

UNIVERSIDADE DE SÃO PAULO

**INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01000 - SÃO PAULO - SP
BRASIL**

1 } **publicações**

2 } IFUSP/P-406

08 AGO 1983



PARTICLE-HOLE RANDOM-PHASE-APPROXIMATION WITH
GOOD ISOSPIN

by

F. Krmpotić

Departamento de Física, Facultad de Ciencias
Exactas, Universidad Nacional de la Plata,
1900 La Plata, C.C. 67, Argentina

C.P. Malta

Instituto de Física, Universidade de São Paulo

K. Nakayama

Institut für Kernphysik, Kernforschungsanlage
Jülich, D-5170 Jülich, West Germany

Maio/1983

PARTICLE-HOLE RANDOM-PHASE-APPROXIMATION WITH GOOD ISOSPIN

F. Krmpotić*

Departamento de Física, Facultad de Ciencias Exactas, Universidad
Nacional de La Plata, 1900 La Plata, C.C. 67, Argentina

C.P. Malta

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01000 São Paulo, SP, Brasil

and

K. Nakayama

Institut für Kernphysik, Kernforschungsanlage Jülich,
D-5170 Jülich, West Germany

ABSTRACT

A simple method is designed for the treatment of a charge-independent Hamiltonian with wave functions with good isospin and for nuclei with $N \neq Z$. Both the particle-hole and the Δ -isobar-hole excitations are considered.

[NUCLEAR STRUCTURE. Collective excitations with good isospin. Tensor equations-of-motion.]

PACS NUMBERS: 21.10.-k, 21.60.Cs, 24.30.Cz

*Member of the Carrera de Investigador Científico del CONICET, Argentina.

In dealing with particle-hole excitations in a nucleus with ground state isospin $T_0 = (N-Z)/2 > 0$ it is convenient to subdivide the shell model orbitals into:

- i) The filled orbits (f), completely filled for both neutrons and protons.
- ii) The valence orbitals (v), filled only for neutrons.
- iii) The empty orbitals (e), containing neither neutrons nor protons.

Thus, there are four types of one particle-one hole (lp-lh) excitations, namely: $f \rightarrow e$, $f \rightarrow v$, $v \rightarrow e$ and $v \rightarrow v$, and two types of Δ isobar-hole-particle excitations which are: $f \rightarrow \Delta$ and $v \rightarrow \Delta$.

For a $f \rightarrow e$ transition we can introduce the coupled particle-hole creation operator

$$C^+(f \rightarrow e; t m_t) = (a_{\frac{1}{2}}^+ \times b_{\frac{1}{2}}^+)_m^t \equiv \sum_m \left(\frac{1}{2} \frac{1}{2} m m_t - m | t m_t \right) a_{\frac{1}{2}, m}^+ b_{\frac{1}{2}, m_t - m}^+ \quad (1)$$

where $t=0,1$ is the isospin carried by the excitation, m_t is its third component and $a_{\frac{1}{2}, \frac{1}{2}}^+$, $a_{\frac{1}{2}, -\frac{1}{2}}^+$, $b_{\frac{1}{2}, \frac{1}{2}}^+$ and $b_{\frac{1}{2}, -\frac{1}{2}}^+$ are, respectively, the creation operators for neutron particles, proton particles, proton holes and neutron holes*. In a second step the operator $C^+(f \rightarrow e; t m_t)$ is coupled with the core tensor state $|T_0 \gg$ (whose $(2T_0+1)$ components $|T_0 M_{T_0} \gg$ are the usual Dirac ket vectors) resulting in an excited state with definite isospin $T=T_0-1$, T_0 and T_0+1 :

*Since we are not interested in any specific particle-hole state, we do not write the single particle states explicitly. Moreover, as the formalism developed here is valid for any type of collective motion, all references to orbital angular momentum, spin and total angular momentum of the resonance will be omitted. The notation is very similar to that of Ref. 1).

.3.

$$|f \rightarrow e, t; T M_T\rangle = (C^+(f \rightarrow e; t) \times |T_0 \gg\rangle)_{M_T}^T$$

$$\equiv \sum_{m_t=-1}^1 (t T_0 m_t, M_T - m_t |T M_T) C^+(f \rightarrow e; t m_t) |T_0, M_T - m_t\rangle \quad (2)$$

(Note that when $M_{T_0} = T_0 - 1$, $C^+(f \rightarrow e; t m_t) |T_0, M_T - m_t\rangle$ gives a 1p-1h state for $m_t = -1$, 2p-2h state for $m_t = 0$ and a 3p-3h state for $m_t = 1$.)

The particle-hole operators for the $f \rightarrow v$ and $v \rightarrow e$ transitions, i.e.,

$$C^+(f \rightarrow v; m_t) = a_{\frac{1}{2}, -\frac{1}{2}}^+ b_{\frac{1}{2}, m_t + \frac{1}{2}}^+ \quad (3a)$$

and

$$C^+(v \rightarrow e; m_t) = a_{\frac{1}{2}, m_t + \frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+ \quad (3c)$$

($m_t = -1, 0$) do not have definite t spins due to the lack of the proton valence states. However, also in this case, eigenstates of the total isospin are easily constructed by noting that the states

$$a_{\frac{1}{2}, -\frac{1}{2}}^+ |T_0 \gg\rangle \equiv |\pi; T_0 - \frac{1}{2} \gg\rangle \quad (4a)$$

and

$$b_{\frac{1}{2}, -\frac{1}{2}}^+ |T_0 \gg\rangle \equiv |v^{-1}; T_0 - \frac{1}{2} \gg\rangle \quad (4b)$$

are tensor states when the operators $a_{\frac{1}{2}, -\frac{1}{2}}^+$ and $b_{\frac{1}{2}, -\frac{1}{2}}^+$ act on the valence region. (The fully aligned components $|\pi; T_0 - \frac{1}{2}, T_0 - \frac{1}{2} \gg\rangle$

.4.

and $|v^{-1}; T_0 - \frac{1}{2}, T_0 - \frac{1}{2} \gg\rangle$ represent, respectively, the nuclei with neutron and proton numbers $(N, Z+1)$ and $(N-1, Z)$. Coupling tensorially the operator $b_{\frac{1}{2}, m}^+$ to the state $|\pi; T_0 - \frac{1}{2} \gg\rangle$ and the operator $a_{\frac{1}{2}, m}^+$ to the state $|v^{-1}; T_0 - \frac{1}{2} \gg\rangle$ one obtains immediately the eigenstates of the total isospin T ,

$$|f \rightarrow v; T M_T\rangle = -(b_{\frac{1}{2}}^+ \times |\pi; T_0 - \frac{1}{2} \gg\rangle)_{M_T}^T \quad (5a)$$

and

$$|v \rightarrow e; T M_T\rangle = (a_{\frac{1}{2}}^+ \times |v^{-1}; T_0 - \frac{1}{2} \gg\rangle)_{M_T}^T \quad (5b)$$

where the isospin can take the values $T = T_0 - 1$ and T_0 and the minus sign was introduced for the sake of convenience. By means of relations (3) the last two states can be also expressed in the form

$$|f \rightarrow v; T M_T\rangle =$$

$$= \sum_{m_t=-1}^0 (\frac{1}{2}, T_0 - \frac{1}{2}, m_t + \frac{1}{2}, M_T - m_t - \frac{1}{2} |T M_T) C^+(f \rightarrow v; m_t) |T_0, M_T - m_t\rangle \quad (6a)$$

and

$$|v \rightarrow e; T M_T\rangle =$$

$$= \sum_{m_t=-1}^0 (\frac{1}{2}, T_0 - \frac{1}{2}, m_t + \frac{1}{2}, M_T - m_t - \frac{1}{2} |T M_T) C^+(v \rightarrow e; m_t) |T_0, M_T - m_t\rangle \quad (6b)$$

In the case of $v \rightarrow v$ transitions we are only

interested in the fully aligned state

$$|v \rightarrow v; T=T_0-1; M_T=T_0-1\rangle = a_{\frac{1}{2}, -\frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+ |T_0 T_0\rangle \equiv C^+(v \rightarrow v; m_t = -1) |T_0 T_0\rangle. \quad (7)$$

For the Δ -h excitations we proceed in the same way. That is, in the case of $f \rightarrow \Delta$ excitations we first construct the coupled Δ -h creation operator of isospin $t=1$ and 2,

$$C^+(f \rightarrow \Delta; t, m_t) = (A_{\frac{3}{2}}^+ \times b_{\frac{1}{2}}^+)_t^{m_t}, \quad (8)$$

where the operator $A_{\frac{3}{2}, m}^+$ creates the Δ^{++} , Δ^+ , Δ^0 and Δ^- isobar states for $m = -\frac{3}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$ and $\frac{3}{2}$, respectively. Next, as in the case of $f \rightarrow v$ transitions, we couple the operator (8) to the tensor ground state $|T_0\rangle\rangle$ and get

$$|f \rightarrow \Delta; T, M_T\rangle = (C^+(f \rightarrow \Delta; t) \times |T_0\rangle\rangle)_{M_T}^T. \quad (9)$$

The second Δ -h creation operator,

$$C^+(v \rightarrow \Delta; m_t) = A_{\frac{3}{2}, m_t + \frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+, \quad (10)$$

($m_t = -2, -1, 0, 1$) does not have a definite isospin and the eigenstates with good isospin are constructed, as in the case of $v \rightarrow e$ transitions, by coupling the operator A_m^+ to the tensor state (4b), i.e.,

$$|v \rightarrow \Delta; T, M_T\rangle = (A_{\frac{3}{2}}^+ \times |v; T_0 - \frac{1}{2}\rangle\rangle)_{M_T}^T \quad (11)$$

or

$$|v \rightarrow \Delta; T, M_T\rangle = \sum_{m_t=-2}^1 \left(\frac{3}{2}, T_0 - \frac{1}{2}, m_t + \frac{1}{2}, M_T - m_t - \frac{1}{2}\right) |T, M_T\rangle C^+(v \rightarrow \Delta; m_t) |T_0, M_T - m_t\rangle. \quad (12)$$

At this point one should remember that the states $|f \rightarrow e, t; T, M_T\rangle$, $|f \rightarrow v; T, M_T\rangle$ and $|v \rightarrow e; T, M_T\rangle$ contain, in addition to $1p$ - $1h$ components, configurations with $2p$ - $2h$ and $3p$ - $3h$, which makes the calculation of the matrix elements in such a basis rather cumbersome (see for example the work of Auerbach and Yeverechyanu²). It is, however, possible to relate the matrix element of states with good T to the $1p$ - $1h$ matrix elements with different M_T 's. Moreover, these last quantities can be easily summed up and the final result appears in the form of matrix elements between the usual $1p$ - $1h$ states, multiplied by a geometrical T_0 -dependent factor which contains correction coming from the $2p$ - $2h$ and $3p$ - $3h$ components. The same procedure can be followed for the Δ -h matrix elements.

Quite recently, Toki³ has elaborated a very ingenious method to establish the above mentioned relationship between the matrix elements. Unfortunately, this method is applicable only within the Tamm-Dancoff approximation (TDA) and an extension to the random-phase approximation (RPA) does not seem to be trivial at all.

Within the RPA, there is the method of tensor-equation of motion (TEM), developed by Rowe and Ngo-Trong¹, in which the $2p$ - $2h$ and $3p$ - $3h$ configurations are automatically included as a result of the tensor coupling of the excitation operator to the tensor ground state. This method, however,

can be applied straightforwardly only for the $f \rightarrow e$ and $f \rightarrow \Delta$ excitations, as in the remaining cases the p-h and Δ -h excitation operators do not have definite t spins. The way to overcome this difficulty is suggested by expressions (2), (5) and (11). That is, the eigenstates $|f \rightarrow v; T \rangle\rangle$ and $|v \rightarrow e; T \rangle\rangle$ and $|v \rightarrow \Delta; T \rangle\rangle$ can be treated, respectively, as result of excitations of rank $\frac{1}{2}$ and $\frac{3}{2}$ coupled to a ground state of rank $T_0 - \frac{1}{2}$. Similarly, the eigenstate $|v \rightarrow v; T \rangle\rangle$ can be considered as result of an excitation of rank zero coupled to a ground state of rank $T_0 + 1$. Therefore, in order to exploit the invariance properties of the Hamiltonian and to be able to make use of the TEM formalism we will introduce auxiliary tensor excitation operators $\zeta^+(\tau)$ of rank $\tau=0, \frac{1}{2}, 1, \frac{3}{2}$ and 2 and auxiliary ground states $|T_0(F) \rangle\rangle$ and $|T_0(B) \rangle\rangle$ for forward and backward going amplitudes, respectively (of course, in the case of $f \rightarrow e$ and $f \rightarrow \Delta$ excitations, the operator $\zeta^+(\tau)$ will coincide with the particle-hole excitation operators (1) and (8) respectively and the ground-state will always be $|T_0 \rangle\rangle$).

Let us first simplify the notation and label different p-h and Δ -h configurations by an index α running from 1 to 8 (see first and second columns of Table I). Next, we define the above mentioned auxiliary quantities (which are listed in the fourth column of Table I) by means of the relations:

$$\zeta^+(\tau^\alpha \mu_\tau) |T_0^\alpha(F) M_{T_0} \rangle = C^+(\alpha; m_\tau = \mu_\tau - T_0 + T_0^\alpha(F)) |T_0 M_{T_0} \rangle = M_{T_0} + T_0 - T_0^\alpha(F) \rangle, \quad (13)$$

$$\zeta(\tau^\alpha \mu_\tau) |T_0^\alpha(B) M_{T_0} \rangle = (-1)^{2\tau} C(\alpha; m_\tau = \mu_\tau + T_0 - T_0^\alpha(B)) |T_0 M_{T_0} \rangle = M_{T_0} + T_0 - T_0^\alpha(B) \rangle,$$

and introduce the RPA excitation operator

$$\Omega_{T_n}^+(\tau^\alpha \mu_\tau) = X_{T_n}(\alpha) \zeta^+(\tau^\alpha \mu_\tau) - (-1)^{\tau^\alpha + \mu_\tau} Y_{T_n}(\alpha) \zeta(\tau^\alpha, -\mu_\tau), \quad (14)$$

where $X_{T_n}(\alpha)$ and $Y_{T_n}(\alpha)$ are, respectively, the forward and backward going amplitudes of the excited state $|T_n \rangle\rangle$ (n-th state with isospin T). From a moment's reflection it can be seen that:

- 1) $X_{T_n}(\alpha) = 0$ ($Y_{T_n}(\alpha) = 0$), for the $f \rightarrow v$, $v \rightarrow e$ and $v \rightarrow v$ transitions and $T_0 = T_0 + 1$ ($T = T_0 - 1$), and
- 2) $X_{T_n}(\alpha) = Y_{T_n}(\alpha) = 0$ for the $v \rightarrow v$ transitions and $T = T_0$.

The properties of the operator $\Omega_{T_n}^+(\tau^\alpha)$ are

$$|T_n \rangle\rangle = \sum_{\alpha} (\Omega_{T_n}^+(\tau^\alpha) \times |T_0^\alpha \rangle\rangle)^T, \quad (15a)$$

$$\Omega_{T_n}(\tau^\alpha) |T_0^\alpha \rangle\rangle = 0. \quad (15b)$$

One should always keep in mind that the operators $\zeta^+(\tau^\alpha)$ act on the forward going ground states $|T_0^\alpha(F) \rangle\rangle$ and the operators $\zeta(\tau^\alpha) = (-1)^{\tau^\alpha} \zeta(\tau^\alpha)$ on the backward going ground states $|T_0^\alpha(B) \rangle\rangle$.

We can now straightforwardly arrive at the tensor equations of motion¹⁾

$$\begin{aligned} & \sum_{\alpha \alpha'} P_T(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; m_\tau) \langle T_0^\alpha \tau_0^\alpha | \left[\Omega_{T_n}(\tau^\alpha \mu_\tau), \Omega_{T_n}^+(\tau^{\alpha'} \mu_\tau') \right] | T_0^{\alpha'} \tau_0^{\alpha'} \rangle \\ & = \omega_{T_n} \sum_{\alpha \alpha'} P_T(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; m_\tau) \langle T_0^\alpha \tau_0^\alpha | \left[\Omega_{T_n}(\tau^\alpha \mu_\tau), \Omega_{T_n}^+(\tau^{\alpha'} \mu_\tau') \right] | T_0^{\alpha'} \tau_0^{\alpha'} \rangle \\ & = \delta_{nn'} \omega_{T_n}, \end{aligned} \quad (16)$$

where $\mu_\tau = m_\tau + T_0 - T_0^\alpha$, $\mu_\tau' = m_\tau + T_0 - T_0^{\alpha'}$, ω_{T_n} is the excitation energy of the state $|T_n \rangle\rangle$ and

$$P_T(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; m_t) = \sum_{T_1} (-1)^{\tau_0^\alpha + \tau_0^{\alpha'} + T_1 + m_t} ((2\tau_0^\alpha + 1)(2T_1 + 1))^{1/2} \\ \times W(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; T_1 T) \frac{(\tau^{\alpha'} \tau_0^{\alpha'}, -m_t - T_0 + \tau_0^\alpha, m_t + T - \tau_0^{\alpha'} | T_1, \tau_0^\alpha - \tau_0^{\alpha'})}{(\tau_0^{\alpha'} T_1 \tau_0^{\alpha'}, \tau_0^\alpha - \tau_0^{\alpha'} | \tau_0^\alpha \tau_0^{\alpha'})} \quad (17)$$

The true excitations operators are defined as

$$O_{T_n}^+(\alpha, m_t) = X_{T_n}(\alpha) C^+(\alpha, m_t) - Y_{T_n}(\alpha) C(\alpha, \bar{m}_t) \quad (18)$$

where

$$C(\alpha, \bar{m}_t) = (-)^{k_\alpha + m_t} C(\alpha, -m_t) \quad (19)$$

and

$$k_\alpha = \begin{cases} t & \text{for the } f \rightarrow e \text{ and } f \rightarrow \Delta \text{ transitions,} \\ 1 & \text{otherwise.} \end{cases}$$

Making use of relations (13), the equations (16)

can be written in terms of the true excitation operators (18)

and (19)

$$\sum_{\substack{\alpha\alpha' \\ m_t}} P_T(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; m_t) \langle T_0 T_0 | [O_{T_n}(\alpha, m_t), H, O_{T_n}^+(\alpha', m_t)] | T_0 T_0 \rangle \\ = \omega_{T_n} \sum_{\alpha\alpha'} P_T(\tau^\alpha \tau_0^\alpha; \tau^{\alpha'} \tau_0^{\alpha'}; m_t) \langle T_0 T_0 | [O_{T_n}(\alpha, m_t), O_{T_n}^+(\alpha', m_t)] | T_0 T_0 \rangle \\ = \delta_{\alpha\alpha'} \omega_{T_n} \quad (20)$$

Substituting the expansion (18) into this last expression one

arrives at the RPA equations

$$\begin{pmatrix} A_T & B_T \\ B_T^+ & D_T \end{pmatrix} \begin{pmatrix} X_{T_n} \\ Y_{T_n} \end{pmatrix} = \omega_{T_n} \begin{pmatrix} X_{T_n} \\ -Y_{T_n} \end{pmatrix} \quad (21)$$

with the submatrices defined as:

$$A_T(\alpha, \alpha') = \sum_{m_t} P_T(\tau^\alpha \tau_0^\alpha(F); \tau^{\alpha'} \tau_0^{\alpha'}(F); m_t) \langle T_0 T_0 | [C(\alpha, m_t), H, C^+(\alpha', m_t)] | T_0 T_0 \rangle \\ = A_T^*(\alpha', \alpha) \quad ,$$

$$B_T(\alpha, \alpha') = \sum_{m_t} P_T(\tau^\alpha \tau_0^\alpha(F); \tau^{\alpha'} \tau_0^{\alpha'}(B); m_t) \langle T_0 T_0 | [C(\alpha, m_t), H, C(\alpha', \bar{m}_t)] | T_0 T_0 \rangle \quad , \\ (22)$$

$$D_T(\alpha, \alpha') = \sum_{m_t} P_T(\tau^\alpha \tau_0^\alpha(B); \tau^{\alpha'} \tau_0^{\alpha'}(B); m_t) \langle T_0 T_0 | [C^+(\alpha, \bar{m}_t), H, C(\alpha', \bar{m}_t)] | T_0 T_0 \rangle \\ = D_T^*(\alpha', \alpha) \quad .$$

In this way we have succeeded in relating the matrix elements

with good T to the lp-lh matrix elements with different

$$M_T = T_0 + m_t \quad .$$

One should notice that $\tau_0^\alpha(F) \neq \tau_0^\alpha(B)$ when the excitation operator $C^+(\alpha, m_t)$ does not have a definite t (for $\alpha=3,4,5$ and 8). As a consequence, the geometrical factors P_T involving these configurations will have different values for the submatrices A_T , B_T and D_T .

Within the TEM formalism the reduced transition matrix elements of a one-body tensor operator W^t of rank t

in isospin space are easily calculated from the expression

$$\begin{aligned} \langle T || W^t || T_0 \rangle &= (-)^{t+T_0-T} (2T+1)^{\frac{1}{2}} \sum_{\alpha m_t} P_T(\tau^\alpha T_0^\alpha; t T_0; m_t) \\ &\times \langle T_0 T_0 | [O_T(\alpha, m_t), W_{M_t}^t] | T_0 T_0 \rangle \end{aligned} \quad (23)$$

Finally as

$$\sum_{\alpha m_t} P_T(\tau^\alpha T_0^\alpha; \tau^\alpha T_0^\alpha; m_t) = 1 \quad (24)$$

eq.(20) leads to the usual RPA normalization condition

$$\sum_{\alpha} (|X_{T_n}(\alpha)|^2 - |Y_{T_n}(\alpha)|^2) = 1$$

Within the TDA our results, both for the p-h and the Δ -h excitations, agree with those obtained by Toki³⁾. We believe, however, that our formalism is more compact and more convenient for a numerical calculation. Moreover, as already mentioned, the extension of Toki's method from TDA to RPA seems to be rather complicated.

Mathematically the method employed in the present work is that of Rowe and Ngo-Trong¹⁾. However, our results coincide with theirs only for the $f \rightarrow e$ and $f \rightarrow \Delta$ transitions, although the last ones were not considered by them. The first important difference appears in the treatment of the $f \rightarrow v$ and $v \rightarrow e$ excitations. In the work of Rowe and Ngo-Trong they carry a zero isospin and are coupled to the ground state with isospin T_0 . As a consequence, the matrix elements involving two configurations of this type are identically equal to zero for

$T = T_0 \pm 1$ (see eq. (17)), which is obviously incorrect. On the contrary, within our formalism the just mentioned excitations are treated as entities of rank $\frac{1}{2}$ and coupled to the ground states with isospins $T_0 - \frac{1}{2}$ and $T_0 + \frac{1}{2}$ for forward and backward going amplitudes, respectively. Another serious disadvantage of the method developed by Rowe and Ngo-Trong is that they do not consider at all the $v \rightarrow v$ configurations, which play a very important role for some charge-exchange states with $T = T_0 - 1$ (in particular, for the Gamow-Teller resonances) through the forward going graphs, and for the states with $T = T_0 + 1$ (for example, in the case of isovector, quadrupole resonances) through the ground state correlations.

Finally let us note that within our formalism the matrix elements with good isospin $T = T_0 + 1$ are always identically equal to those of lp-lh with $M_T = T_0 + 1$ for both the forward and backward going amplitudes.

Explicit results for the matrix elements and transition probabilities, as well as, a few numerical results will be presented in a forthcoming paper.

ACKNOWLEDGMENT

One of us (F.K.) wants to acknowledge the support of the German-Argentine Scientific Exchange Program which allowed him to visit Germany. He is also grateful to the members of the Institut für Kernphysik, KFA, Jülich and of the Instituto de Física, USP, São Paulo, for kind hospitality. The authors are pleased to acknowledge many extremely helpful discussions with J. Speth, V. Klempt and F. Osterfeld.

REFERENCES

- 1) D.J. Rowe and Ngo-Trong, Rev. Mod. Phys. 47 (1975) 471.
- 2) N. Auerbach and A. Yeverechyahu, Nucl. Phys. A332 (1979) 173.
- 3) H. Toki, preprint Michigan State University, 1981.

TABLE I - The excitation operators $C^+(\alpha, m_L)$ and $\zeta^+(\tau^\alpha m_L)$, their tensor ranks t^α and τ^α and the corresponding ground state isospins $T_0^\alpha(F)$ and $T_0^\alpha(B)$ for the forward and backward going amplitudes, respectively.

| | α | $C^+(\alpha, m_L)$ | $\zeta(\tau^\alpha \mu_T)$ | t^α | τ^α | $T_0^\alpha(F)$ | $T_0^\alpha(B)$ |
|-----|----------|--|-----------------------------|------------|---------------|---------------------|---------------------|
| f→e | 1 | $(a_{\frac{1}{2}}^+ \times b_{\frac{1}{2}}^+)_{m_L = \mu_T}^{t=\tau}$ | | 0 | 0 | T_0 | T_0 |
| | 2 | | | 1 | 1 | T_0 | T_0 |
| f→v | 3 | $a_{\frac{1}{2}, -\frac{1}{2}}^+ b_{\frac{1}{2}, m_L + \frac{1}{2}}^+$ | $-b_{\frac{1}{2}, \mu_T}^+$ | - | $\frac{1}{2}$ | $T_0 - \frac{1}{2}$ | $T_0 + \frac{1}{2}$ |
| v→e | 4 | $a_{\frac{1}{2}, m_L + \frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+$ | $a_{\frac{1}{2}, \mu_T}^+$ | - | $\frac{1}{2}$ | $T_0 - \frac{1}{2}$ | $T_0 + \frac{1}{2}$ |
| v→v | 5 | $a_{\frac{1}{2}, -\frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+$ | 1 | - | 0 | $T_0 - 1$ | $T_0 + 1$ |
| f→Δ | 6 | $(A_{\frac{3}{2}}^+ \times b_{\frac{1}{2}}^+)_{m_L = \mu_T}^{t=\tau}$ | | 1 | 1 | T_0 | T_0 |
| | 7 | | | 1 | 2 | T_0 | T_0 |
| v→Δ | 8 | $A_{\frac{3}{2}, m_L + \frac{1}{2}}^+ b_{\frac{1}{2}, -\frac{1}{2}}^+$ | $A_{\frac{3}{2}, \mu_T}^+$ | - | $\frac{3}{2}$ | $T_0 - \frac{1}{2}$ | $T_0 + \frac{1}{2}$ |