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STATISTICAL ASPECTS OF MULTISTEP PROCESSES IN NUCLEAR REACTIONS

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STATISTICAL ASPECTS OF MULTISTEP PROCESSES IN NUCLEAR REACTIONS†

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ABSTRACT: Several statistical aspects of nuclear multistep processes are discussed. The connection between the cross-section auto-correlation function and the average number of maxima in the excitation function is emphasized in the case of multistep compound (pre-compound) processes. The energy averaged angular distributions of transitions to specific final states proceeding through these processes are discussed in light- and heavy-ion induced reactions.

I. REVIEW OF RECENT DEVELOPMENT

In recent years the basic mechanisms underlying nuclear reactions have been profoundly reexamined in view of the increasing experimental evidence in support of new types of processes that lie, in complexity, between the usual direct and compound ones.

The theoretical description of these processes is rendered difficult as a result of their being more complicated than the simple direct processes, usually describable within DWBA or coupled channels theory, and yet less complicated than the usual compound processes normally accounted for by the statistical Hauser-Feshbach theory. This necessarily implies that the description of these preequilibrium processes must, somehow, contain both the statistical features, dominant in compound reactions, and some coherent effects (e.g., peaking in the small-angle region) that characterize direct processes.

One possible way of simplifying the theoretical description of pre-equilibrium reactions is to separate the average cross-section into two well-defined and different pieces; one describing that part of the process which is forward peaked (called multistep direct part by the MIT group[1]) and the other, symmetrical about 90°, considered as a generalized Hauser-Feshbach cross-section that describes what is called precompound or multistep compound part. In particular, this last part has been the subject of extensive theoretical discussion in the last several years. Principally, three theories have been advanced[1),2,3)], the common feature of which is their final result

summarized as a generalized Hauser-Feshbach expression for the average cross-section. This expression is given as a sum of N distinct terms related to the contributions from the N different classes of compound doorways resonances assumed populated in the process of the formation of the compound nucleus. The system is allowed to decay to the open channels from any of these stages. From the characteristics of these decay processes one should be able to learn something about the nature of the compound nucleus configurations through which the trapped incident flux percolates.

In Ref. (3) a major rôle is given to the statistical fluctuations around the average cross-section (Ericson's fluctuations) in providing the above mentioned information about the different CN stages. Through a careful study of the cross-section auto-correlation function, it is suggested that one may be able to extract at least several distinct correlation widths attached to the different lifetimes of these stages.

Clearly, for these statistical analyses to be viable, one is forced to restrict oneself to transitions leading to well-separated discrete states of the residual nucleus. The excitation functions of these transitions are expected to exhibit clear statistical fluctuations, for not too high incident energies.

The situation may be better understood through a discussion of the energy spectrum. We show in Fig. 1 a typical spectrum of emitted particles in a light ion induced reaction. This figure constitutes the prototype one usually used[1)] to describe a nuclear reaction at not too small energies. The preequilibrium portion of the spectrum is seen to be in the continuum region.

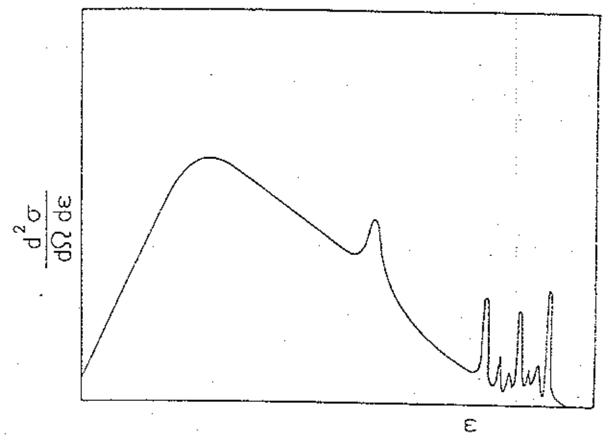

Fig. 1. A schematic plot showing a typical nuclear reaction spectrum. The individual peaks at the end of the spectrum represents direct transitions to discrete states in the residual nucleus. The broad bump indicates the evaporation component.

At somewhat smaller energies, the form of the spectrum changes, as even the discrete part of the spectrum becomes compound-nucleus dominated (90^o - symmetrical angular distribution). A possible picture of the spectrum at these lower energies is shown in Fig. 2. As is clearly implied,

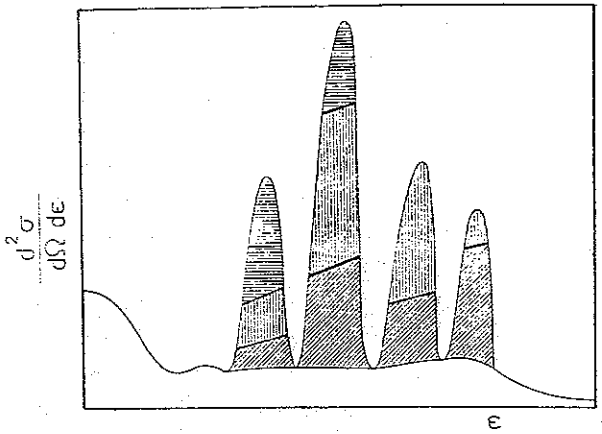

Fig. 2. A schematic plot showing the spectrum of emitted particles at lower energies. The individual peaks represent compound transitions to discrete states in the residual nucleus. The different components in each peak represent the contributions from the different stages.

individual peaks in the spectrum receive contributions from the decay of the compound system at the different stages through which it passes on its way to equilibrium. The different shaded areas in the individual peaks represent the contributions from the different pre-compound configurations.

One would therefore expect that the average cross-section for a given transition from channel c to channel c' is (all formulae below refer to the contribution of a given partial wave)

$$\langle \sigma_{cc'}^{fl.} \rangle = \sum_n \sigma_{n,cc'}^{fl.} \tag{1}$$

Similarly, the S-matrix auto-correlation function may be written as a sum of contributions

$$C_{cc'}^{(s)}(\varepsilon) = \sum_n \frac{\sigma_{n,cc'}^{fl.}}{1 + i\varepsilon/\Gamma_n} \tag{2}$$

In Eqs. (1) and (2), $\sigma_{n,cc'}^{fl.}$ is the contribution to the transition $c \to c'$ arising from the nth class of overlapping doorways. These expressions can be rigorously derived from basic statistical theory as was done in Ref. (2) ans (3). The importance of the sum-over-poles form of $C^{(s)}(\varepsilon)$ in describing the time development of the pre-compound reaction has been discussed in[4] a summary of which may be found in Kirk McVoy's talk at this conference. The Γ_n's are related to the inverses of the average lifetimes of the different classes of compound nucleus configurations.

Recently several measurements have been made on light ion systems, the aim of which is the extraction of $\sigma_n^{fl.}$ and Γ_n through a careful analysis of both σ^{fl} and the cross-section auto-correlation function $C_{cc'}(\varepsilon) = | C_{cc'}^{(s)}(\varepsilon)|^2$. In particular, the systems $^{27}Al(^3He,p)$ at $E^* = 25 - 30$ MeV, $^{25}Mg(^3He,p)$ and $^{25}Mg(^3He,\alpha)$ at $E^* = 30 - 40$ MeV, $^{27}Al(^3He,\alpha)$ at $E^* = 25 - 30$ MeV and $^{27}Al(p,\alpha)$ at $E^* = 28 - 35$ MeV, have been analysed[4] within the multistep compound model represented through Eqs. (1) and (2). In all these cases it was found that at least two distinct correlation widths, representing two different configurations of the equilibrating compound systems, were present.

II. DOUBLE-CHECKING THE RESULTS OBTAINED THROUGH $C_{cc'}(\varepsilon)$ BY THE NUMBER OF MAXIMA METHOD

Though quite convincing, the evidence obtained from the auto-correlation analysis (or the basically equivalent spectral density method[5]), discussed in the previous section, concerning the hierarchy of overlapping doorway classes contributing to the multistep compound process, has to be double checked. A way of achieving this is through a generalization of the Brink and Stephen number of maxima method[6]. These authors have shown in the one-class case that a simple counting of the number of maxima in a statistical excitation function directly supplies the average correlation width attached to that class.

Recently, this method has been discussed in the context of multi-class compound processes[7]. The average number of maxima per unit energy is given by[8]

$$\bar{n} = \frac{1}{\pi \varepsilon_0} \tan^{-1} \sqrt{4 \frac{C(0) - C(\varepsilon_0)}{C(0) - C(2\varepsilon_0)} - 1} \tag{4}$$

where $C(\varepsilon_0)$ is given by Eq. (3). In the one-class case the above expression for $\bar{n}$ reduces to the Brink result by taking the limit $\varepsilon_0 \to 0$ $\bar{n} = 0.55/\Gamma$. Considering the case of two classes in the limit $\varepsilon_0 \to 0$ gives

$$\bar{n} = \frac{0.55}{\Gamma_{eff.}(\sigma_1, \sigma_2, \Gamma_1, \Gamma_2)} \tag{5}$$

where

$$\Gamma_{eff.}(\sigma_1, \sigma_2, \Gamma_1, \Gamma_2) = \left\{ \frac{\sigma_1^2/\Gamma_1^4 + \sigma_2^2/\Gamma_2^4 + 2\sigma_1\sigma_2\left[\Gamma_1^{-4} + \Gamma_2^{-4} + \Gamma_1^{-2}\Gamma_2^{-2} - \Gamma_1^{-3}\Gamma_2^{-1} - \Gamma_1^{-1}\Gamma_2^{-3}\right]}{\sigma_1^2/\Gamma_1^2 + \sigma_2^2/\Gamma_2^2 - 2\sigma_1\sigma_2\left[\Gamma_1^{-1}\Gamma_2^{-1} - \Gamma_1^{-2} - \Gamma_2^{-2}\right]} \right\}^{-1/2} \tag{6}$$

may be considered as an effective correlation width of the two-class system. It is easy to show that Γ_{eff} satisfies the inequality

$$\Gamma_1 \leq \Gamma_{eff.} \leq \Gamma_2 \tag{7}$$

which is quite reasonable on physical grounds. Clearly before applying the above formulae for experimental excitation functions, several corrections have to be included. The most important of these is the error bar correction discussed by Bizzeti and Maurenzig[8].

The number of maxima method along the lines set above was applied[7a] to the multistep compound reaction $^{25}Mg(^3He,p)$ studies in Ref. (4). The calculation of Ref. (7a) included the error-bar correction in an approximate way. The result of $\bar{n}$ (see table 1) were found to be consistent with the multistep nature of the reaction as described by the generalized cross-section auto-correlation function.

TABLE I - Number of maxima extracted from the excitation functions of $^{25}Mg(^3He,p)$ of Ref. (4b), in the energy range $E_{^3He}$ = 8 to 16 MeV (column 2), compared with the theoretical predictions (column 3). The values of Γ_1, Γ_2, σ_1, σ_2 used in Eq. (3) are those indicated in Ref. (4b). If only the larger Γ (200 keV) would be present, $\bar{n}$ would be 6; on the other hand, for Γ = 50 keV one would obtain $\bar{n}$ = 25. The quantity δn in the last column denotes the width of the distribution of the number of maxima n whose average is $\bar{n}$. We have used the estimate $\delta n = \sqrt{\bar{n}}$.

Transition	n_{exp}	$(\bar{n} \pm \delta n)_{th}$
p_0	15	19 ± 4
p_1	16	21 ± 5
p_2	22	22 ± 5
p_3	21	20 ± 4
p_4	21	20 ± 4

III. MULTISTEP COMPOUND HEAVY-ION REACTIONS

Clearly multistep compound processes are not restricted to light-ion induced reactions. In heavy ion reactions multistep processes are in most cases actually the rule rather than the exception. This feature arises from the rather large number of degrees of freedom that are involved in heavy-ion induced reactions.

Recently several excitation functions of heavy-ion induced nuclear transitions were analysed with the generalized cross-section auto-correlation function, Eqs. (2) and (3). It was clearly demonstrated that at least two classes of overlapping resonances seem to participate in the reaction $^{12}C(^{15}N,\alpha)$ at $E^* \simeq 22 - 30$ MeV[9]. Similar behaviour was found in the reaction $^{12}C(^{16}O,\alpha)$[10]. Though the equilibrated compound nuclear stage (associated with the smaller correlation width) is well understood, the nature of the shorter-lived stage associated with the larger Γ is not well understood in this system. This is to be contrasted with light-ion reactions where a simple exciton description of tehse stages is adequate. This clearly points to the need of another physical quantity aside from Γ, that would furnish more information about the pre-compound stage in the heavy-ion reaction.

In a recent work[11], it was suggested that a careful study of the fusion cross-section in the region of its maximum value, may supply some information.

Here, we would like to suggest that the energy-averaged angular distribution to well-separated final states may supply further information. The angular distribution, in principle, directly reflects the distribution of the partial cross-sections $\sigma^{f\ell}_{n,cc'}(J)$ in angular momentum space. In the light-ion induced reactions mentioned so far, $\sigma^{f\ell}_{n,cc'}(J)$ has broad distributions centered at not too large value of J. Thus the summed partial cross-section $\sum_n \sigma^{f\ell}_{n,cc'}(J)$ would have an even broader distribution. In such situations the average angular distribution exhibits a rather structureless feature symmetrical about 90^0. This angular dependence approaches the classical phase space $\frac{1}{\sin\theta}$ - dependence at higher energies (higher values of the position in J - space of the maximum of $\sigma^{f\ell}_{n,cc'}(J)$).

It would therefore seem improbable that the angular distributions of statistically emitted particles in a light-ion induced reaction would carry any additional useful information concerning the multistep nature of the transitions involved.

The situation changes dramatically in heavy-ion induced reactions. In view of the rather well-localized $\sigma^{f\ell}(J)$ in these systems, one expects transitions to low spin states in the residual nucleus to exhibit regular angle-oscillations whose period is inversely proportional to the position of the maximum of the $\sigma(J)$.

In Figure (3) we show the $\sigma(J)$ representing the reaction $^{16}O + {}^{12}C \rightarrow {}^{24}Mg + \alpha$ proceeding through the equilibrated compound nucleus ^{28}Si as calculated within the conventional Hauser-Feshbach theory. The window-like behaviour of σ_J arises naturally from the combined effect of the inverse of the density of states that enter in $1/\sum T$ and the final channel transmission coefficient. The resulting angular distribution to low-spin final states is shown in Fig. (4). One may describe the cross-section as composed of three factors: an over-all $1/\sin\theta$, regular oscillation with a period given by $\frac{\pi}{L_0}$, with L_0 being the center of

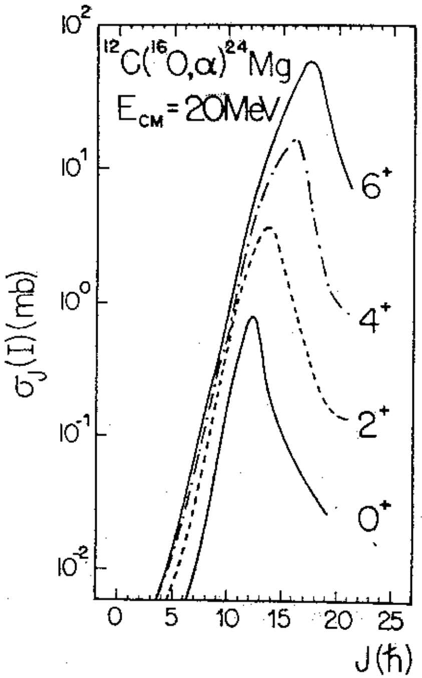

Fig. 3. Partial cross sections $\sigma_J(I)$ for the excitation of final states with spin I = 0, 2, 4 and 6, (calculated with the code STATIS . . . . The excitation energy $E^* = 0$ has been considered in all the cases (J represents the CN angular momentum) from Ref. (12).

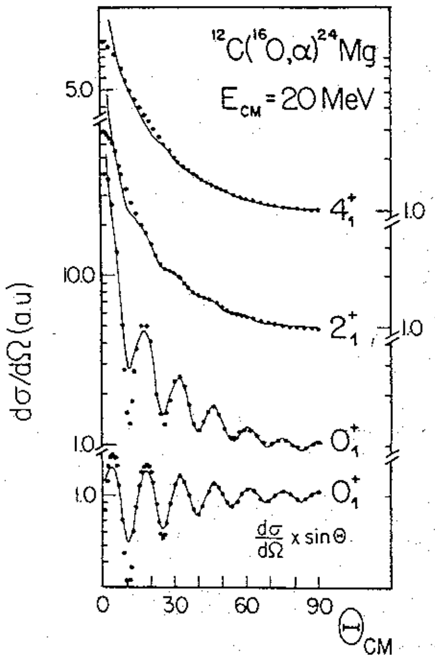

Fig. 4. Differential cross sections for the $^{12}C(^{16}O,\alpha)^{24}Mg$ (E_{cm}=20.0MeV) calculated using eq. (8) (solid line) and the code STATIS (points). The J-window parameters used were: - ground-state (α = 1.45, L_0 = 12.5, ΔL_0 = 2.7, I = 0);- first excited state (α = 1.45, L_2 = 13.0, ΔL_2 = 2.7, I = 2); second excited state (α = 1.45, L_4 = 15.0, ΔL_4 = 2.7, I = 4). Curves were normalized to unity at θ_{cm} = 90° from Ref. (12).

gravity (position of maximum) of the "statistical" window σ_J, and a damping function that gradually reduces the magnitude of these oscillations.

In Ref. (12) a simple model for the heavy-ion compound angular distribution was constructed using as an input the window function referred to above. The expression derived for $\frac{d\sigma}{d\Omega}$, using a derivative of a Fermi function form for σ_J, is

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin\theta}\left\{1 + \frac{(-)^I}{(2I+1)}\left[\sin 2L\theta \, \frac{2\pi\Delta L\,\theta}{\sinh 2\pi\Delta L\theta} + \sin 2L(\pi-\theta)\frac{2\pi\,\Delta L\,(\pi-\theta)}{\sinh 2\pi\Delta L(\pi-\theta)}\right]\right\} \quad (8)$$

where A_I is just the angle-integrated cross-section, I the spin of the residual nucleus state, L_0 the center-of-gravity of the J-window and ΔL_0 its width.

It is interesting to observe that the oscillations due to the second term on the RHS of Eq. (8) become more damped for higher I due to the factor $(2I+1)^{-1}$. The above simple expression for $\frac{d\sigma}{d\Omega}$ has been compared to exact Hauser-Feshbach calculation and the results were found to be excellent (Fig. 4)[12].

The exitence of more than one class of overlapping resonances in the heavy-ion system implies the existence of more than one statistical window whose shapes are close to that of Fig. (3). The center of gravity of the doorway J-window would presumably be higher than that of the equilibrated system. The reason being that the composite heavy ion system representing the non-equilibrium stage is more likely to be of a molecular shape whose effective moment of inertia is bigger than that of a sphere that represents the equilibrated system. In a two-class case the angular distribution would be given by[13]

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin\theta}\left\{(1+B) + \frac{(-)^I}{2I+1}\left[\left[\sin 2L_1\theta\,\frac{2\pi\Delta L_1\theta}{\sinh 2\pi\Delta L_1\theta} + \sin 2L_1(\pi-\theta)\,\frac{2\pi\Delta L_1(\pi-\theta)}{\sinh 2\pi\Delta L_1(\pi-\theta)}\right]B + \left[\sin 2L_2\theta\,\frac{2\pi\Delta L_2\theta}{\sinh 2\pi\Delta L_2\theta} + \sin 2L_2(\pi-\theta)\,\frac{2\pi\Delta L_2(\pi-\theta)}{\sinh 2\pi\Delta L_2(\pi-\theta)}\right]\right]\right\} \quad (9)$$

where 1 and 2 refer to the doorway (pre-equilibrium) and compound classes, respectively. The parameter B represents the relative contribution of the doorway class to the transition in question. This parameter may be traced to the ratio $\frac{\sigma_1}{\sigma_2}$ which is determined from the correlation function analysis.

In Fig. (5) we show the result of a calculation of $\frac{d\sigma_I}{d\Omega}$ using Eqs. (9) (two class) and (8) (one class). Clear changes in the period of the oscillations as well as the general shape of the cross-section, are seen to occur as a result of the contribution of the "doorway class".

Whereas the cross-section cross correlation function supplies in the two-class case discussed above, Γ_1, Γ_2 and $\frac{\sigma_1}{\sigma_2}$, the angular distribution may furnish, through the parameters L_1 and ΔL_1, useful information concerning the nature of the pre-equilibrium heavy-ion composite-configuration, in particular its density of states[13]. It would be quite interesting to measure complete angular distributions of heavy-ion induced transitions that are dominated by the pre-equilibrium class of doorways. This dominance may be easily decided upon through the generalized Ericson's fluctuation analysis described in sections I and II.

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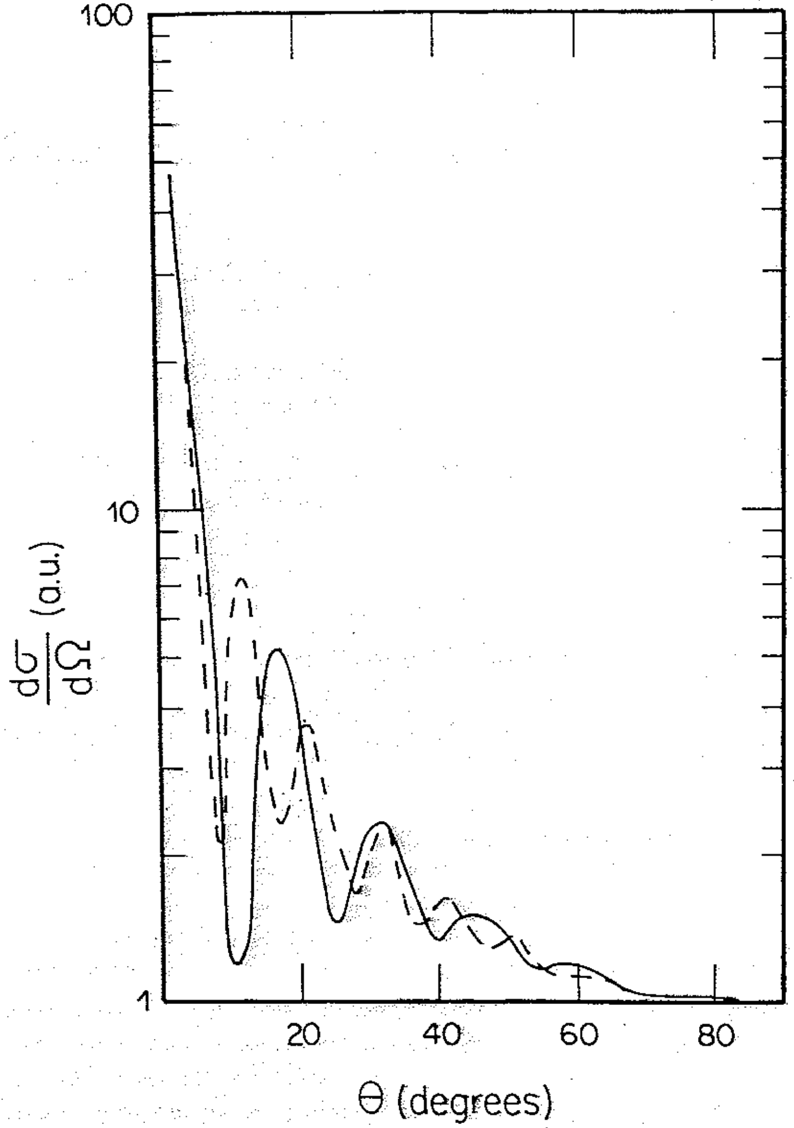

Fig. 5. The angular distribution of the emitted α-particle in the reaction $^{12}C(^{16}O,\alpha)$ to the $I = 0$ state in ^{24}Mg. Solid curve represents the pure compound emission ($B = 0$) (Eq. [5] and the dashed curve the result with $B = 4$. The parameters were $L_1 = 18.5$, $\Delta L_1 = \frac{2.7}{2.9}$, $L_2 = 12.5$, $\Delta L_2 = 2.7$.

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