relative coordinates and E_R is the energy of the residual system. The $(\pm)$ is the usual convention to indicate that ψ satisfies outgoing or incoming boundary conditions, respectively.

The phase space decomposition described above may be expressed as

$$|\psi^{\pm}_{E_R}> = A|\psi^{\pm}_{E_R}> + P|\psi^{\pm}_{E_R}> \tag{2}$$

where the operator $A = |A><A| \equiv |u_nC><u_nC|$, with $P = 1-|A><A|$ and u_n is the single neutron orbital bound to the target core C which describes the parent state.

Substitution of equation (2) into (1) followed by operation on the left first by A, then by P gives two coupled equations

$$(E_R-H_{AA})\ A|\psi^{\pm}_{E_R}> = H_{AP}\ P|\psi^{\pm}_{E_R}> \tag{3}$$

$$(E_R-H_{PP})\ P|\psi^{\pm}_{E_R}> = H_{PA}\ A|\psi^{\pm}_{E_R}> \tag{4}$$

where the convention $H_{AA} = AHA$ and $H_{PA} = PHA$ etc. has been adherred to.

The non-resonant continuum states, denoted by $\varphi^{\pm}_{E_R}$ are obtained from

$$(E_R-H_{PP})\ |\varphi^{\pm}_{E_R}> = 0 \tag{5}$$

and

$$<A|\varphi^{\pm}_{E_R}> = 0 \tag{6}$$

In terms of these the solution of equation (4) is

$$P|\psi^{\pm}_{E_R}> = |\varphi^{\pm}_{E_R}> + \frac{1}{(E^{\pm}_{E_R}-H_{PP})}\ H_{PA}\ A|\psi^{\pm}_{E_R}> \tag{7}$$

where $\frac{1}{(E^{\pm}_{E_R}-H_{PP})}$ is the Green's function for H_{PP}.

The second term on the RHS of equation (7) involves the amplitude $\langle A|\psi^{\pm}_{E_R}\rangle$ which can be found to be, from equations (3) and (7)

$$\langle A|\psi^{\pm}_{E_R}\rangle = \left[E_R - H_{AA} - \langle A|H\frac{P}{E^{\pm}_{E_R}-H_{pp}}H|A\rangle\right]^{-1}\langle A|H|\varphi^{\pm}_{E_R}\rangle \qquad (8)$$

It thus shows resonant behavior and corresponds to virtual outgoing waves originating from the normalized analog state $|A\rangle$. The total state of equation (2) may now be written as,

$$|\psi^{\pm}_{E_R}\rangle = |\varphi^{\pm}_{E_R}\rangle + \left[1 + \frac{1}{E^{\pm}_R-H_{pp}}H_{PA}\right]|A\rangle\langle A|\psi^{\pm}_{E_R}\rangle \qquad (9)$$

For the problem on hand, this analysis of $|\psi^{\pm}_{E_R}\rangle$ was used in connection with the final proton plus target state in a standard DWBA (d,n) stripping amplitude. This leads to a non-resonant (background) contribution

$$T^{B}_{dn,p} = \langle\chi^{-}_{n}\varphi^{-}_{E_R}|V_{pn}|\chi^{+}_{d}\phi_{d}\rangle \qquad (10)$$

and a resonant contribution

$$T^{R}_{dn,p} = \left[\langle\chi^{-}_{n}u_{n}|V_{pn}|\chi^{+}_{d}\phi_{d}\rangle + \langle\chi^{-}_{n}\bar{\varphi}^{-}_{E_R}|V_{pn}|\chi^{+}_{d}\phi_{d}\rangle\right]\langle\psi^{-}_{E_R}|A\rangle \qquad (11)$$

where χ^{-}_{n} and χ^{+}_{d} are the neutron and deuteron distorted waves, ϕ_d is the deuteron wave function, and we used the substitution

$$\left[1 + \frac{1}{E^{\pm}_R-H_{pp}}H_{PA}\right]|A\rangle \rightarrow u_n(r) + \bar{\varphi}^{\pm}_{E_R}(r) \qquad (12)$$

which omits explicit reference to the core variables, since they are integrated out to one in the DWBA stripping amplitude.

The resonant amplitude $\langle\psi^{-}_{E_R}|A\rangle$, equation (8), will give rise to a peak in the neutron spectrum at the energy for which $E_R \approx \langle A|H|A\rangle$. The width of this peak is related to that of the analog resonant state. In general, interference effects between the resonant and non-resonant amplitudes will be present in the excitation function.

The first matrix element in equation (11) has exactly the structure of a standard DWBA stripping matrix element leading to a bound state in the residual nucleus with the parent state description. The second matrix element contains three continuum states and involves radial integrals which converge slowly. The phase space construction imposes the orthogonality relations

$$\langle u_n|\varphi^{+}_{E_R}\rangle = 0 \qquad \text{and} \qquad \langle u_n|\bar{\varphi}^{+}_{E_R}\rangle = 0 \qquad (13)$$

In practice equations (4) and (5) may be solved by considering instead the homogeneous equations

$$(E_R-H)\,|\varphi^{+(\lambda)}_{E_R}\rangle = \lambda|u_n\rangle \qquad (14)$$

$$(E_R-H)\,|\bar{\varphi}^{+(\alpha)}_{E_R}\rangle = \alpha|u_n\rangle + PH|u_n\rangle \qquad (15)$$

where H is the optical model Hamiltonian for the proton + ^{12}C system.

The orthogonality conditions serve to determine λ and α in a relatively simple way since, e.g., the overlap

$\langle u_n | \varphi^{+(\lambda)}_{E_R} \rangle$ is a linear function of λ. Moreover, the choice described above for $|u_n\rangle$ leads to

$$PH|u_n\rangle = PV_c|u_n\rangle \qquad (16)$$

where V_c is the Coulomb field of the core or target nucleus. A computer code TABOO[9] exists for calculating the $u_n(r)$ and for solving the inhomogeneous equations subject to the orthogonality conditions (13) to obtain the functions $\varphi^{+}_{E_R}$ and $\tilde{\varphi}^{+}_{E_R}$.

A computer code TUNEL[10] which incorporates the procedures of Vincent and Fortune[4] calculates the radial integral for the second matrix element from a point on the real axis r_p to infinity along imaginary axes $V^{\pm}$, as they suggest. The point r_p is chosen so that in the space beyond it (the outer space), only the second matrix element contributes to the radial integral. The DWUCK4 code[11] is supplied with the calculated resonance form factors up to the point r_p (this is mostly the first matrix element) and performs the integration along the real axis from $r=0$ to $r=r_p$ in the usual manner. The values of the integral in the outer space for the different partial waves is then summed with the appropriate inner space integrals of the DWBA calculation. The inner space integrals are referred to as "uncompensated" and the total space integrals as "compensated".

3. THE TEST CASE

We chose as a test case an angular distribution at $E_n = 13.0$ MeV of the neutrons from the reaction $^{12}C(d,n_1)^{13}N$. Time of flight neutron spectra have been measured for this case

at the University of São Paulo Pelletron Laboratory by H. Schelin et al.[8]. This first excited state in ^{13}N at 2.365 MeV excitation energy is unbound by 0.421 MeV. The single particle description views this state as a $2s_{1/2}$ continuum proton coupled to a ^{12}C core. The parent state is the first excited state in ^{13}C at 3.09 MeV excitation energy and is bound by 1.85 MeV. The single particle description is a $2s_{1/2}$ neutron bound to a ^{12}C core. Since the L-transfer in the stripping process is zero, this is an especially convenient case for testing the analysis.

In obtaining the neutron spectra the experimental energy spread (100 keV) was larger than the resonance width $\Gamma = 32$ keV[12]. Thus it was impossible to see any interference effects between the non-resonant background and the resonant part of the stripping amplitude in these spectra. However, they clearly showed that the background contribution in the region of the n_1 peak is indeed extremely small (if not zero) suggesting that the resonant stripping amplitude alone accounts for the integrated yield under the neutron peak. Because of resolution limitations, this is directly compared to the calculated doubly differential cross-section $d^2\sigma/dE_R d\Omega$ integrated over the resonant line. Since the resonance is narrow, the slow energy dependence of $\tilde{\varphi}_{E_R}$ is neglected in the energy integrations over intervals of the order of the resonance width.

4. RESULTS

The form factors $(u_n + \tilde{\varphi}_{E_R})$ calculated by the computer code TABOO for the 2.365 MeV resonance in ^{13}N are

shown by the full curves of figure 1a and 1b. The dashed curve of figure 1a is the parent state form factor.

The DWBA stripping differential cross sections calculated with the code DWUCK4[11] for the potentials of table 1 but using the form factors of figures 1a and 1b (full curves) with the radial integration terminating at r_p=24.0 F and r_p=34.0 F respectively, are shown in figure 2. The sensitivity of the predicted cross sections to the upper limit of the inner space radial integral is quite pronounced.

Figure 3 shows the predicted cross sections where the appropriate outerspace integrals (beginning at r_p=24.0 F and r_p=34.0 F) have been summed to the DWBA integrals. These compensated integrals give similar predictions for the differential cross section as one would expect. It should be mentioned that the number of partial waves included was such to guarantee the convergence of the radial integral with respect to this parameter.

The differential cross sections calculated for the partitioning radius 34.0 F with the radial integral compensated to include total space is shown again in figure 4 along with the differential cross section calculated by using just the parent state form factor, i.e., the u_n term of equation (11). The striking similarity of the two cross sections shows the insensitivity of the calculations to the second matrix element of equation (11), i.e., the term involving the proton wave.

The results of figure 4 suggest that the DWBA stripping analysis to isolated analog resonances is quite well accounted for by using the form factor u_n which describes the single neutron orbital of the parent state. This simplifies considerably the calculations since the usual DWBA codes may be used without complicated alterations.

As an illustration, an energy averaged angular distribution for the n_1 neutrons at E_d=12.4 MeV was chosen from the work of H. Schelin[8] for analysis. The energy averaged distribution was used in order to insure the validity of adding the direct and compound nuclear mechanisms incoherently. Justification of this energy averaging technique is given in detail by P.E. Hodgson[13,14]. The results are shown in figure 5. The dash curve is the DWBA prediction using the parent state neutron orbital to calculate the form factor and the dash-point curve is the result of the Hauser Feshbach statistical model prediction for this channel. The Hauser Feshbach calculations take no explicit account of the resonant nature of the residual state. Details of the optical parameters used for calculating these cross sections are reported in reference 8).

The solid curve of figure 5 is the 'best fit' to the experimental cross section indicated by the solid points. The curve was obtained using a direct reduction factor of 0.28 and a Hauser Feshbach reduction factor of 0.65. Discussion of these results is also given in reference 8) and will not be repeated here.

5. CONCLUSION

Stripping to isolated analog resonances may be analyzed with the standard DWBA techniques if the unbound proton form factor is replaced by the parent state bound neutron form factor. This conclusion is based on the fact that the non-resonant breakup amplitude does not contribute significantly to the stripping amplitude over the resonance itself.

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TABLE 1 - Optical parameters for DWBA code at E_d = 13.0 MeV.

	Volume Real W-S (MeV)	Surface Imaginary × 4 (MeV)	R_{ov}(F)	R_{os}(F)	a_v(F)	a_s(F)
Neutron	45	44	1.32	1.32	0.57	0.34
Deuteron	118	30.9	0.90	1.67	0.90	0.57
No spin-orbit terms used						

FIGURE CAPTIONS

Figure 1a - The solid curves represent the real part of the form factor for the resonant state in ^{13}N at E_x= 2.365 MeV which was calculated from the analog state projection formalism. The dash curve represents the parent state form factor which results from a DWBA stripping analysis.

Figure 1b - The solid curves represent the imaginary part of the form factor for the resonant state in ^{13}N at E_x = 2.365 MeV calculated from the analog state projection formalism.

Figure 2 - Behavior of the radial integral for two different values of the upper integration limit chosen along the real r-axis. The dash curve is for r_{max}= 24.0 F and the solid curve is for r_{max}=34.0 F. The resonance state form factor was used in the standard DWBA analysis.

Figure 3 - Repeat of the calculations of Figure 2 where the values of each radial integrals was corrected by the appropriate outerspace contribution. This outerspace contribution was obtained by integration along the contours of Vincent and Fortune. The dots and solid curve refer to r_{max}=24.0 F along the real axis and the triangles to r_{max}=34.0 F.

Figure 4 - The solid curve represents the DWBA differential cross section obtained using the analog resonance state form factor with the DWBA radial integral evaluated for all space. The triangles represent the cross section calculated using the bound parent state form factor.

Figure 5 - Illustration of the application of the analysis to the energy averaged cross section at $\langle E_d \rangle$ = 12.4 MeV for the reaction $^{12}C(d,n_1)^{13}N$. The dash curve is the DWBA prediction. The dash-point curve is the Hauser Feshbach prediction. The solid curve is the "best fit" to the experimental points (solid circles) which was obtained with a DWBA reduction factor 0.28 and a Hauser Feshbach reduction factor 0.65.

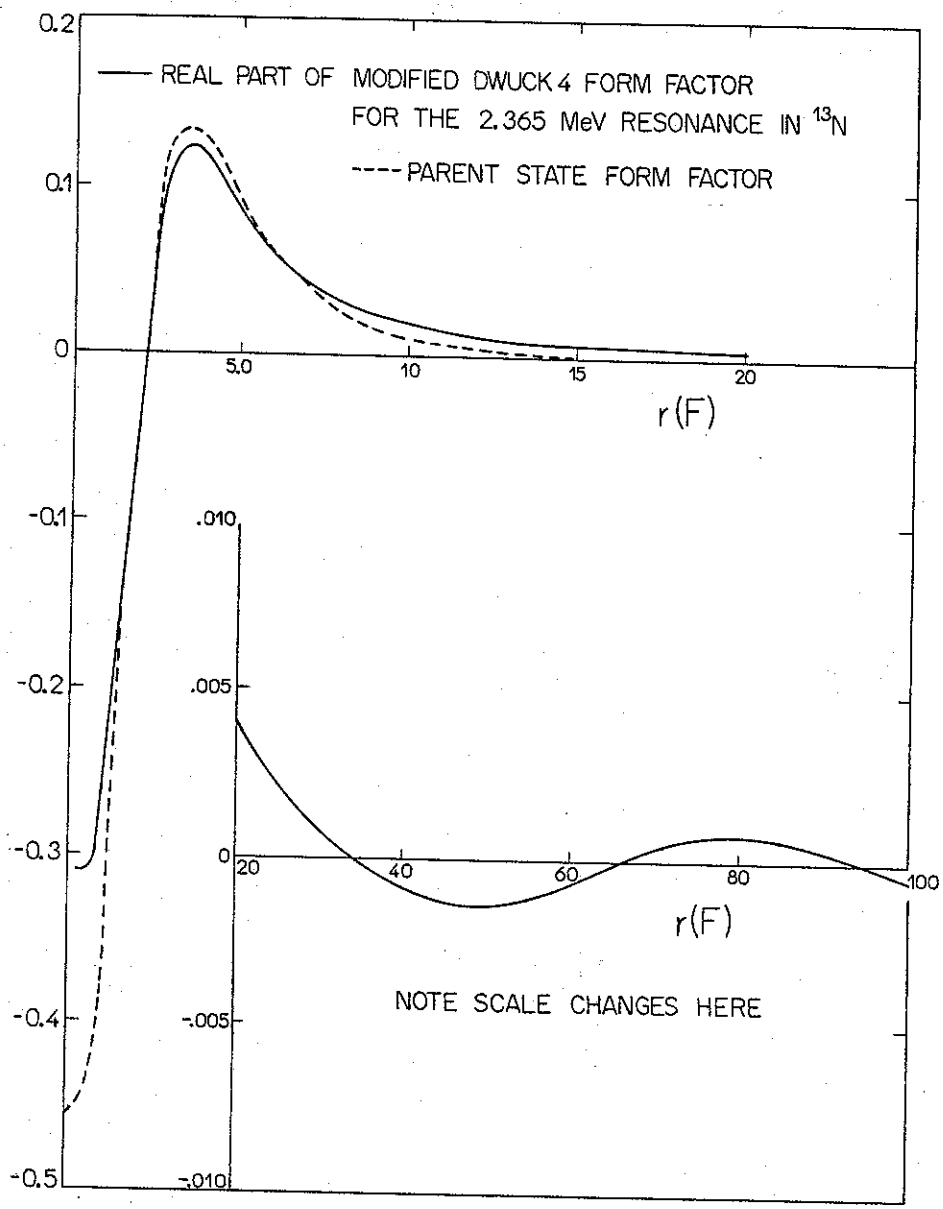

Figure 1a

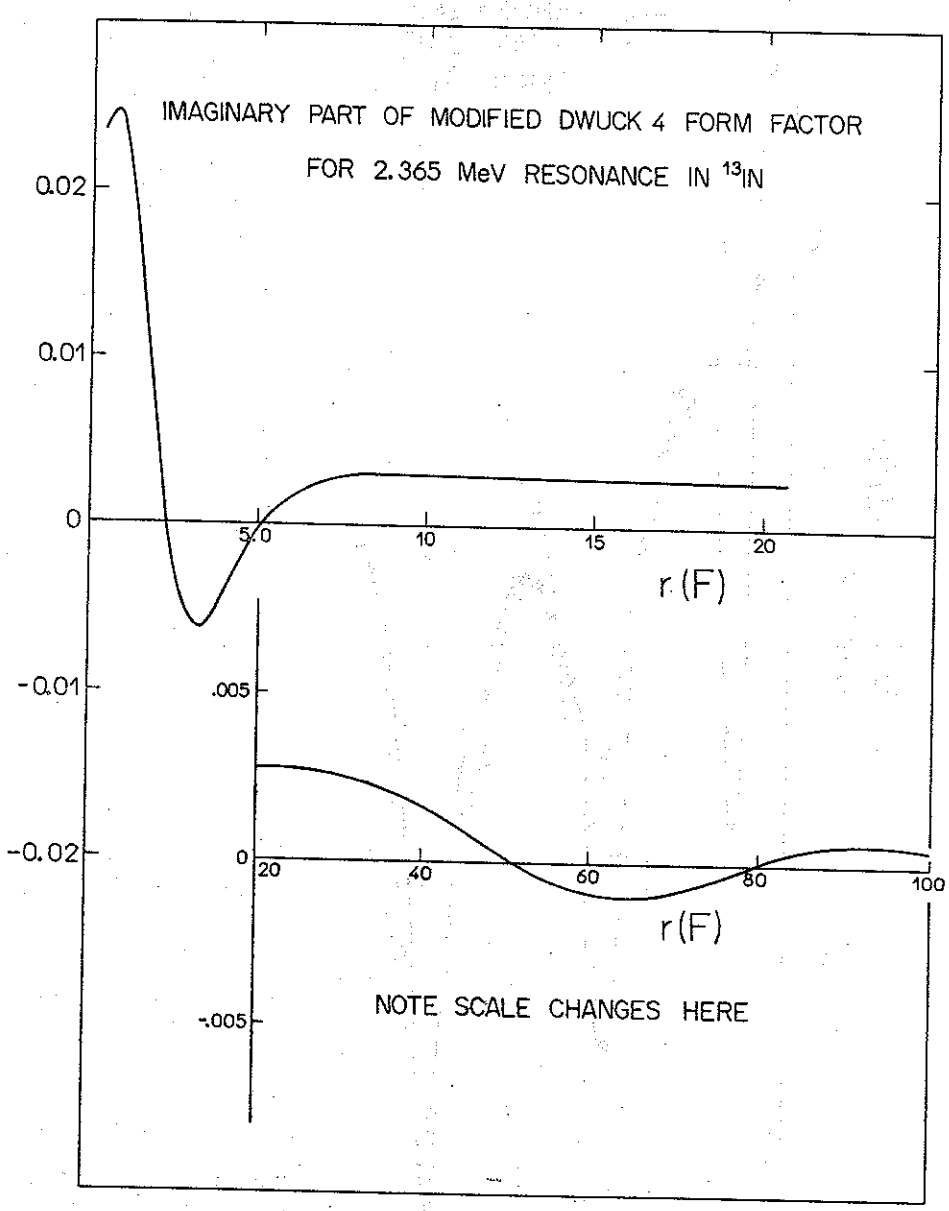

Figure 1b

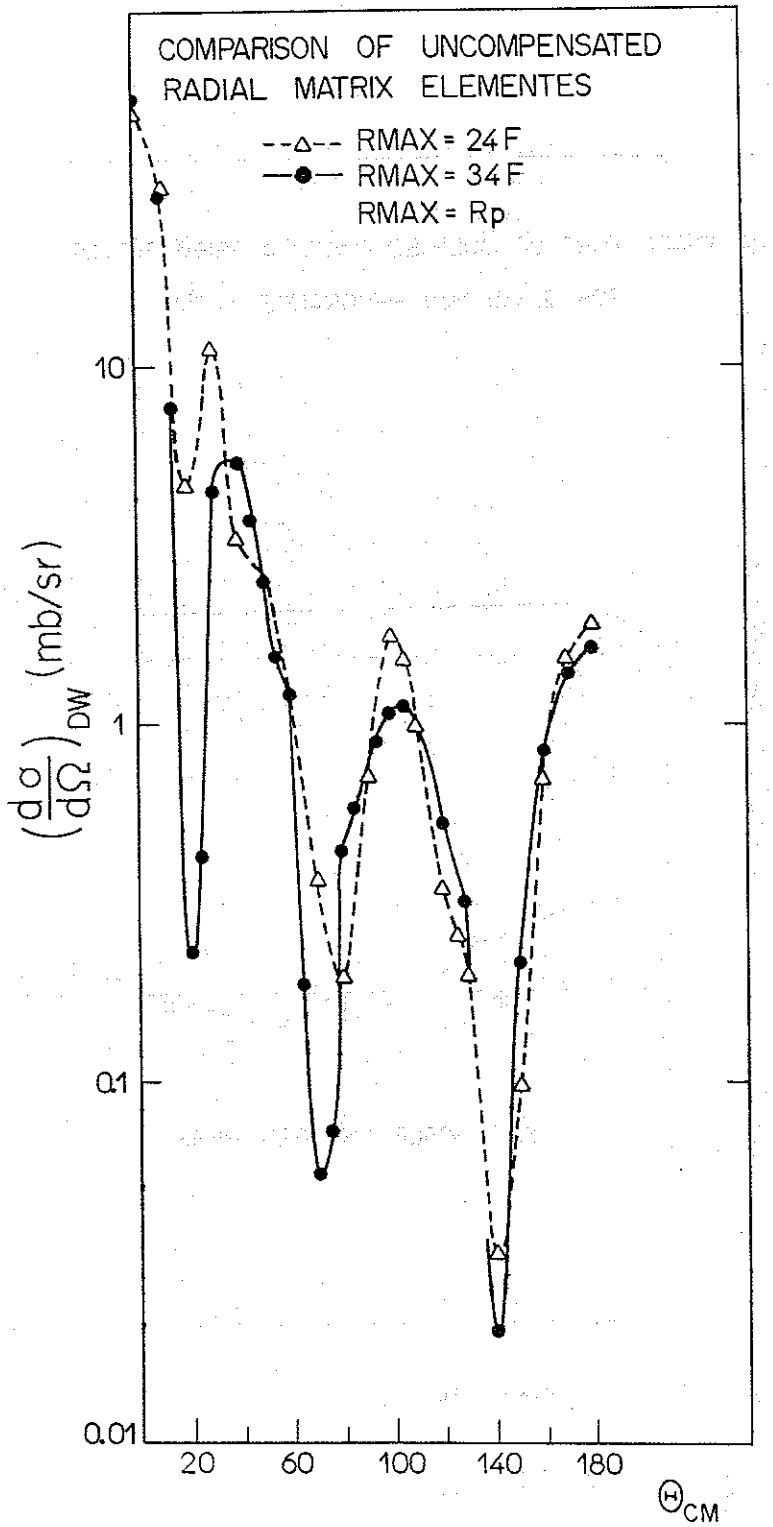

Figure 2

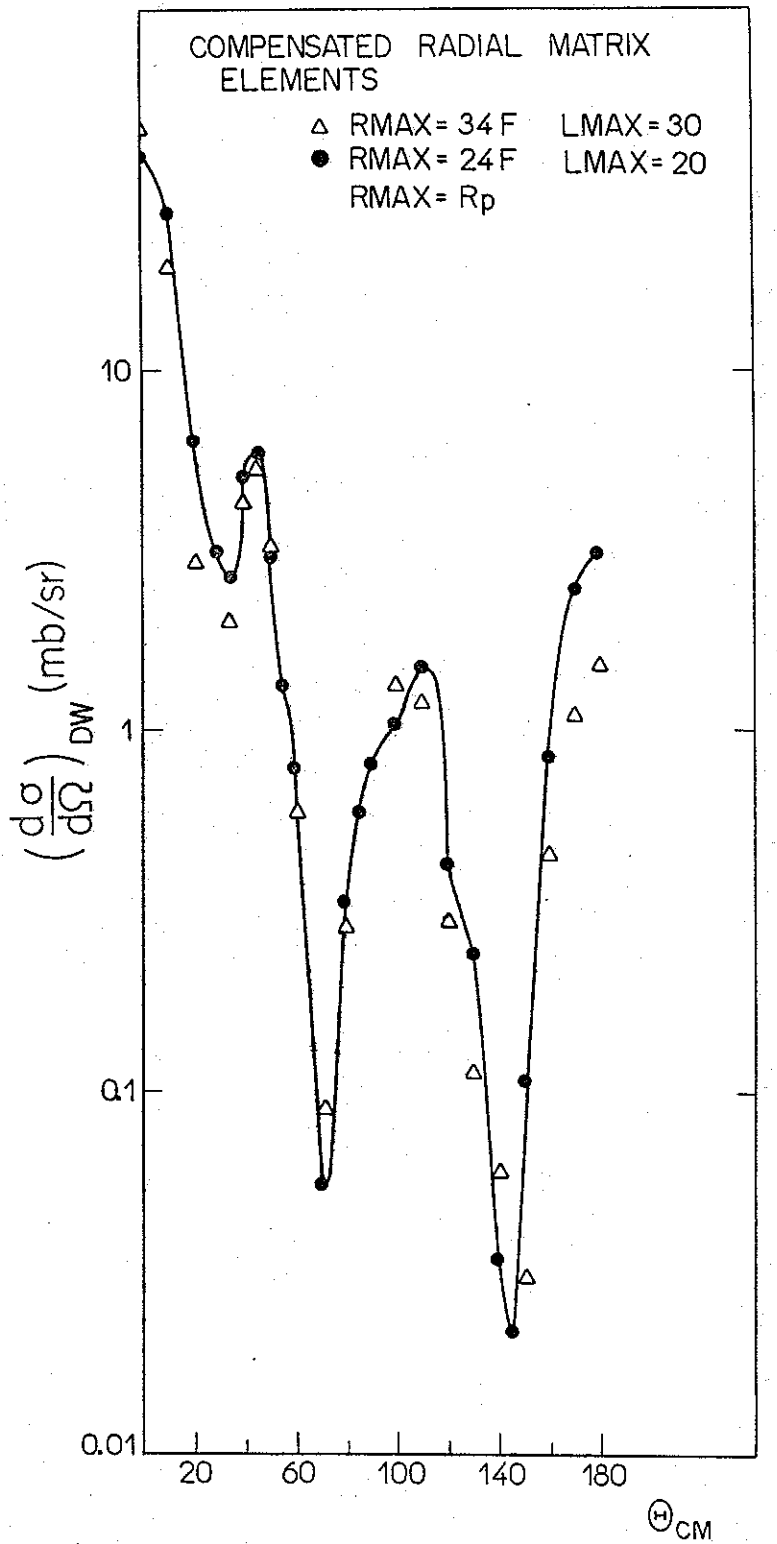

Figure 3

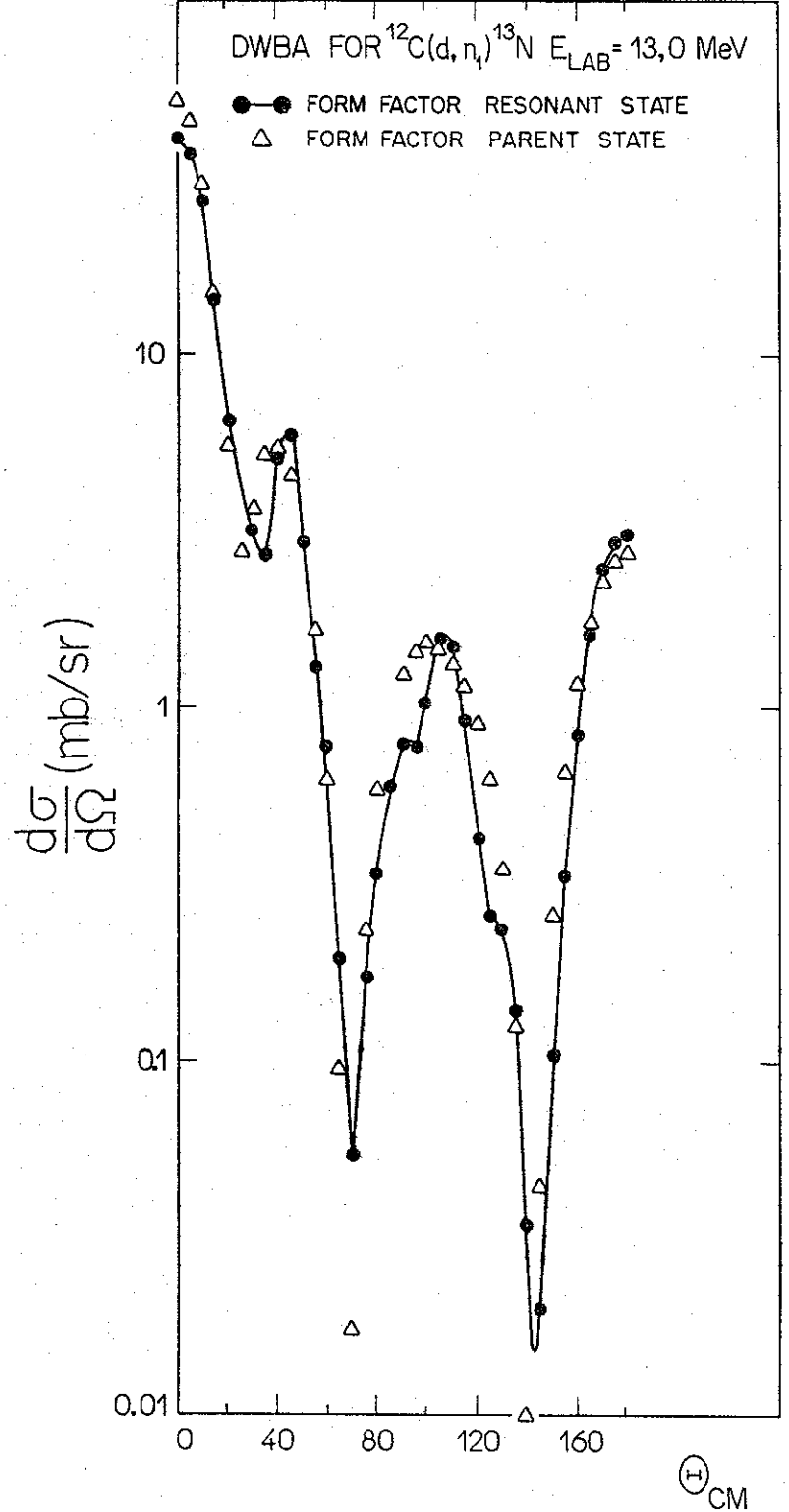

Figure 4

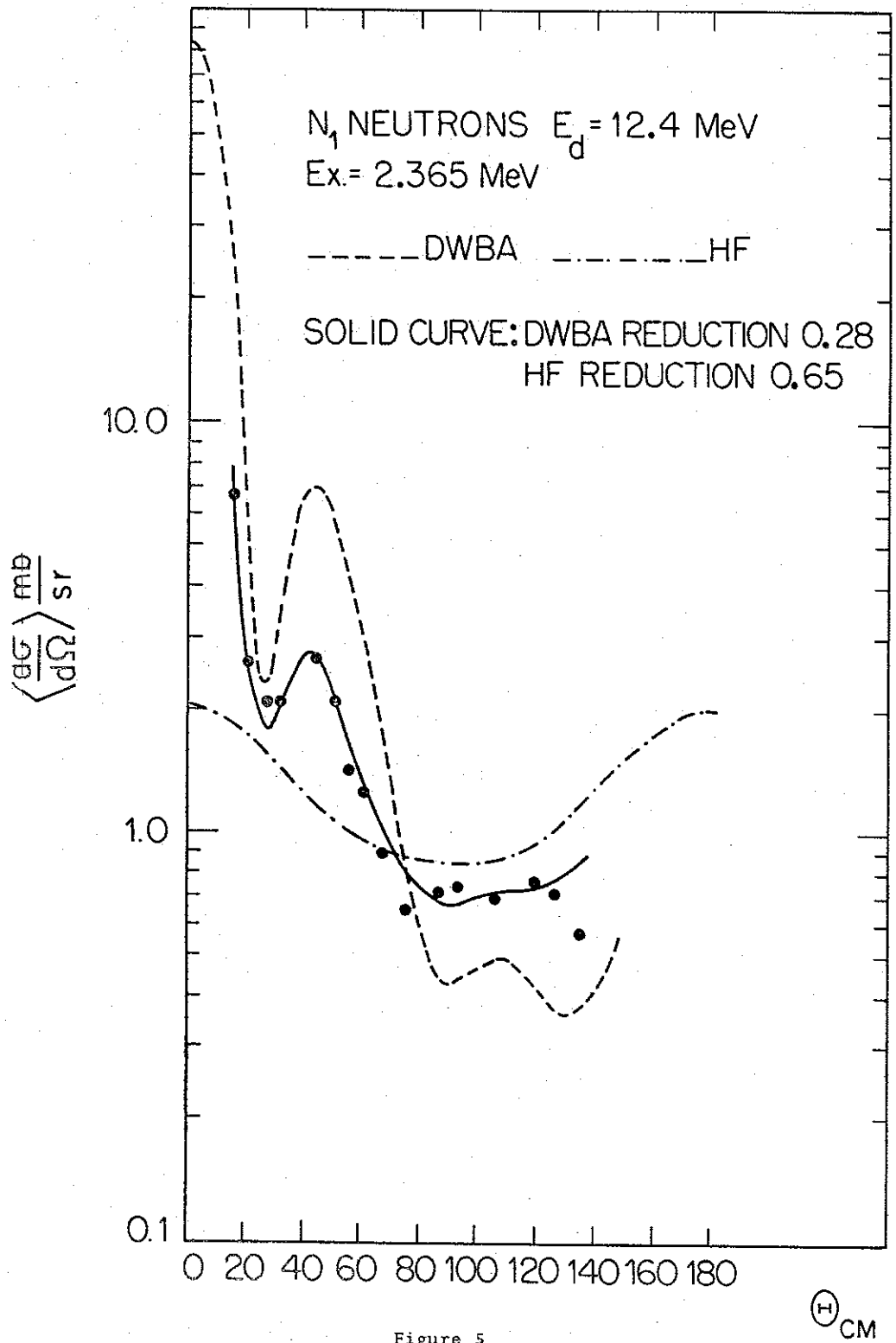

Figure 5