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IFUSP/P - 407

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Maio/1983

# ON THE S-WAVE COMPONENT OF THE TWO-PION EXCHANGE THREE-NUCLEON FORCE*

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## ABSTRACT

A critical assessment of the role of the pion-nucleon form-factor in the two-pion exchange three-body force shows that its s-wave component is mostly repulsive.

PACS numbers: 21.30+y , 21.40+d

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*Work supported by FAPESP and CNPq, Brazilian Agencies.



The importance of three-body forces for trinucleon systems is becoming well established nowadays. Indeed, it is hoped that these forces can account for the differences between experimental values of trinucleon observables and those calculated by means of two-nucleon potentials(1). In this work we examine two particular aspects of the two-pion exchange three-body force ($\pi\pi$E-3BF). First, we show that $\pi$N scattering lengths are not a reliable input for constructing the potential. Second, we argue that the presence of spurious terms associated with the use of form-factors can alter completely the most important features of the potential.

Since the pioneering work of Fujita and Miyazawa(2) it has been realized that this force contained terms originated from both s and p waves in the intermediate $\pi$N amplitude. The strength of the s wave component of the three-nucleon force ($W_s$) was originally assumed to be proportional to the isospin even $\pi$N scattering length, which is rather small. This way of treating the problem inaugurated a tradition where the terms of the force due to p waves were considered to be largely dominant. This tradition lasted until the derivation of the Tucson potential(2), where the contributions of intermediate s and p waves were shown to be comparable. This change took place because in that work the $\pi$N amplitude has been treated by means of chiral symmetry, ensuring that it was suitable for describing off-shell pions. This is a crucial point, as we discuss below.

The relativistic amplitude for the process $\pi^a(k)N(p) \to \pi^b(k')N(p')$ can be written as

$$T^{ab}_{\pi N} = \bar{U}(p')\left[(A^+ + \frac{\not{k}+\not{k}'}{2} B^+)\delta_{ab} + (A^- + \frac{\not{k}+\not{k}'}{2} B^-) i\, \epsilon_{bac}\tau_c\right]U(p) \qquad (1)$$

where $A^\pm$ and $B^\pm$ can be determined from experiment and depend on the variables $\nu=(k+k').(p+p')/4m$ and $t=(k-k')^2$. Some combinations of $A^\pm$ and $B^\pm$ are of particular interest. Here we consider the amplitude $C^+$, that is related to the isospin symmetric $\pi N$ scattering length, the $\pi N$ $\sigma$-term and the s wave $\pi\pi$E-3BF. It is defined as $C^+(\nu,t) = A^+(\nu,t) + B^+(\nu,t)/(1-t/4m^2)$.

Among the various contributions to $T^{ab}_{\pi N}$ is that of a nucleon propagating forward in time, which must not be included in the $\pi\pi$E-3BF, since it corresponds to an iteration of the two-body potential. The subtraction of this contribution is denoted by the symbol (~) on top of the appropriate quantity. Dispersion relations can be used to extrapolate the amplitudes $A^{\pm}$, $B^{\pm}$ and $C^{\pm}$ to regions below threshold, where they can be expanded as a power series of $\nu$ and $t$[4]. The expansion of $\tilde{C}^+$, for instance, is given by $\tilde{C}^+(\nu,t)=\sum_{ij} c^+_{ij}\, \nu^{2i}\, t^j$. The coefficients needed in this work have the following experimental values[5]: $c^+_{00}=-1.50\ \mu^{-1}$, $c^+_{01}=1.14\ \mu^{-3}$, $c^+_{10}=1.12\ \mu^{-3}$, $c^+_{02}=0.036\ \mu^{-5}$, $c^+_{20}=0.200\ \mu^{-5}$, where $\mu$ is the pion mass.

At threshold, where $\nu=\mu$ and $t=0$, the amplitude $C^+$ is related to the isospin even scattering length by

$$a^+_0 = \frac{m}{4\pi(m+\mu)}\, C^+_0 = \frac{m}{4\pi(m+\mu)}\left(-\frac{g^2\mu^2}{4m^3}+\tilde{C}^+_0\right) = \frac{m}{4\pi(m+\mu)}\left(-\frac{g^2\mu^2}{4m^3}+c^+_{00}+c^+_{10}\,\mu^2+c^+_{20}\mu^4\right) \qquad (2)$$

The term proportional to $g^2$ is the nucleon-pole contribution.

The $\pi N$ process corresponding to the $\pi\pi$E-3BF, on the other hand, is characterized by different values of the kinematical variables, whose orders of magnitude are the following[3,6,7]: $|k|\sim|k'|\sim|p|\sim|p'|\sim\mu$, $p_0\sim p_0'\sim m$, $|k_0|\sim|k_0'|\sim\mu^2/m$. Hence we have $\nu\sim\mu^2/m$, $t\sim\mu^2$ and the "non-relativistic" value of $\tilde{C}^+$ is given by $\tilde{C}^+_{nr}= c^+_{00} - c^+_{01}\ (\vec{k}^2+\vec{k}'^2-2\vec{k}.\vec{k}')$. The s wave contribution in the intermediate $\pi N$ system comes from the function $\tilde{C}^+_s= c^+_{00} - c^+_{01}(\vec{k}^2+\vec{k}'^2)$, which is very different from that employedin eq.(2). First, because it does not include the nucleon contribution; second, because it corresponds to another combination of the expansion coefficients $c^+_{ij}$. Therefore the smallness of the isospin symmetric $\pi N$ scattering length cannot be taken as an indication that the contribution of s waves to the $\pi\pi$E-3BF is correspondingly small. In refs. (3,7)



chiral simmetry has been used to establish theoretically the values of the coefficients $c^+_{00}$ and $c^+_{01}$.

The function $W_s$ , the s wave component of the $\pi\pi$E-3BF, is obtained by evaluating the permutations of the Feynman diagram depicted in fig.1, where the definitions of the kinematical variables can be found. This calculation has been done in refs.(3,7) and will not be reproduced here. We just quote the result in momentum space

$$W^{23}_s = -\frac{1}{4m^2}\,(\vec{\tau}^{(2)}.\vec{\tau}^{(3)})\,(\vec{\sigma}^{(2)}.\vec{k})\,(\vec{\sigma}^{(3)}.\vec{k}')\left[c^+_{00}-c^+_{01}(\vec{k}^2+\vec{k}'^2)\right]\frac{g}{\vec{k}^2+\mu^2}\,\frac{g}{\vec{k}'^2+\mu^2}\,. \qquad (3)$$

In this expression $\vec{\tau}^{(i)}$ and $\vec{\sigma}^{(i)}$ are respectively the isospin and the spin operators acting on nucleon i , whereas g represents the $\pi N$ coupling constant. The potential in momentum space can be rewritten as

$$W^{23}_s = -\frac{1}{4m^2}\,(\vec{\tau}^{(2)}.\vec{\tau}^{(3)})\,(\vec{\sigma}^{(2)}.\vec{k})\,(\vec{\sigma}^{(3)}\vec{k}')\Big[(c_{00}+2\mu^2c^+_{01})\,\frac{g}{\vec{k}^2+\mu^2}\,\frac{g}{\vec{k}'^2+\mu^2} \qquad (4)$$

$$-\,c^+_{01}\left(g\,\frac{g}{\vec{k}'^2+\mu^2}+\frac{g}{\vec{k}^2+\mu^2}\,g\right)\Big]\ .$$

The next step towards the final form of $W_s$ consists in the evaluation of the Fourier transform of this equation. Before doing this, however, it is convenient to introduce $\pi N$ form factors, in order to ensure that the expressions in configuration space do not diverge when the internucleon distances vanish. From a formal point of view, this can be done by allowing the coupling constant to become momentum dependent: $g \to g(k^2) = g\,\overline{G}(k^2)$, where the function $\overline{G}$ is such that $\overline{G}(\mu^2)=1$. It is worth pointing out that this way of introducing form factors is <u>not</u> prescribed by chiral symmetry. Rather, it corresponds to phenomenological corrections to the results obtained through the use of this symmetry.

The potential in configuration space is given by

$$W^{23}_s = \left(\frac{g\mu}{2m}\right)^2\left(\frac{1}{4\pi}\right)^2(\vec{\tau}^{(2)}.\vec{\tau}^{(3)})\,(\vec{\sigma}^{(2)}.\vec{\nabla})\,(\vec{\sigma}^{(3)}.\vec{\nabla}')\ \times$$

$$\times\left\{(c^+_{00}+2\mu^2c^+_{01})U(r)U(r') - \mu^2c^+_{01}\left[G(r)U(r')+U(r)G(r')\right]\right\} \qquad (5)$$

In this expression the functions $U(r)$ and $G(r)$ are, respectively, the Fourier transforms of $(4\pi/\mu)\ \overline{G}(\vec{k}^2)/(\vec{k}^2+\mu^2)$ and $(4\pi/\mu^3)\ \overline{G}(\vec{k}^2)$.

The function $G(r)$ corresponds to a nucleon that is not point-like, since it is proportional to the Fourier transform of the form factor. In the remainder of this work we will be mostly concerned with its role in practical calculations, as well as with its physical significance. For the purpose of the present discussion we consider the following definite form(6,7,8): $\overline{G}(k^2)=(\Lambda^2-\mu^2/\Lambda^2-k^2)^2$.

In order to illustrate the importance of $G(r)$ for the trinucleon system, we display in fig.2 the equipotential plots for the expectation value of $W_s$ between totally antisymmetric spin and isospin states, that is given by

$$\langle W_s^{23}\rangle = -\left(\frac{g\mu}{2m}\right)^2\left(\frac{1}{4\pi}\right)^2 \mu^2 \cos(\vec{r},\vec{r}\,') \times$$

$$\times \left\{(c_{00}^{+}+2\mu^2 c_{01}^{+})\ \frac{1}{\mu}\frac{U(r)}{\partial r}\,\frac{1}{\mu}\frac{U(r')}{\partial r'} - \mu^2 c_{01}^{+}\left[\frac{1}{\mu}\frac{\partial G(r)}{\partial r}\,\frac{1}{\mu}\frac{\partial U(r')}{\partial r'} + \frac{1}{\mu}\frac{\partial U(r)}{\partial r}\,\frac{1}{\mu}\frac{\partial G(r')}{\partial r'}\right]\right\} \quad (6)$$

Following the work of Brandenburg and Glöckle(8), we construct these diagrams by fixing the positions of two nucleons and using the third one as a probe. We consider the fixed internucleon distance to be $x=0.88$ fm, corresponding to the minimum of the two-body potential, and adopts the value $\Lambda=5\ \mathrm{fm}^{-1}$. The plot of fig.2a describes the full potential, given by eq.(6), whereas in fig.2b the function $G(r)$ has been set equal to zero. These plots are strikingly different. This fact is unexpected, since form factors should correspond to short distance effects, but in this case they determine the shape of the potential. Inspecting the diagrams we note that the potential of fig.2a favours the triangular configuration, whereas that of fig.2b has much less structure and is mostly repulsive. These features have definite consequences for the trinucleon binding energy. In table 1 we display the values of the contribution of $W_s$ to the binding



energy of $^3H$ and $^3He$, evaluated by means of the hyperspherical harmonic method(9,7). In this calculation we have considered only the fully symmetric s wave ground state, since we are mostly interested in the qualitative features of the problem.

The importance of $G(r)$ for $W_s$ can also be determined directly form eq.(6). The coefficients $c_{00}^{+}$ and $c_{01}^{+}$ are of the same order of magnitude and hence the contribution of $G(r)$ relative to that of $U(r)$ can be studied by means of the function $R(r)=\ln(|\partial G/\partial r|/|\partial U/\partial r|)$. The plot of this function is shown in fig.3, where we note that the influence of the form factor is not confined to small internucleon distances.

The preceding discussions show that the function $G(r)$ determines the most important features of $W_s$ and that its influences extend far beyond the small distance region, where they should remain confined to. These results are even more disturbing when we remind ourselves that form factors correspond to corrections of the amplitude in the region of high momenta. For instance, in the parametrization of $\overline{G}(\vec{k}^2)$ used in this work we have $\vec{k}^2/\Lambda^2 \sim \mu^2/m^2 \sim 1/50$. On the other hand, the form of $W_s^{23}$ given by eq.(5) has been derived under the assumption that the nucleons are non-relativistic. Thus the effects introduced by form factors are , in principle, of the same order of magnitude as others neglected throughout the calculation of $W_s$. That relativistic corrections dominate $W_s$ is a clear indication of an inconsistency.

When form factors are not introduced into the problem, the function G is given by $G(r)=(4\pi/\mu^3)\ \delta^3(\vec{r})$. This expression shows that the terms proportional to $G(r)$ in eq.(5) represent contact interactions between two nucleons, corresponding to permutations of the diagram of fig.4a. In this figure the $\sigma$ has been represented as a propagating particle for the sake of clarity.

In fact, it corresponds to a contact interaction, that can be formally obtained by ascribing a very large mass to the $\sigma$.

When we consider form factors, the function $\overline{G}(\vec{k}^2)$ is not equal to one and we have "contact" interactions between extended objects. In order to make this statement more precise, we consider the dynamical content of the $\pi$N form factor. Within the context of the chiral SU(2)xSU(2) group, it corresponds to diagrams such as those of fig.4b. So, by "contact" interactions between extended objects we mean the processes represented in fig.4c.

The inclusion of the function G(r) in the potential means that one is considering forces that do not correspond to the <u>propagation</u> of two pions. This procedure poses various problems. For instance, the dynamical content of form factors used in practical calculations is usually hidden behind a parametrization. This makes it difficult to understand which are the Feynman diagrams one is including in ones calculation. Of course, diagrams such as those depicted in fig.4c should be evaluated at some stage of the research program on three-body forces. However, their inclusion should be the result of explicit calculations, using an appropriate dynamics such as chiral symmetry. Moreover, in this research program, the study of many other processes such as pion-rho, rho-rho, pion-omega, three-pion exchanges should precede those of fig.4c, since they correspond to forces of shorter range. The introduction of dynamical effects through the use of G(r) is problematic in yet another sense. The function G(r) describes "contact" interactions between nucleons tha are not point-like. However, these "contact" interactions are prevented by the short distance repulsion between nucleons and hence their effects are expected not to be large.

In conclusion, the three-body potential $W_s$, given by eq.(5), is composed of two types of terms. One of them, containing the Fourier transform of the form factor, describes a "contact" interaction between two of the nucleons and the propagation of



just one pion. The other one, containig only functions U, corresponds to the propagation of two pions. The considerations produced in this work show that only the latter can be interpreted as a $\pi\pi$E-3BF. Thus the s wave component of the three-body potential can be written as

$$W_s^{23} = \frac{C_s}{\mu^2}\,(\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})(\vec{\sigma}^{(2)}\cdot\vec{\nabla})(\vec{\sigma}^{(3)}\cdot\vec{\nabla}')\;U(r)U(r') \qquad (7)$$

where $C_s = (g\mu/2m)^2(1/4\pi)^2\,\mu^2\,(c_{00}^{+} + 2\mu^2 c_{01}^{+})$. When the experimental results of ref.(5) are used, we obtain the following value for the strength parameter: $C_s = (0.68 \pm 0.08)$ MeV. This value is different from the corresponding ones in refs.(3) and (7) because those works are based on expansions of the amplitude $\bar{C}^{+}(\nu,t)$ containing only terms linear in t. In this case the $\pi$N $\sigma$-term, defined by the relation $\sigma = f_\pi^2\,\tilde{C}^{+}(0,2\mu^2)$, can be written as $\sigma = f_\pi^2\,(c_{00}^{+}+2\mu^2 c_{01}^{+})$ and $C_s$ becomes proportional to $\sigma$. On the other hand, the data of ref.(5) show that a more precise expression is $\sigma = f_\pi^2\,(c_{00}^{+}+2\mu^2 c_{01}^{+}+4\mu^4 c_{02}^{+})$, showing that the proportionality between $C_s$ and $\sigma$ is just an approximate one.

The results presented in this work show that the s wave component of the three-body force due to the exchange of two pions is mostly repulsive. This means that it tends to increase the gap between experimental and two body force calculations of trinucleon properties. Unfortunately.

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TABLE 1

Influence of G(r) on trinucleon binding energy

| Nucleus | Potential $\langle W_s \rangle$ (eq.(14)) | Δ BE (MeV) |
|---|---|---|
| $^3H$ | | +0.612 |
| | G(r) = G(r') = 0 | -0.151 |
| $^3He$ | | +0.592 |
| | G(r) = G(r') = 0 | -0.146 |

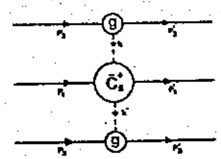

Fig.1 - Feynman diagrams corresponding to $W_s$.

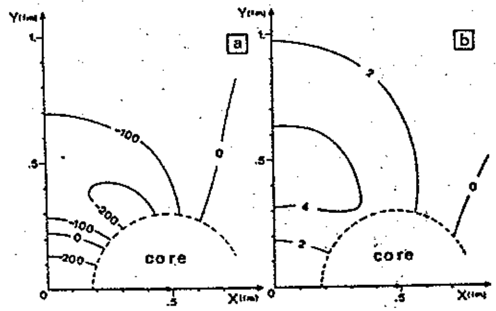


Fig.2 - Equipotential plots for $W_s$, given by eq.(6); a - full equation; b - G(r)=0. Energies in MeV, distances in fm; conventions are those of ref.(6).

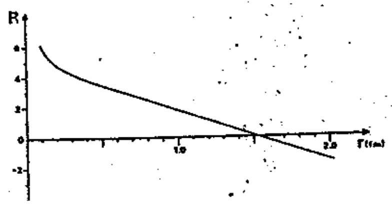

Fig.3 - The direct influence of the form factor.

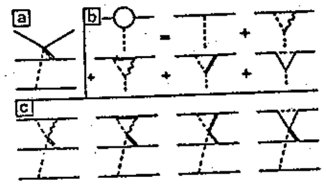

Fig.4 - The "contact" interaction between extended nucleons. NUcleons, deltas, pions, rhos and sigmas are represented respectively by full, thick, broken, wavy and double lines.