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# ON SOME FEATURES OF $CP^{n-1}$ MODELS WITH FERMIONS

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## ABSTRACT

We present the bound state S-matrix of the $CP^{n-1}$ model coupled to fermions in a minimal or supersymmetric way, and discuss the relation to the low energy features in the supersymmetric case.



1. The $CP^{n-1}$ model displays, classically, an infinite series of conservation laws[(1)]. It is well known that they do not survive quantization, neither the non-local[(2)] nor the local ones[(3)(4)]. However, if the $CP^{n-1}$ - bosons interact with fermions in a minimal or supersymmetric way, the (first) non-local conservation law is restored to leading order[(5)], and, in fact, to all orders[(6)] in $1/n$. This is sufficient to define a factorizable S matrix, as done in refs. (7) and (8). Here, we shall use the conserved non-local charge to derive this S matrix and draw conclusions about the bound states, whose scattering amplitudes are derived from those of the partons by well known methods[(9)(10)]. As expected, the resulting S matrix, although intersting from a purely mathematical point of view, does not exhibit pair production. Moreover, in the supersymmetric case, it is possible to present an effective Lagrangian displaying interesting physical features[(11)]. In fact, the non-local charge turns out to be a kind of hidden symmetry, because it is written originally in terms of the integrated $z$ and $\psi$ fields.

2. The minimal model defined by the lagrange density

$$\mathcal{L} = \overline{D_\mu z}\, D_\mu z + i\,\overline{\psi}\,(\partial\!\!\!/ - iA\!\!\!/)\,\psi \tag{1}$$

has a conserved Noether current

$$J^{ij}_\mu = z^i \overleftrightarrow{D}_\mu \bar{z}^j \tag{2}$$

and the first quantum non-local charge

$$Q^{ij} = \lim_{\delta\to 0} \iint_{|y_1-y_2|\geq\delta} dy_1 dy_2\, J_0^{ik}(t,y_1) J_0^{kj}(t,y_2)\, \varepsilon(y_1-y_2) -$$

$$- \frac{1}{\pi} \ln(\mu\delta) \int dy\, J_1^{ij}(t,y) + \int_{-\infty}^{\infty} (z^i \bar{\psi} \gamma_1 \psi \bar{z}^j)(t,y)\, dy \qquad (3)$$

is conserved [6].

This expression can be put in terms of asymptotic fields. The current $J_\mu$ cannot be obtained by simple substitution of the asymptotic fields. However, we can take the asymptotic current as being that one which has the same commutation relations and same vacuum expectation values [12].

After some work we have

$$Q^{ij}_{\substack{in\\(out)}} = \binom{+}{-} \frac{1}{n} \int d\beta(p_1) d\beta(p_2)\, \tilde{\varepsilon}(p_1-p_2) : (b^{i\dagger}(p_1) b^k(p_1) -$$

$$- a^{k\dagger}(p_1) a^i(p_1) + d^{i\dagger}(p_1) d^k(p_1) - c^{k\dagger}(p_1) c^i(p_1)) (a^{j\dagger}(p_2) a^k(p_2) -$$

$$- b^j(p_2) b^k(p_2) + c^{j\dagger}(p_2) c^k(p_2) - d^{j\dagger}(p_2) d^k(p_2)) : +$$

$$+ \frac{1}{i\pi} \int d\beta(p)\, \ln \frac{p^0+p'}{m} : \Big\{ (b^{i\dagger}(p) b^j(p) - a^{j\dagger}(p) a^i(p) +$$

$$- d^i(p) d^{j\dagger}(p) - c^{j\dagger}(p) c^i(p)) + \frac{\delta^{ij}}{n} (b^{\ell\dagger}(p) b^\ell(p) -$$

$$- a^{\ell\dagger}(p) a^\ell(p) + d^{\ell\dagger}(p) d^\ell(p) - c^{\ell\dagger}(p) c^\ell(p)) \Big\} : \qquad (4)$$

where $d\beta(p) = \frac{dp'}{2\pi 2p^0}$.

As a result of the equality of the "in" and "out" charges, the exact S-matrix turns out to be of class II of ref. [13], without bound states [7], such that



$$t_1(\theta) = \frac{\Gamma\left(\frac{1}{2} + \frac{\theta}{2\pi i}\right) \Gamma\left(\frac{1}{2} + \frac{1}{n} - \frac{\theta}{2\pi i}\right)}{\Gamma\left(\frac{1}{2} + \frac{\theta}{2\pi i}\right) \Gamma\left(\frac{1}{2} + \frac{1}{n} + \frac{\theta}{2\pi i}\right)} \qquad (5)$$

3. The supersymmetric model is more interesting. The non local charge can be again defined (for details see (5) and (6). Again we can obtain the factorization equations as a result of the equality of the "in" and "out" charges. The result for asymptotic boson states $b(\theta)$ and fermion states $f(\theta)$ is:

$$\langle b_\beta(\theta_1') b_\delta(\theta_2') | b_\alpha(\theta_1) b_\gamma(\theta_2) \rangle =$$

$$= \delta(\theta_1-\theta_1')\delta(\theta_2-\theta_2')\, {}_{\alpha\gamma}V_{\beta\delta}(\theta) + \delta(\theta_1-\theta_2')\delta(\theta_2-\theta_1')\, {}_{\alpha\gamma}V_{\delta\beta}(\theta) \qquad (6)$$

$$\langle f_\beta(\theta_1') f_\delta(\theta_2') | f_\alpha(\theta_1) f_\gamma(\theta_2) \rangle =$$

$$= \delta(\theta_1-\theta_1')\delta(\theta_2-\theta_2')\, {}_{\alpha\gamma}U_{\beta\delta}(\theta) - \delta(\theta_1-\theta_2')\delta(\theta_2-\theta_1')\, {}_{\alpha\gamma}U_{\delta\beta}(\theta) \qquad (7)$$

$$\langle f_\beta(\theta_1') b_\delta(\theta_2') | f_\alpha(\theta_1) b_\gamma(\theta_2) \rangle =$$

$$= \delta(\theta_1-\theta_1')\delta(\theta_2-\theta_2')\, {}_{\alpha\gamma}C_{\beta\delta}(\theta) - \delta(\theta_1-\theta_2')\delta(\theta_2-\theta_1')\, {}_{\alpha\gamma}D_{\beta\delta} \qquad (8)$$

with $V$, $U$, and $D$ given by ${}_{\alpha\gamma}V_{\beta\delta}(\theta) = v_1(\theta)\delta_{\alpha\beta}\delta_{\gamma\delta} + v_2(\theta)\delta_{\alpha\delta}\delta_{\gamma\beta}$ (analogously for $U$, $u_1$, $u_2$ ; $C$, $c_1$, $c_2$ ; $D$, $d_1$, $d_2$) and, the backward particle-antiparticle scattering amplitudes vanish. Moreover,

$$v_1(\theta) = \frac{\sin\frac{\pi}{2}\left(\frac{\theta}{\pi i} - \frac{2}{n}\right)}{\sin\frac{\theta}{2i}}\, c_1(\theta) \qquad (9)$$

$$u_1(\theta) = \frac{\sin\frac{\pi}{2}\left(\frac{\theta}{\pi i}+\frac{2}{n}\right)}{\sin\frac{\theta}{2i}}\, c_1(\theta) \qquad (10)$$

$$d_1(\theta) = -\frac{\sin\frac{\pi}{n}}{\sin\frac{\pi}{2i}}\, c_1(\theta) \qquad (11)$$

$$c_1(\theta) = \frac{\Gamma\left(\frac{\theta}{2\pi i}+\frac{1}{n}\right)\Gamma\left(1-\frac{\theta}{2\pi i}\right)}{\Gamma\left(1-\frac{\theta}{2\pi i}+\frac{1}{n}\right)\Gamma\left(\frac{\theta}{2\pi i}\right)} \cdot \frac{\Gamma\left(\frac{\theta}{2\pi i}-\frac{1}{n}\right)\Gamma\left(1-\frac{\theta}{2\pi i}\right)}{\Gamma\left(1-\frac{\theta}{2\pi i}-\frac{1}{n}\right)\Gamma\left(\frac{\theta}{2\pi i}\right)} \qquad (12)$$

4. The bound state S matrix can now be written down immediately[(9),(10)]. The mass spectrum is given by

$$m_\ell = m\frac{\sin\frac{\ell\pi}{n}}{\sin\frac{\ell}{n}} \quad , \qquad \ell = 1,2,\ldots\ n-1 \qquad (13)$$

We write, symbolically, for the bound state of $p$ fermions

$$\pi_{\alpha_1\ldots\alpha_p}(\Sigma\theta_i/p)\rangle = \frac{1}{\sqrt{p}}\prod_{i=1}^{p}|f_{\alpha_i}(\theta_i)\rangle \qquad (14)$$

and for the bound state of p-2 fermions and 1 boson

$$|\Sigma_{\alpha_1\ldots\alpha_p}((\theta_1+\ldots+\theta_p)\rangle = \frac{1}{\sqrt{p}}\left\{|b_{\alpha_1}(\theta_1)f_{\alpha_2}\ldots f_{\alpha_p}(\theta_p)\rangle+\ldots+\right.$$

$$\left.+\ |f_{\alpha_1}(\theta_1)\ldots f_{\alpha_{p-1}}(\theta_{p-1})b_{\alpha_p}(\theta_p)\rangle\right\} =$$

$$= \frac{1}{\sqrt{p}}\sum_{\alpha=1}^{p}|\alpha\rangle \qquad (15)$$

On states composed of these and of a single further fermion resp. boson state, the S matrix acts according to



$$S|\pi_{\alpha_1\ldots\alpha_{n-1}}(\theta_1)f_{\alpha_n}(\theta_2)\rangle = \frac{1}{\sqrt{n-1}}\sum{}'\prod_{j=1}^{n-1}U_{jn}\prod_{i=1}^{n-1}|f_{\alpha_i'}\,f_{\alpha_n'}\rangle$$

resp

$$S|\pi_{\alpha_1\ldots\alpha_{n-1}}(\theta_1)b_{\alpha_n}(\theta_2)\rangle = \frac{1}{\sqrt{n-1}}\sum{}'\left\{\prod_{j=1}^{n-1}R_{jn}\prod_{i=1}^{n-1}|f_{\alpha_i'}\,b_{\alpha_n'}\rangle\ +\right.$$

$$\left. +\sum_{\alpha'=1}^{n-1}\prod_{j=1}^{n-1}R_{jn}^{\alpha'}|f_{\alpha_j'}\,f_{\alpha_n'}\rangle\right\} \qquad (16)$$

where $\sum'$ means the sum over all permutations in the isospin indices $\alpha_i'$. Writing out the products in (16), we get

$$U_{jn} = u_\beta \qquad (17)$$

$$R_{jn} = c_\beta \qquad (18)$$

$$R_{jn}^{\alpha'} = \begin{cases} u_\beta & \text{if} \quad j<\alpha' \qquad (19a) \\ c_\beta & \text{if} \quad j\geq\alpha' \qquad (19b)\end{cases}$$

($\beta=1$ if $\alpha_i'=\alpha_i$ and $\beta=2$ if $\alpha_i'\neq\alpha_i$).

Other amplitudes can be handled similarly. Thus the bound state

$$B_\delta = \varepsilon_{\delta_{\alpha_1\ldots\alpha_{n-1}}}\pi_{\alpha_1\ldots\alpha_{n-1}} \qquad (20)$$

can be identified with the antiboson, and the bound state.

$$F_\alpha = \varepsilon_{\alpha\alpha_1\ldots\alpha_{n-1}}\Sigma_{\alpha_1\ldots\alpha_{n-1}} \qquad (21)$$

with the antifermion.

It is interesting to point out that in the case of the $CP^1$ model, some new features arise. First, in the absence of fermions, the S-matrix for bound-states of two partons is identical with that for the 0(3) nonlinear σ-model. Moreover, in the $CP^1$ model the S-matrix for the partons is also factorizable, because that particular model is anomaly-free(14)(15). If fermions are coupled minimally, then the bound state disappears, because so does the long range force between the partons. If they are coupled supersymmetrically, then the bound-states should reproduce the S-matrix for the supersymmetric 0(3) nonlinear σ-model(16).

5. In comparison to the recent paper from D'Adda et al.(11), it is intersting to point out that in the low-energy region the structure of their model is such as to admit pair production. This is a consequence of the fact that in their model, the Adler anomaly has higher order corrections. It is an open problem to verify if the non-local charge anomaly has also the same corrections, however we think it has not, because the non-oocal symmetry is much more rigid, and consequently more difficult to be maintained in a modified Lagrangian. This would give a very interesting picture, where only for high energy the non-local symmetry constraints the model to be factorizable, whereas it should be modified in the low energy region.

We should perhaps also say that in our model, although the Adler anomaly remains constrained by Adler Bardeen's theorem, this seems not to be the case, for the trace anomaly, which is given by



$$
\begin{aligned}
N_2(\theta^{\mu}_{\mu}) &= N_2\left[-\bar{z}D_{\mu}D^{\mu}z + \bar{\psi}\,\not{D}\,\psi\right] = \\
&= N_2\left[+m^2\bar{z}z + m\bar{\psi}\psi\right] = \\
&= m^2\,N_0\left[\bar{z}z\right] + m\,N_1\left[\bar{\psi}\psi\right] + \text{anomalous contributions}
\end{aligned}
\tag{22}
$$

where

$$\theta_{\mu\nu} = \bar{z}D_{\mu}D_{\nu}z + \frac{1}{2}\bar{\psi}(\gamma_{\mu}D_{\nu} + \gamma_{\nu}D_{\mu})\psi$$

and $N_{\delta}$ means normal product in the sense of Zimmermann(17) (see also (18)), In this case anomalous contributions are highly non trivial, in constrast to Adlers anomaly, and the first non zero term already involves integration over the (non trivial) auxiliary particle propagators. We do not know yet how to deal with this anomaly.

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