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IFUSP/P-434

NOTE ON A VERSION OF THE SIGMA NON-LINEAR MODEL IN 3+1 DIMENSIONS

by

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Outubro/1983

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ABSTRACT

We generalize the model proposed by Cremmer and Scherk to represent the Sigma Non-linear Model in 3+1 dimensions and quantize it via functional path integration method.

1. INTRODUCTION

When we couple supergravity to scalar (A) (pseudo-scalar-B) fields[1] the complete lagrangian becomes multiplied by an exponential factor of the form $\exp -\frac{1}{6}(A^2+B^2)$, which reveals its non-polynomial character. If we proceed in quantizing this kind of theory, via functional path integration method, the exponential factor represents a serious hindrance in this respect. It becomes desirable to impose the (geometrical) constraint $A^2+B^2 = \text{constant}$. This constraint is of the same kind as the one satisfied by the Non-linear Sigma Model[2,3]. Cremmer and Scherk[3] obtained the supersymmetric Non-linear Model in 3+1 dimensions by starting with a N component Wess-Zumino scalar multiplet and imposing the necessary constraints so as to make the theory supersymmetric.

The Non-linear Sigma Model in 1+1 dimensions is renormalizable and presents important properties such as assymptotic freedom, dimensional transmutation and dynamical mass generation, which turns it into a good laboratory for the study of 3+1 dimensional models such as QCD. On the other hand, in 3+1 dimensions, it is non-renormalizable (by usual power counting arguments) so that it is necessary to introduce a cut-off in the ultraviolet region, regarded as an independent parameter.

We will show that if we improve the lagrangian due to Cremmer and Scherk then the N component Nambu = Jona-Lasinio Model[4] appears, in part, as the supersymmetric partner of the Non-linear Sigma Model. The Nambu = Jona-Lasinio Model represented, at the beginning of the sixties, a pioneering work in relation to the study of the mass generation in terms of symmetry breaking, via quartic fermion interaction. Eguchi[5] argues that because of the high divergence structure, four fermion interaction theories[4,6] are indeed equivalent to renormalizable theories.

We propose to study the generalization of the model proposed by Cremmer and Scherk in order to permit supersymmetric mass generation. In this way we sum a four fermion scalar interaction term (plus a four fermion pseudo-scalar interaction in order to maintain supersymmetry) to their lagrangian. Mass generation occurs, on the one side, for the bosonic fields, due to the geometric constraint, and on the other side, for the fermionic fields, due to the presence of the four fermion scalar interaction term. The resultant model is very similar to the supersymmetric Non-linear Sigma Model in 1+1 dimensions[7].

This paper is organized as follows: in part 2 we introduce the model which is quantized, via functional path integration, so that we obtain an effective action in part 3. In part 4 we show the occurence of supersymmetric mass generation and finally in part 5 we draw conclusions.

2. THE MODEL

The following lagrangian

$$L = \underset{(a)}{\frac{1}{2}\partial^a\phi_i^* \partial_a\phi_i} + \underset{(b)}{\frac{1}{2}\bar{\psi}_i \not\partial \psi_i} + \frac{1}{8}\underset{(c)}{\left[\phi_i^* \overleftrightarrow{\partial}_a \phi_i + \bar{\psi}_i \gamma_a \gamma_5 \psi_i\right]^2} -$$

$$- \underset{(d)}{\frac{1}{8}(\bar{\psi}_i\psi_i)^2} + \underset{(e)}{\frac{1}{8}(\bar{\psi}_i\gamma_5\psi_i)^2} \tag{2.1}$$

(*) See Appendix for conventions.

and the constraints

$$\text{a)}\quad \phi_i^*\phi_i = 1 \tag{2.2a}$$

$$\text{b)}\quad \tilde{\phi}_i\psi_i = 0 \tag{2.2b}$$

$$\text{c)}\quad \tilde{\phi}_i^2 = 0 \tag{2.2c}$$

$$\text{d)}\quad \tilde{\phi}_i\gamma^a\psi_i = 0 \tag{2.2d}$$

is supersymmetric under the transformations

$$\delta\phi_i = \bar{\varepsilon}(1-\gamma_5)\psi_i \tag{2.3a}$$

$$\delta\psi_i = \gamma^a\left[\partial_a\tilde{\phi}_i - \frac{1}{2}\gamma_5\tilde{\phi}_i(\phi_j^*\overleftrightarrow{\partial}_a\phi_j + \bar{\psi}_j\gamma_a\gamma_5\psi_j)\right]\varepsilon + $$

$$+ \frac{1}{2}(\bar{\psi}_j\phi_j)\tilde{\phi}_i\varepsilon + \frac{1}{2}(\bar{\psi}_j\gamma_5\psi_j)\tilde{\phi}_i\gamma_5\varepsilon \tag{2.3b}$$

where the N component bosonic field is defined by $\phi_i \equiv A_i+iB_i$, such that A_i is a scalar and B_i a pseudo-scalar field. Also the fermionic ψ_i N component field is a Majorana spinor, and $\tilde{\phi}_i \equiv A_i+\gamma_5B_i$.

It should be noticed that the supersymmetric transformations (2.3a/b) are on-shell. This means that if we go off-shell there are necessary auxiliary fields. We can show that, by improving $\delta\psi_i$ as

$$\delta'\psi_i = \tilde{F}_i\,\varepsilon \tag{2.4}$$

where $\tilde{F}_i$ $(\equiv F_i + i\gamma_5 G_i)$ satisfies the relations

$$\bar{F}_i \, \tilde{\phi}_i = 0 \tag{2.5a}$$

$$\bar{F}_i \, \gamma^a \tilde{\phi}_i = 0 \quad , \tag{2.5b}$$

and transforms supersymmetrically as

$$\delta \bar{F}_i = \bar{\varepsilon}(1 - \gamma_5) \not{D} \, \psi_i + \ldots \tag{2.6}$$

$(D_a \equiv \partial_a + i \gamma_5 A_a$ and $A_a = -\frac{i}{2} [\phi_i^* \overleftrightarrow{\partial}_a \phi_i + \bar{\psi}_i \gamma_a \gamma_5 \psi_i])$

then the lagrangean, plus $-F_i^2/2$ and $-G_i^2/2$, is supersymmetric. The auxiliary fields F_i and G_i do not appear in the effective action so that they do not become dynamical at the quantum level.

Because of the large number of constraints, the theory is well defined only if N is big.

3. FUNCTIONAL PATH INTEGRAL QUANTIZATION

We proceed in quantizing the theory by using the functional path integration method. The Green function functional generator is:

$$Z(J_i, J_i^*, T_i) = N^{-1} \int \prod_{i,j,k} [d\phi_i^*][d\phi_j][d\psi_k] \prod_x \delta\, [\phi_i^* \phi_i - 1] \times$$

$$\times \delta \Big[[\phi_i^* + \phi_i + \gamma_5 (\phi_i - \phi_i^*)] \psi_i \Big] \delta \Big[[\phi_i + \phi_i^* - \gamma_5 (\phi_i - \phi_i^*)] \psi_i \Big] \times$$

$$\times \delta \Big[\phi_i^2 + \phi_i^{*2} \Big] \delta \Big[\phi_i^2 - \phi_i^{*2} \Big] \exp i \int dx^4 \Big\{ \frac{1}{2} \partial^a \phi_i^* \partial_a \phi_i + \frac{1}{2} \bar{\psi}_i \not{\partial} \psi_i +$$

$$+ \frac{1}{8} \Big[\phi_i^* \overleftrightarrow{\partial}_a \phi_i + \bar{\psi}_i \gamma_a \gamma_5 \psi_i \Big]^2 - \frac{1}{8} (\bar{\psi}_i \psi_i)^2 + \frac{1}{8} (\bar{\psi}_i \gamma_5 \psi_i)^2 +$$

$$+ J_i^* \phi_i + \phi_i^* J_i + \frac{\bar{T}_i \psi_i}{2} + \frac{\bar{\psi}_i T_i}{2} \Big\} \tag{3.1}$$

where $A_i = \frac{1}{2} (\phi_i + \phi_i^*)$ and $B_i = \frac{1}{2} (\phi_i - \phi_i^*)$. The constraint (2c), if written in terms of ϕ_i and ϕ_i^* has the form $\phi_i^2 + \phi_i^{*2} + \gamma_5 (\phi_i^2 - \phi_i^{*2}) = 0$, which is equivalent to the constraints $\phi_i^2 + \phi_i^{*2} = 0$ and $\phi_i^2 - \phi_i^{*2} = 0$ taken separatelly.

Now, in order to perform gaussian functional integration over ϕ_i and ψ_i, we introduce auxiliary fields.

If

$$V_a \equiv \phi_i^* \overleftrightarrow{\partial}_a \phi_i + \bar{\psi}_i \gamma_a \gamma_5 \psi_i \tag{3.2}$$

we have that

$$\exp \frac{i}{8} \int dx^4 \, V_a^2 = N^{-1} \int [d\lambda_a] \exp i \int dx^4 \Big[\lambda_a^2 + \frac{i}{\sqrt{2}} V^a \lambda_a \Big] . \tag{3.3}$$

Similarly for the $(\bar{\psi}_i \psi_i)^2$ and $(\bar{\psi}_i \gamma_5 \psi_i)^2$ terms, we obtain

$$\exp -\frac{i}{8} \int dx^4 (\bar{\psi}_i \psi_i)^2 = N^{-1} \int [d\sigma] \exp i \int dx^4 \Big[\sigma^2 + \frac{\sigma}{\sqrt{2}} (\bar{\psi}_i \psi_i) \Big] \tag{3.4}$$

and

$$\exp \frac{i}{8} \int dx^4 (\bar{\psi}_i \gamma_5 \psi_i)^2 = N^{-1} \int [d\pi] \exp i \int dx^4 \Big[\pi^2 + \frac{i}{\sqrt{2}} \pi (\bar{\psi}_i \gamma_5 \psi_i) \Big] \tag{3.5}$$

After expontiating the constraints, which appear in the measure of the integration, we write Z as:

$$Z = N^{-1}\int \prod_{i,j,k} [d\phi_i^*][d\phi_j][d\psi_k][d\alpha][dc_1][dc_2][d\lambda_a][d\sigma][d\pi] \times$$

$$\times [dk_1][dk_2] \exp i \int dx^4 \left\{ \frac{1}{2}\partial^a \phi_i^* \partial_a \phi_i + \frac{1}{2}\bar{\psi}_i \not\partial \psi_i + \lambda_a^2 + \right.$$

$$+ \frac{i}{\sqrt{2}} v^a \lambda_a + \sigma^2 + \frac{\sigma}{\sqrt{2}}(\bar{\psi}_i\psi_i) + \pi^2 + \frac{i}{\sqrt{2}}\pi(\bar{\psi}_i\gamma_5\psi_i) +$$

$$+ \frac{\alpha}{\sqrt{N}}[\phi_i^*\phi_i - 1] + \frac{1}{2\sqrt{N}}\bar{\psi}_i[(1+\gamma_5)c_1 + (1-\gamma_5)c_2]\phi_i +$$

$$+ \frac{1}{2\sqrt{N}}\phi_i^*[\bar{c}_1(1-\gamma_5) + \bar{c}_2(1+\gamma_5)]\psi_i + \frac{1}{2\sqrt{N}}[(k_1+k_2)\phi_i^2 +$$

$$\left. + (k_1-k_2)\phi_i^{*2}] + \text{source terms}\right\} . \qquad (3.6)$$

Finally we use

$$\exp\frac{i}{2N}\int dx^4 (k_1-k_2)\phi_i^{*2} = N^{-1}\int[d\beta]\exp i\int dx^4\left[\frac{\beta^2}{2} + i\left(\frac{k_1-k_2}{N}\right)^{1/2}\beta\phi_i^*\right] \qquad (3.7a)$$

and

$$\exp\frac{i}{2N}\int dx^4 (k_1+k_2)\phi_i^{2} = N^{-1}\int[d\beta^*]\exp i\int dx^4\left[\frac{\beta^{*2}}{2} - i\left(\frac{k_1+k_2}{N}\right)^{1/2}\phi_i\beta^*\right] \qquad (3.7b)$$

Putting (3.7a/b) in (3.6) and integrating over ϕ_i and ψ_i, gives us that

$$Z(J_i, J_i^*, T_i) = N^{-1}\int [d\alpha][d\beta][d\beta^*][dk_1][dk_2][d\sigma][d\pi][d\lambda_a][dc_1] \times$$

$$\times [dc_2] \exp\left\{i\, S_{eff} - \frac{1}{2}\bar{T}_i \Delta_F^{-1} T_i - J_i^{*\prime}\Delta_B^{-1} J_i^{\prime}\right\} \qquad (3.8)$$

where

$$J_i^{\prime} \equiv J_i + \frac{i}{2\sqrt{N}}[\bar{c}_1(1-\gamma_5) + \bar{c}_2(1-\gamma_5)]\Delta_F^{-1}T_i + i\left(\frac{k_1-k_2}{N}\right)\beta \qquad (3.9)$$

$$\Delta_F \equiv -i\not\partial + \sqrt{2}\gamma_a\gamma_5\lambda^a - i\sqrt{2}\sigma + \sqrt{2}\pi\gamma_5 \qquad (3.10)$$

$$\Delta_B \equiv \frac{1}{2}\left[i\Box + 2\sqrt{2}\lambda^a\partial_a - 2i\lambda_a^2 - \frac{2i\alpha}{\sqrt{N}} + \frac{1}{N}[\bar{c}(1-\gamma_5) + \bar{c}_2(1+\gamma_5)] \times\right.$$

$$\left.\times \Delta_F^{-1}[(1+\gamma_5)c_1 + (1-\gamma_5)c_2]\right] \qquad (3.11)$$

and

$$S_{eff} = -i\left\{N\left[\frac{1}{2}\mathrm{Tr}\,\ell n\,\Delta_F - \mathrm{Tr}\,\ell n\,\Delta_B\right] - \int dx^4\left[\sigma^2 + \pi^2 - \frac{\alpha}{\sqrt{N}} + \right.\right.$$

$$\left.\left. + \frac{\beta^2}{2} + \frac{\beta^{*2}}{2}\right]\right\} . \qquad (3.12)$$

Rescaling the fields $\phi_i \to \phi_i^{\prime} = \left(\frac{N}{g_o}\right)^{1/2} i$; $\psi_i \to \psi_i^{\prime} = \left(\frac{N}{g_o}\right)^{1/2}\psi_i$; $\lambda_a \to \lambda_a^{\prime} = \frac{1}{\sqrt{N}}\lambda_a$; $\sigma \to \sigma^{\prime} = \sqrt{g_o}\,\sigma$ and $\pi \to \pi^{\prime} = \sqrt{g_o}\,\pi$, gives us for the effective action

$$S_{eff} = -i\left\{N\left[\frac{1}{2}\mathrm{Tr}\,\ell n\left[-i\not\partial + \sqrt{\frac{2}{N}}\lambda_a\gamma_5\gamma^a - i\sqrt{\frac{2}{N}}\sigma + \sqrt{\frac{2}{N}}\pi\gamma_5\right] - \right.\right.$$

$$- \mathrm{Tr}\,\ell n\left[i\Box + 2\sqrt{\frac{2}{N}}\lambda^a\partial_a - \frac{2i}{N}\lambda_a^2 - \frac{2i\alpha}{\sqrt{N}} + \right.$$

$$\left.\left.\left. + \frac{1}{N}\left[[\bar{c}_1(1-\gamma_5) + \bar{c}_2(1+\gamma_5)]\Delta_F^{-1}[(1+\gamma_5)c_1 + (1-\gamma_5)c_2]\right]\right]\right]\right\} -$$

$$- \int dx^4\left[\frac{\sigma^2}{g_o} + \frac{\pi^2}{g_o} - \frac{\sqrt{N}}{g_o}\alpha + \frac{\beta^2}{2} + \frac{\beta^{*2}}{2}\right] \qquad (3.13)$$

4. 1/N EXPANSION AND MASS GENERATION

As it can be depicted from the effective action given by (3.13), we write it in terms of a 1/N expansion

$$S_{eff} = \sum_{i=1}^{\infty} N^{1-i/2} S_i \qquad (4.1)$$

where S_i is the "i" point function involving all (auxiliary) fields that appear in it. The lagrangian proposed by Cremmer and Scherk[3], given by the terms (a), (b) and (c) of expression (2.1), does not permit a supersymmetric mass generation. The constraint (2.2a) gives rise, at the quantum level, of a mass term for the ϕ_i field, which is equivalent to a rescaling of the (infinite) normalization factor that appears in $Z(J_i, J_i^*, T_i)$. But this fact means that supersymmetry is broken, since we do not have a mechanism to provide a mass for the fermionic ψ_i field. The addition of the terms (d) and (e) in (2.1) permits the generation of the fermionic mass. Nambu and Jona-Lasinio[4] noticed that the mass generation for a fermionic field occurs due to some interaction between massless bare fermions fields in a direct analogy with the quasi-particles that emerge in the context of super condutivity due a phonon mediated interaction between the electrons[8]. Its 1+1 dimensional analogue, the Gross-Neveu Model[9], displays a mass generation mechanism due to the global chiral symmetry breaking of the one loop effective potential for the auxiliar $\underline{\sigma}$ field (introduced in order to eliminate the quartic fermionic interaction and to perform gaussian integration). Since this mass is not introduced artificially but appears as a dynamical result of the four fermion interaction, at the quantum level, it is called dynamical mass generation.

The hope is to display an analog formalism in 3+1 dimensions, which would discard the presence of the Higg mechanism and as a consequence the Higgsparticles.

In our case, we present a formal supersymmetric procedure to generate masses. It is formal until this stage because the renormalizability of the model rests as an open question.

The linear term for the σ field, obtained in the 1/N expansion of the effective action, is given by

$$- i \frac{\sqrt{N}}{2} \, \mathrm{Tr} \left\{ \sqrt{2} \, \frac{i\sigma}{i\not\partial} \right\} \qquad (4.2)$$

If $N \to \infty$ this term oscilates strongly in the exponential so that the 1/N expansion is badly defined. In order to eliminate it we define a new (shifted) field $\underline{\sigma'}$ as

$$\sigma'(x) \equiv \sigma(x) - \sqrt{\frac{N}{2}} \, \sigma_o \qquad (4.3)$$

such that its vacuum expectation value is zero $\left[\langle \sigma' \rangle_o = 0 \right]$, and σ_o is constant. Putting (4.3) into the expression of the effective action (3.13) results in the appearence of a mass term for the ψ_i field, that is

$$\sigma_o = M \qquad (4.4)$$

In terms of $\underline{\sigma'}$ the linear term turns into

$$\Gamma^{\sigma'} = - i \sqrt{N} \left\{ \frac{\mathrm{Tr}}{2} \left[\frac{-i\sqrt{2}\,\sigma'}{-i(\not\partial + M)} \right] - \int dx^4 \, 2M \, \frac{\sigma'}{\sigma_o} \right\} \qquad (4.5)$$

Defining

$$F \equiv -i(\not\partial + M) \qquad (4.6a)$$

and

$$\langle x|F^{-1}|y\rangle \equiv \frac{1}{(2\pi)^4}\int \frac{dk^4\, e^{ik(x-y)}}{\not k - iM} \qquad (4.6b)$$

gives us for (4.5), in momentum space

$$\Gamma^{\sigma'}(p^2,M^2) = -i\sqrt{N}\left\{ tr\left[-i\frac{\sqrt{2}}{2}\tilde{\sigma}'(0)\int\frac{dk^4(\not k+iM)}{k^2+M^2}\right] - \sqrt{2}\,\frac{M}{g_o}\,\tilde{\sigma}'(0)\right\} \qquad (4.7)$$

where trace (tr) operates over spinorial indices and

$\tilde{\sigma}'(p) = \frac{1}{(2\pi)^4}\int dx^4\, e^{-ipx}\,\sigma(x)$, etc.

Now, imposing the vanishing of (4.7), so that 1/N expansion becomes well behaved, implies, after a Pauli-Villars regularization in terms of a ultraviolet cut-off Λ, in that

$$\frac{1}{2\pi^2 g_o} = \Lambda^2 - M^2\,\ell n\left(\frac{\Lambda^2}{M^2}+1\right) \qquad (4.8)$$

On the other hand, the linear term of the $\underline{\alpha}$ field is

$$\Gamma^{\alpha} = -i\sqrt{N}\left[Tr\,\frac{2i\alpha}{i(\Box - m^2)} + \int dx^4\,\frac{\alpha}{g_o}\right] \qquad (4.9)$$

where we took into account the ϕ_i field mass generation

through the geometric constraint (2.2a), as explained earlier. After defining

$$B \equiv i(\Box - m^2) \qquad (4.10a)$$

and

$$\langle x|B^{-1}|y\rangle = \frac{i}{(2\pi)^2}\int\frac{dk^4\, e^{ik(x-y)}}{k^2+m^2} \qquad (4.10b)$$

we obtain for (4.9)

$$\Gamma^{\alpha}(p^2,m^2) = -i\sqrt{N}\left\{-2\tilde{\alpha}(0)\int\frac{dk^4}{k^2+m^2} + \frac{\tilde{\alpha}(0)}{g_o}\right\} . \qquad (4.11)$$

The vanishing of expression (4.11) gives us, after regularization:

$$\frac{1}{2\pi g_o} = \Lambda^2 - m^2\ell n\left(\frac{\Lambda^2}{m^2}+1\right) . \qquad (4.12)$$

By comparing (4.8) and (4.12) we conclude that the bosonic and fermionic generated masses are equal

$$m = M \qquad (4.13)$$

which means that supersymmetry is not broken.

5. CONCLUSIONS

The story does not come here to an end since we did only study the $(\sqrt{N})$ order terms for the $\underline{\sigma}$ and $\underline{\alpha}$ fields.

For π and λ_a we can show that their linear terms vanish. Also the quadratic terms for all fields have to be investigated. As Eguchi[5] showed, this $\mathcal{O}(1)$ order terms do give rise to kinetical forms for the fields, thus making them dynamical entities. The 1/N expansion is well defined since we make perturbative calculus around the true vacuum, by performing the adequate shifting of the σ field, giving rise to a mass term for ψ_i.

We wonder, finally, if this kind of procedure can lead us to the study of the dynamically quantum enhancement of the compound fields that appear in SO(8) supergravity associated to the local SU(8) symmetry, though its manifold is non-compact[11].

ACKNOWLEDGMENTS

I would like to thank E. Abdalla, E. Marino, V. Rivelles and I. Ventura for helpfull conversations and FAPESP for finantial support.

APPENDIX

The gama matrices, in the Majorana representation, are of the following form:

$$\gamma_1 \equiv \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \qquad \gamma_2 \equiv \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_3 \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma_4 \equiv \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \tag{A.1}$$

and satisfy

$$\left\{\gamma_a , \gamma_b\right\} = 2\,\delta_{ab} \tag{A.2}$$

where δ_{ab} is the Kroneker delta.

Fierz transformations are written as:

$$(\bar{\psi}_A M \psi_B)(\bar{\psi}_C N \psi_D) = -\frac{1}{4}\sum_j (\bar{\psi}_A M O_j N \psi_D)(\bar{\psi}_C O_j \psi_B) \tag{A.3}$$

such that

$$O_j \equiv (\mathbb{1}, \gamma_a, 2i\,\sigma_{ab}, i\gamma_5\gamma_a, \gamma_5) \tag{A.4}$$

and

$$\sigma_{ab} \equiv \frac{1}{4}\left[\gamma_a, \gamma_b\right] \quad . \tag{A.5}$$

Also we adopt the summation convention

$$\sum_{i=1}^{N} \phi_i^* \phi_i \equiv \phi_i^* \phi_i \quad , \quad \text{etc..} \tag{A.6}$$

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