

IFUSP/P 408  
B.L.F. - U.S.

# UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA  
CAIXA POSTAL 20516  
01000 - SÃO PAULO - SP  
BRASIL

08 AGO 1983



# publicações

2 IFUSP/P-408

THE AVERAGE ANGULAR DISTRIBUTION OF EMITTED  
PARTICLES IN MULTI-STEP COMPOUND PROCESSES

by

R. Bonetti, B.V. Carlson, M.S. Hussein and  
A. Szanto de Toledo

Instituto de Física, Universidade de São Paulo

Junho/1983

THE AVERAGE ANGULAR DISTRIBUTION OF EMITTED  
PARTICLES IN MULTI-STEP COMPOUND PROCESSES<sup>†</sup>

R. Bonetti\*, B.V. Carlson, M.S. Hussein and A. Szanto de Toledo

Instituto de Física, Universidade de São Paulo  
Caixa Postal 20.516, São Paulo, S.P., Brazil

ABSTRACT

A simple model for the differential cross-section that describes the angular distribution of emitted particles in heavy-ion induced multi-step compound reactions, is constructed. It is suggested that through a careful analysis of the deviations of the experimental data from the pure Hauser-Feshbach behaviour may shed light on the physical nature of the pre-compound, heavy-ion configuration.

<sup>†</sup>Supported in part by the CNPq - Brazil, University of Milano and INFN Sezione di Milano - Italy.

\*Permanent address: Istituto di Fisica Generale Applicata, University of Milano.

May/1983

Light-ion induced multi-step compound reactions have recently been investigated both theoretically<sup>1)</sup> and experimentally<sup>2)</sup> through the statistical analysis of the cross-section. This is primarily achieved with the generalized cross-section auto-correlation function, which in the absence of direct reactions, attains the form<sup>1)</sup>

$$C_{cc'}(E) = \left| \sum_n \frac{\sigma_{n,cc'}^{fl.}}{1 + iE/\Gamma_n} \right|^2 \quad (1)$$

In Eq. (1),  $\Gamma_n$  refers to the correlation width of the  $n^{\text{th}}$ -class of overlapping resonances  $\sigma_{n,cc'}^{fl}$ , the corresponding fluctuation cross-section for the transition  $c \rightarrow c'$ . The total average fluctuation cross-section is simply related to the  $\sigma_n^{fl}$ 's, through<sup>1,3)</sup>

$$\sigma_{cc'}^{fl} = \sum_n \sigma_{n,cc'}^{fl} \quad (2)$$

where the reference to a given partial wave is implicit in both Eqs. (1) and (2).

The experimental results of ref. (2) have shown that at least two distinctly different  $\Gamma_n$ 's seem to be present in the systems studied. This clearly shows that in these reactions at least two or more compound nuclear configurations having different life times ( $\frac{\hbar}{\Gamma_n}$ ) are populated during the transition from channel  $c$  to channel  $c'$ . An important aspect of the pre-compound process, which has so far not been given due attention, is the angular momentum ( $J$ ) distribution of the different components  $\sigma_{n,cc'}^{fl}$  of the average fluctuation cross-section, Eq. (2). In light-ion induced pre-compound

.3.

reactions, the angular distributions of emitted particles are insensitive to the details of this J-distribution of the  $\sigma_{n,cc}^{f\ell}(J)$ . This is due, in part, to the rather wide J-character of the fluctuation cross-section in these cases<sup>3)</sup>. Accordingly, the cross-sections exhibit a structureless  $1/\sin\theta$  behaviour.

In the present Letter, we suggest that the angular distribution of heavy-ion induced multi-step compound reactions, can be sensitive to the detailed J-distribution of the  $\sigma_n^{f\ell}$ , and may, accordingly, be used to extract further useful information about the pre-compound stages. We also suggest that this information may be used to further our understanding of the fusion cross-section<sup>4)</sup>.

Some evidence has recently been presented in support of a picture of light-heavy-ion compound reactions similar in physical content to that given above for light-ion induced pre-compound reactions. Existing data on the reactions  $^{12}\text{C}(^{15}\text{N},\alpha)$  at  $E^* = 22-30$  MeV<sup>5)</sup> and  $^{12}\text{C}(^{16}\text{O},\alpha)$  at  $E^* = 25-35$  MeV<sup>6)</sup>, have been reanalyzed<sup>7)</sup> using Eqs. (1) and (2). Two quite different correlation widths associated with two distinct classes of resonances, were found to dominate the individual transitions. Although the nature of the class characterized by the smaller  $\Gamma$ , is well understood and can be adequately described within the usual Hauser-Feshbach model<sup>8)</sup>, the shorter-lived heavy-ion pre-compound (composite) class, is not as well-understood. Clearly, one needs more information, than that supplied by  $C(\epsilon)$ , in order to pin down the statistical properties of this "composite" HI configuration. The angular distribution of transitions dominated by the composite system, may furnish part of this information.

The behaviour of the experimental angular distribution

.4.

of a given HI-induced transition via the equilibrated compound nucleus indicates that the contributions of the partial cross section,  $\sigma_{CN}^{f\ell}(J)$  are well localized in angular momentum space<sup>9)</sup>. Transitions to low-spin states in the residual nucleus exhibit two important features of this localization: the center of gravity of the window,  $L$  and its width,  $\Delta L$ . The period of the regular oscillations seen in  $\frac{d\sigma}{d\Omega}$  is just  $\frac{\pi}{L}$  whereas the gradual damping of these undulations as  $\theta$  increases is directly related to  $\Delta L$ . The over-all flying-wheel dependence,  $1/\sin\theta$ , is also present.

The above mentioned facts were successfully exploited in Ref. (10) to construct a simple, "statistical window", model for the HI-induced compound differential cross-section. The expression found using a specific parametrized form of  $\sigma_J$  (derivative of a Fermi function) is<sup>10)</sup>

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin\theta} \left\{ 1 + \frac{(-)^I}{2I+1} \left( \sin 2L\theta \frac{2\pi \Delta L \theta}{\sinh 2\pi \Delta L \theta} + \sin 2L(\pi-\theta) \frac{2\pi \Delta L (\pi-\theta)}{\sinh 2\pi \Delta L (\pi-\theta)} \right) \right\} \quad (3)$$

where  $I$  refers to the spin of the residual nucleus and  $A_I$  is an over-all normalization related to the angle-integrated cross-section).

Comparisons between exact Hauser-Feshbach calculation and the statistical window model showed that Eq. (3) is an excellent approximation over almost the full angular range (except for angles in the range  $0 < \theta < \frac{1}{L}$ , where the approximation used in deriving Eq. (3) break down). Short of having an exact theory of the differential cross-section for

heavy-ion induced multi-step compound reactions, we therefore feel justified in using and, further extending, the statistical window model to describe these processes.

We thus describe each of the  $\sigma_{n,cc}^{fl}(J)$ 's that appear in Eqs. (1) and (2) with a suitable statistical window function, constructed following the same guidelines used in Ref. (10). The resulting differential cross-section would then contain as many "oscillation-damping" terms (such as the one appearing in the second term of the RHS of Eq. (3)) as there are classes.

For a two-class dominated reaction we have

$$\frac{d\sigma_I}{d\Omega} = \frac{A_I}{\sin \theta} \left\{ (1+B) + \frac{(-)^I}{2I+1} \left[ B \left( \sin 2L_1 \theta \frac{2\pi \Delta L_1 \theta}{\sinh 2\pi \Delta L_1 \theta} \right. \right. \right. \\ \left. \left. \left. + \sin 2L_1(\pi-\theta) \frac{2\pi \Delta L_1(\pi-\theta)}{\sinh 2\pi \Delta L_1(\pi-\theta)} \right) \right. \right. \\ \left. \left. + \left( \sin 2L_2 \theta \frac{2\pi \Delta L_2 \theta}{\sinh 2\pi \Delta L_2 \theta} \right. \right. \right. \\ \left. \left. \left. + \sin 2L_2(\pi-\theta) \frac{2\pi \Delta L_2(\pi-\theta)}{\sinh 2\pi \Delta L_2(\pi-\theta)} \right) \right] \right\} \quad (4)$$

where 1 and 2 refer to the pre-equilibrium (pre-compound) and equilibrium (compound) stages, respectively. Depending on how different the parameters of the two windows are,  $\frac{d\sigma_I}{d\Omega}$  may exhibit important deviations from the normal one-class behaviour of Eq. (3). We show in Fig. (1) one such deviation. The period of oscillations has clearly diminished as a result of

the larger value of  $L_1$  over  $L_2$ .

We should mention that the center of gravity of the pre-equilibrium J-window is expected, on physical ground, to be larger than that of the equilibrated stage. The composite heavy-ion system representing the pre-equilibrium stage is more likely to be of a "quasi-molecular" shape characterized by an effective moment of inertia larger than that of the sphere that represents the equilibrated compound nucleus. The fact that  $L_1$  is larger than  $L_2$  has another immediate consequence. The total anisotropy defined through

$$R \equiv \frac{\sigma(0^\circ)}{\sigma(90^\circ)} - 1 \quad (5)$$

will be larger in a pre-equilibrium dominated reaction. This can easily be seen for the case  $I=0$ , which gives, in the two-class case being discussed,

$$R = \left\{ \frac{\pi}{2} \left[ B(2L_1+1)^2 + (2L_2+1)^2 \right] \cdot \frac{1}{[B(2L_1+1) + (2L_2+1)]} \right\} - 1 \quad (6)$$

In obtaining Eq. (6) we have assumed a very small  $\Delta L_1$  and  $\Delta L_2$ . The result does not change much if we relax this assumption. For very large  $B$  (dominance of class 1), one has  $R \approx \frac{\pi}{2}(2L_1+1) - 1$ . In the opposite limit  $B \ll 1$ ,  $R \approx \frac{\pi}{2}(2L_2+1) - 1$ .

At this point it is worth while comparing the above formula for the  $I=0$  anisotropy with the corresponding one for a light-ion induced reaction. Owing to the wide J-distribution of the windows one obtains in that case

$$R_{LI} \approx \left\{ \frac{2}{3} \pi \left[ B \left[ (L_1+1)^3 - \frac{1}{4} (L_1+1) \right] + (L_2+1)^3 - \frac{1}{4} (L_2+1) \right] \cdot \frac{1}{B (L_1+1)^2 + (L_2+1)^2} \right\} - 1 \quad (7)$$

The different forms of  $R$  given in Eqs. (6) and (7) reflects clearly the different nature of the J-windows: narrow in heavy-ion induced reactions and wide in light-ion induced reactions.

The effect of the pre-compound stage on other angle-dependent physical quantities, e.g. the angular cross-correlation function, has been discussed in Ref. (11). The coherence angles attached to different transitions may come out quite different, depending on the relative strength of the pre-compound contribution. The differences in the coherence angles reflect the differences in the widths of the contributing classes.

This is easily seen through an examination of the angular cross correlation function, defined by<sup>12)</sup>

$$C_{cc'}(\theta, \theta') = \frac{\left\langle \frac{d\sigma_{cc'}(\theta)}{d\Omega_{c'}} \frac{d\sigma_{cc'}(\theta')}{d\Omega_{c'}} \right\rangle_I - \left\langle \frac{d\sigma_{cc'}(\theta)}{d\Omega_{c'}} \right\rangle_I \left\langle \frac{d\sigma_{cc'}(\theta')}{d\Omega_{c'}} \right\rangle_I}{\left\langle \frac{d\sigma_{cc'}(\theta)}{d\Omega_{c'}} \right\rangle_I \left\langle \frac{d\sigma_{cc'}(\theta')}{d\Omega_{c'}} \right\rangle_I} \quad (8)$$

Restricting the calculation to pure compound processes and ignoring spin effects, we obtain<sup>11)</sup>

$$C_{cc'}(\theta-\theta') \approx \frac{\left| \sum_n \left( \sum_J (2J+1) \sigma_{n,cc'}^{fl}(J) \cos(L_n(\theta-\theta')) \right) \right|^2}{\left( \sum_J (2J+1) \sum_n \sigma_{n,cc'}^{fl}(J) \right)^2} \quad (9)$$

$$= \left| \sum_n b_n \cos(L_n(\theta-\theta')) F_{cc'}(\Delta L_n(\theta-\theta')) \right|^2 \quad (10)$$

where  $L_n$  and  $\Delta L_n$  represent the position of center of gravity and the width of the partial fluctuation cross section  $\sigma_n^{fl}$  of class  $n$ , respectively and  $b_n$  represents the relative contribution of class  $n$ . The function  $F_{cc'}(\Delta L_n(\theta-\theta'))$  attains a unit value at  $\theta=\theta'$  and drops gradually to zero at large values of the difference  $\theta-\theta'$ . In the particular case of channels  $c, c'$  couple strongly to a given class of doorways,  $n$ , only one term in the expression for  $C(\theta-\theta')$  ( $b_n=1$ , Eq. (10)) would then contribute. In this case the coherence angles may be determined from

$$\left( F(\Delta L_n(\theta-\theta')_{coh.}) \right)^2 = 1/2 \quad (11)$$

leading to

$$(\theta-\theta')_{coh., n} = \frac{A_n}{\Delta L_n} \quad (12)$$

with  $A_n$  being a constant determined by the details of the form of  $\sigma_n^{fl}(J)$ .

In the more general case of several classes that couple equally strongly to the channels, then the coherence angle, which is defined as the angle at which the envelope of  $C(\theta-\theta')$  becomes 0.5, is determined by  $L_n$  and  $\Delta L_n$  of all the classes contributing.

It is clear therefore that a careful experimental investigation of the angle-dependent physical quantities in HI multi-step compound reactions, may reveal important information

about the width and center of gravity of the pre-equilibrium, composite, stages. These parameters in turn carry information about the physical nature of the composite system, e.g. its moment of inertia.

A possible picture of the pre-compound stage discussed above may be obtained from that of a composite system at the grazing radius involving towards equilibrium. Such a description can be considered an extension of the present picture of deeply inelastic and quasi-fusion collisions. An important physical quantity that enters in such a description is the density of states of the composite system. As a first approximation to this one can use a convolution of the densities of the two participating nuclei. This yields a density of states that describes a system that has reached equilibrium in energy and angular momentum, with the other degrees of freedom still relaxing. Further, a simple calculation of this density shows a slower decrease with angular momentum than the corresponding density of the equilibrated system<sup>12)</sup>. This fact directly leads to a pre-compound J-window centered at a higher J-value than the equilibrated statistical window, in accord with our expectations.

In conclusion, we have presented in this Letter a simple model for the averaged differential cross-section that describes the angular distribution of emitted particles in a heavy-ion induced multi-step compound nuclear reaction. Through a careful study of the deviations from the normal Hauser-Feshbach behaviour, one may be able to extract the parameters of the pre-compound statistical windows and eventually learn something about the underlying composite configuration. This, in turn, could shed light on the physical origin of the limitation to the fusion cross section at  $E/E_B \approx 2$ .

## REFERENCES

- 1) W.A. Friedman, M.S. Hussein, K.W. McVoy and P.A. Mello, Phys. Rep. 77 (1981) 47. See also, D. Agassi, H.A. Weidenmüller and G. Mantzouranis, Phys. Rep. 22 (1975) 145.
- 2) L. Colli Milazzo, R. Bonetti and A. Garegnani, Lett. Al Nuovo Cimento, 29 (1980) 496;  
R. Bonetti, L. Colli Milazzo, M. Melanotte, A. De Rosa, G. Inghima, E. Perillo, M. Sandoli, V. Russo, N. Saunier and F. Shahin, Phys. Rev. C25 (1982) 717; and to be published.
- 3) H. Feshbach, A.K. Kerman and S.E. Koonin, Ann. Phys. (NY), 125 (1980) 429.
- 4) O. Civitarese, B.V. Carlson, M.S. Hussein and A. Szanto de Toledo, Phys. Lett. B in press (IFUSP/P-365).
- 5) J. Gomez del Campo, M.E. Ortiz, A. Dacal, J.L.C. Ford, R.L. Robinson, P.H. Stelson and S.T. Thornton, Nucl. Phys. A262 (1976) 125;  
J. Gomez del Campo, J.L.C. Ford, R.L. Robinson, M.E. Ortiz, A. Dacal and E. Andrade, Nucl. Phys. A297 (1978) 125.
- 6) M.L. Halbert, F.E. Durham, C.D. Moak and A. Zucker, Phys. Rev. 162 (1967) 916.
- 7) R. Bonetti, L. Colli Milazzo, M. Melanotte and M.S. Hussein, Phys. Rev. C25 (1982) 1406;  
R. Bonetti, L. Colli Milazzo, M. Melanotte and F. Conti, to be published.
- 8) See, e.g., A. Szanto de Toledo, M. Schrader, E.M. Szanto and H.V. Klapdor, Phys. Rev. C19 (1979) 555.
- 9) P. Braun-Munzinger and J. Barrette, Phys. Rev. Lett. 44 (1980) 713;  
A. Szanto de Toledo and M.S. Hussein, Phys. Rev. Lett. 46 (1981) 985.

- 10) M.S. Hussein and A. Szanto de Toledo, Phys. Lett. 107B (1981) 173.
- 11) M.S. Hussein, in Proceedings of the 3<sup>rd</sup> International Conference on Nuclear Reaction Mechanisms, (E. Gadioli, Ed.) 28 (1982) 40.
- 12) D.M. Brink, R.O. Stephen and N.W. Tanner, Nucl. Phys. 54 (1964) 577.
- 13) B.V. Carlson, O. Civitarese, M.S. Hussein and A. Szanto de Toledo, to be published.

FIGURE CAPTION

FIG. 1 - The angular distribution of the emitted  $\alpha$ -particle in the reaction  $^{12}\text{C}(^{16}\text{O},\alpha)$  to the  $I=0$  state in  $^{24}\text{Mg}$ . Solid curve represents the pure compound emission ( $B=0$ ) (Eq. (5)) and the dashed curve the result with  $B=4$ . The parameters were  $L_1 = 18.5$ ,  $\Delta L_1 = \frac{2.7}{2.9}$ ,  $L_2 = 12.5$ ,  $\Delta L_2 = 2.7$ .

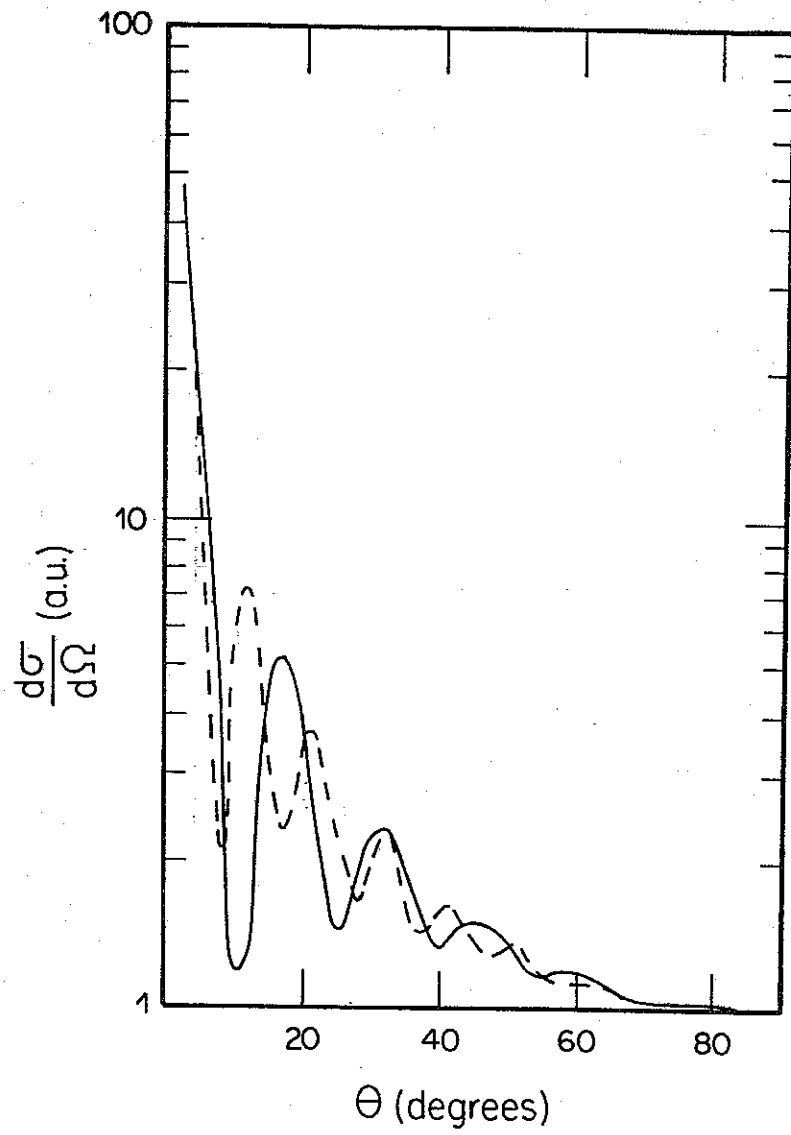


Fig. 1