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ON A NONRELATIVISTIC EXACT U(1) SYMMETRIC S MATRIX AND THE NONLINEAR SCHRÖDINGER EQUATION

by

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ON A NONRELATIVISTIC EXACT U(1) SYMMETRIC S MATRIX, AND THE NONLINEAR SCHRÖDINGER EQUATION

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ABSTRACT

We relate the U(1) symmetric factorizable galilean invariant S matrices to the exact solution of the nonlinear Schrödinger equation, using perturbation theory techniques and to some well known solutions of the Calogero system.

1. INTRODUCTION

Factorizable exact S matrices is by now a well known chapter of two dimensional field theory[(1),(2)]. However the nonrelativistic limit remains as a case of minor importance and our aim is to show that well established methods of quantum field theory apply as well in that limit. As previously mentioned in (1), the nonrelativistic factorizable S matrix is obtained remembering that the rapidity (θ) is superseded by nonrelativistic velocity, $\theta \to p/m$, and crossing is no more required. In this way we constructed solutions with U(1) symmetry and identify by means of perturbation theory the solution of the nonlinear Schrödinger model. R.Sogo et all[(3)]

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had obtained this result as a special limit of z_k symmetry.

2. NONRELATIVISTIC FACTORIZED S MATRIX WITH THE U(1) SYMMETRY

We consider a field ϕ and its conjugate ϕ^* and a U(1) symmetry:

$$U(1)|\phi\rangle = e^{i\beta}|\phi\rangle \quad , \quad 0 \le \beta < 2\pi \quad . \tag{2.1}$$

The S matrix is defined as follows. We call

$$|1,\theta_1,\theta_2\rangle = |\phi(\theta_1)\phi(\theta_2)\rangle \quad , \tag{2.2a}$$

$$|2,\theta_1,\theta_2\rangle = |\phi^*(\theta_1)\phi^*(\theta_2)\rangle \quad , \tag{2.2b}$$

$$|3,\theta_1,\theta_2\rangle = |\phi(\theta_1)\phi^*(\theta_2)\rangle \quad , \tag{2.2c}$$

$$|4,\theta_1,\theta_2\rangle = |\phi^*(\theta_1)\phi(\theta_2)\rangle \quad , \tag{2.2d}$$

so that

$$|i,\theta_1,\theta_2\rangle_{in} = S_{ij}|j,\theta_1,\theta_2\rangle_{out} \quad . \tag{2.3}$$

Using charge conjugation and U(1) symmetry, one's sees that S has the form

$$\hat{S}(\theta) = \begin{pmatrix} S(\theta) & 0 & 0 & 0 \\ 0 & S(\theta) & 0 & 0 \\ 0 & 0 & S_r(\theta) & S_t(\theta) \\ 0 & 0 & S_t(\theta) & S_r'(\theta) \end{pmatrix} \tag{2.4}$$

We take now the factorization equations[1] to be (the arguments as being respectively θ , $\theta+\theta'$ and θ')

$$S\, S_t\, S_r = S_t\, S\, S_r + S_r'\, S_r\, S_t \quad , \tag{2.5a}$$

$$S_r'\, S_r\, S_r' = S_r\, S_r'\, S_r \quad , \tag{2.5b}$$

$$S_r'\, S_t\, S = S_t\, S_r'\, S_r + S_r'\, S\, S_t \quad . \tag{2.5c}$$

The above system can be easily dealt by an usual procedure, the result for the S matrix being

$$S_r(\theta) = S_r'(\theta) \quad , \tag{2.6}$$

$$S(\theta) = \frac{\sin(\nu+b\theta)}{\sin\nu} S_r(\theta) \quad , \tag{2.7a}$$

$$S_t(\theta) = \frac{\sin b\theta}{\sin\nu} S_r(\theta) \quad , \tag{2.7b}$$

unitarity giving us

$$S_r(\theta) S_r(-\theta) = \frac{\sin^2\nu}{\sin^2\nu - \sin^2 b\theta} \quad . \tag{2.7c}$$

If we make b and $\nu \to 0$, with $\lambda = -b/\nu$ fixed, we get

$$S(\theta) = (1+\lambda\theta) S_r(\theta) \quad , \tag{2.8a}$$

$$S_t(\theta) = \lambda\theta\, S_r(\theta) \quad , \tag{2.8b}$$

$$S_r(\theta) S_r(-\theta) = \frac{1}{1-\lambda^2\theta^2} \quad . \tag{2.8c}$$

The minimal solution for $S_r(\theta)$ can be written as

$$S_r^{min}(\theta) = -\frac{\sin\nu}{\sin b\theta} \frac{\Gamma(-\frac{\nu}{\pi}-\frac{b\theta}{\pi})\Gamma(1+\frac{\nu}{\pi}-\frac{b\theta}{\pi})}{\Gamma(-\frac{b\theta}{\pi})\Gamma(1-\frac{b\theta}{\pi})} \tag{2.9}$$

or (for $b/\nu = -\lambda$ fixed):

$$S_r^{min}(\theta) = -\frac{1}{1-\lambda\theta} \tag{2.10}$$

3. NONLINEAR SCHRÖDINGER MODEL

The nonlinear Schrödinger equation (with mass $m = \frac{1}{2}$) can be obtained from the Lagrangian density

$$\mathcal{L} = \frac{i}{2}\phi^* \overleftrightarrow{\partial_0}\, \phi - |\partial_1\phi|^2 - c(\phi^*\phi)^2 \quad . \tag{3.1}$$

or equivalently with help of an auxiliary field σ ,

$$\mathcal{L} = \frac{i}{2}\phi^* \overleftrightarrow{\partial_0}\, \phi - |\partial_1\phi|^2 + \sigma\sigma^* - \sqrt{c}(\sigma\phi^* + \sigma^*\phi^2) \quad . \tag{3.2}$$

Using a perturbation theory in the parameter c we can verify that the model has no particle production and its factorizable S matrix is given by (2.8) and (2.10) (see analogous calculation for the massive thirring model and Sine-Gordon equation in (4) and (5)).

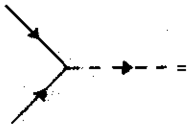
The ϕ propagator is given by

$$\longrightarrow \quad D^{\phi}(k) = \langle\tilde{\phi}(k)\tilde{\phi}^*(-k)\rangle = \frac{-1}{k^0 - (k^1)^2 + i\varepsilon} \quad . \tag{3.3}$$

while for σ we have

 $$D^{\sigma}(k) = \frac{1}{1 - \frac{ic}{\sqrt{2k^0 - (k')^2}}} \quad , \qquad (3.4)$$

and a vertex $\sigma^*\phi^2$

 $$= \quad (4ic)^{\frac{1}{2}} \qquad (3.5)$$

There are 3 contributions for particle pair production, which can be seen to cancel, namely those in fig. 1, plus permutations

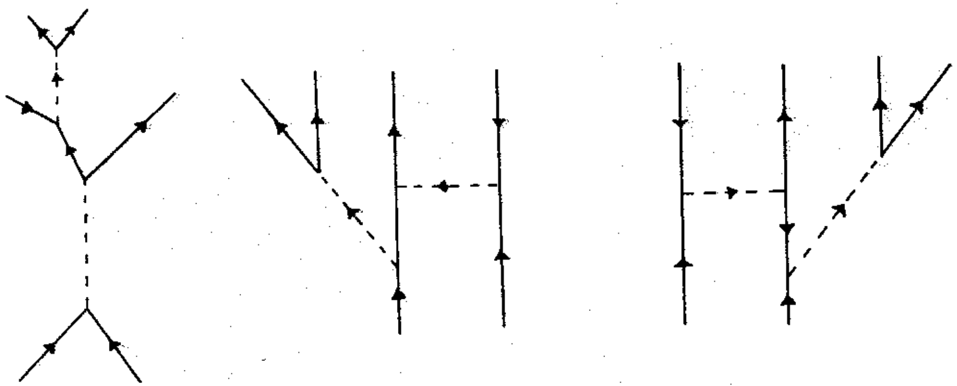


Fig. 1

The explicit S matrix is shown in fig. 2

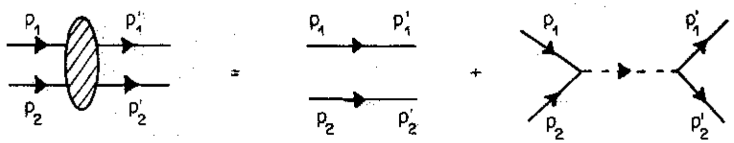


Fig. 2

$$\langle p_1' p_2' | S | p_1 p_2 \rangle = \langle p_1' p_1' | p_1 p_2 \rangle + (2\pi)^2 \, \delta(p_1' + p_2' - p_1 - p_2) 4ic \, D^{\sigma}(p_1 + p_2) \qquad (3.6)$$

Substituting the propagator the eq. (3.6) can be summarized in a simple form

$$\langle p_1' p_2' | S | p_1 p_2 \rangle = \frac{1 + ic/(p_1^1 - p_2^1)}{1 - ic/(p_1^1 - p_2^1)} \langle p_1' p_2' | p_1 p_2 \rangle \quad . \qquad (3.7)$$

If we recall that

$$p^1 - p^1 = m(\theta_1 - \theta_2) = \frac{1}{2}\theta \quad , \qquad (3.8)$$

We have the same form as the exact solution (2.8), by choosing

$$\lambda = - im/c = - i/2c \quad . \qquad (3.9)$$

As already mentioned by Zamolodchikov and Zamolodchikov[(1)], the S matrix (2.9) can be identified with those of Calogero system[(6)]. In that case the potencials are given by Weierstrass's p-function (see also (7)). Solution (2.9) describes scattering by a Pöschl-Teller potential whose complete solution can be found in (8).

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