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INSTITUTO DE FÍSICA
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DYNAMICAL MASS GENERATION FOR THE GRAVITINO IN SIMPLE N=1 SUPERGRAVITY

by

Radu S. Jasinschi
Instituto de Física, Universidade de São Paulo

and

Alexander W. Smith
Centro Brasileiro de Pesquisas Físicas
Rua Xavier Sigaud, 150, 22290, Rio de Janeiro, RJ, Brasil

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DYNAMICAL MASS GENERATION FOR THE GRAVITINO IN SIMPLE N=1 SUPERGRAVITY

Radu S. Jasinschi‡

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brasil

Alexander W. Smith‡‡

Centro Brasileiro de Pesquisa Físicas
Rua Xavier Sigaud,150,CEP 22290,Rio de Janeiro,R.J.,Brasil

We show that the gravitino mass can be dynamically generated in the simple N=1 supergravity because of the presence of quartic scalar gravitino interaction term.

In supergravity theories coupled to matter fields, like the $(2,\frac{3}{2})+(\frac{1}{2},0^+,0^-)$ model, the gravitino acquires mass through the well known super Higgs effect [1,2,3]. In this sense we are led to think that simple N=1 supergravity describes a massless gravitino as the superpartner of the massless graviton (tetrad). On the other hand the gravitino mass, appearing in models of supergravity coupled to matter fields, sets a bound to the supersymmetry breaking scale [4,5].

We will show that even in the simple N=1 supergravity model the gravitino becomes massive, but now as a result of a dynamical mechanism. By adopting the second order formalism

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in the Einstein-Cartan theory of supergravity, the spin connection depends on the graviton (tetrad) and on the gravitino, in the latter case quadratically. After some little algebra it can be shown that the gravitino sector is given only by quadratic and quartic interaction terms. There is a resemblance between the gravitino sector of simple N=1 supergravity and non linear spinor theories as, for example, the Jona-Lasinio-Nambu model [6]. Jona-Lasinio and Nambu showed that the presence of a quartic fermion interaction term is responsible, at the quantum level, for fermionic mass generation, in an analogy with superconductivity [7] where the energy gap produced by the attractive phonon-mediated interaction between electrons produces correlated pairs of electrons, the Cooper pair. The energy gap in superconductivity plays the same role as the mass gap in the Dirac theory of the electrons.

Because of the quartic fermion interaction term it is necessary to introduce a dimensionfull coupling constant (dimension of $[\text{mass}]^{-2}$). In the case of simple N=1 supergravity this constant is κ ($\kappa = (16\pi G_N)^{1/2}$ where G_N is the Newton constant, $G_N = 6.67 \cdot 10^{-8}\ \text{cm}^3/\text{gs}^2$). It will be shown that κ^2 satisfies a relation analogous to the energy gap equation [6] in superconductivity relating it to the dynamical generated gravitino mass and to the ultraviolet cutt-off Λ.

The simple N=1 supergravity, given by the lagrangian

$$L = -\frac{e}{2\kappa^2} R(e,\psi) - \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\,\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\lambda \psi_\rho + \frac{1}{3} A_\mu^3 - \frac{1}{3}(S^2+P^2) \tag{1}$$

where

$$R(e,\psi) = e_a^\mu\, e_b^\nu\, R_{\mu\nu}^{ab}(e,\psi) \tag{2a}$$

$$R^{ab}_{\mu\nu}(e,\psi) = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + \omega^{ac}_\mu \omega^{b}_{\nu c} - \omega^{ac}_\nu \omega^{b}_{\mu c} \tag{2b}$$

$$\omega^{ab}_\mu = \omega^{ab}_\mu(e) + \frac{\kappa^2}{4}\left[\overline{\psi}_\mu \gamma^a \psi^b - \overline{\psi}_\mu \gamma^b \psi^a + \overline{\psi}^a \gamma_\mu \psi^b\right] \tag{2c}$$

and

$$D_\mu = \partial_\mu + \frac{1}{2}\omega^{ab}_\mu \sigma_{ab} \quad , \tag{2d}$$

can be written as

$$\begin{aligned} L = & -\frac{e}{2\kappa^2} R(e) + \frac{e}{4}\left\{\frac{1}{e}\partial_\mu\left[e^\mu_a e^\nu_b e\right]\left[\overline{\psi}_\nu \gamma^a \psi^b - \overline{\psi}_\nu \gamma^b \psi^a + \overline{\psi}^a \gamma_\nu \psi^b\right]\right\} - \\ & - \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho}\, \overline{\psi}_\mu \gamma_5 \gamma_\nu (\partial_\lambda + \frac{1}{2}\omega^{ab}_\lambda(e)\, \sigma_{ab})\psi_\rho - \\ & - \frac{11}{16}\kappa^2 e\left[(\overline{\psi}_b \psi^b)^2 - (\overline{\psi}_b \gamma_5 \psi^b)^2\right] + \frac{33}{64}\kappa^2 e\, (\overline{\psi}_b \gamma_5 \gamma_c \psi^b)^2 + \\ & + \frac{1}{3} A^2_\mu - \frac{1}{3}(S^2 + P^2) \end{aligned} \tag{3}$$

where we chosed the gravitino gauge[‡1]

$$\gamma^\mu \psi_\mu = 0 \tag{4}$$

[‡1] The Majorana gamma matrix representation is

$$\gamma_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & -i\sigma_3 \\ i\sigma_3 & 0 \end{pmatrix}$$

satisfying

$$\{\gamma_a , \gamma_b\} = 2\,\delta_{ab}$$

Also

$p_\mu = (p_i , p_4)$ so that $p_\mu p^\mu = p_i^2 + p_4^2$.

The quantization of the theory is made through functional path integration, so that the Greens function functional generator is

$$\begin{aligned} Z(\xi^\mu_a, J^\mu) = N^{-1} \int [d\psi_\mu][de^a_\mu]\, \delta(\not{\psi})\,\delta(\text{grav.}) \exp i \int dx^4 e \Big[L + L_{fix} + \\ + L_{ghost} + \frac{1}{2}\xi^\mu_a e^a_\mu + \frac{1}{2} J^\mu \psi_\mu + \text{h.c.}\Big] \end{aligned} \tag{5}$$

Since we are only interested in integrating over the gravitino ψ_μ, we just mention the graviton (tetrad) gauge fixing as $\delta(\text{grav.})$ and after linearizing e^a_μ, we only use its flat-space background tetrad in all expressions of the effective action thus obtained. The gravitino ghost is given by the Nielsen-Kallosh ghost [9] as discussed in ref. [10].

In order to functionally integrate over ψ_μ we have to introduce auxiliary fields so as to eliminate the quartic gravitino terms. So

$$\begin{aligned} Z = N^{-1} \int & [d\psi_\mu][de^a_\mu][d\sigma][d\pi][d\lambda_a]\, \delta(\not{\psi})\,\delta(\text{grav.}) \times \\ \times \exp i \int & dx^4 e \Big\{ -\frac{1}{2}\overline{\psi}_\mu \Big[\frac{i}{e}\varepsilon^{\mu\nu\lambda\rho}\gamma_5 \gamma_\nu \partial_\lambda - 3i\, \omega^{\mu\rho}_\lambda(e)\gamma^\lambda - \\ & - \frac{i}{2e}\left[\partial_\lambda (e^{\lambda a} e^{\mu b} e)[\gamma_a e^\rho_b - \gamma_b e^\rho_a]\right] + \partial_\lambda\left[e^{\lambda a} e^{\nu b}_b\right] e^\mu_a e^\rho_b \gamma_\nu - \\ & - i\sqrt{11}\,\kappa\sigma\,\delta^{\mu\rho} + \sqrt{11}\,\kappa\gamma_5 \pi\,\delta^{\mu\rho} + \frac{\sqrt{33}}{2}\kappa\gamma_5\gamma_d \lambda^d \delta^{\mu\rho}\Big]\psi_\rho + \\ & + i\sigma^2 + i\pi^2 + i\lambda^2_d + L_{fix} + L_{ghost} + \frac{1}{2}\xi^\mu_a e^a_\mu + \frac{1}{2}J^\mu \psi_\mu + \text{h.c.}\Big] \end{aligned} \tag{6}$$

Using for $\delta(\not{\psi})$

$$\delta(\not{\psi}) = \int [d\zeta] \exp \frac{i}{2} \int dx^4 e\left[(\overline{\psi}_\mu \gamma^\mu \zeta) - (\overline{\zeta}\gamma^\mu \psi_\mu)\right] \tag{7}$$

and performing the functional integration over ψ_μ , gives us:

$$Z = N^{-1}\int [de^a_\mu][d\sigma][d\pi][d_{\lambda d}][d\zeta]\,\delta(\text{grav.})\exp i\Big\{S_{eff} + \int dx^4 e\Big[L_{fix} +$$

$$+ L_{ghost} - \frac{1}{2}\Big[\big[-(\bar\zeta\,\gamma^\lambda\,\Delta^{-1}_{\lambda\mu}) + (\bar J^\lambda\,\Delta^{-1}_{\lambda\mu})\big]\,\Delta^{\mu\nu}\big[(\Delta^{-1}_{\nu\rho}\,\gamma^\rho\zeta) + (\Delta^{-1}_{\nu\rho}\,J^\rho)\big]\Big]\Big] \qquad (8)$$

where

$$S_{eff} = -\frac{i}{2}\,\mathrm{Tr}\,\ell n\,\Delta_{\mu\nu} + \int dx^4\, e\,\big[\sigma^2 + \pi^2 + \lambda d^2\big] \qquad (9)$$

and

$$\Delta_{\mu\nu} = \frac{i}{e}\,\varepsilon_{\mu\rho\lambda\nu}\,\gamma_5\,\gamma^\rho\,\partial^\lambda - 3\,i\,\omega^\lambda_{\mu\nu}\,\lambda_\lambda -$$

$$-\frac{i}{2}\,\frac{1}{e}\,\Big[\partial_\lambda\big[e^{\lambda a}\,e_\mu{}^b\,e\big]\big[\gamma_a\,e_{\nu b} - \gamma_b\,e_{\nu a}\big] + \partial_\lambda\big[e^\lambda_a\,e^\rho_b\big]\;e^a_\mu\,e^b_\nu\,\gamma_\rho -$$

$$- i\sqrt{11}\,\kappa\,\sigma\,\delta_{\mu\nu} + \sqrt{11}\,\kappa\,\gamma_5\,\pi\,\delta_{\mu\nu} + \frac{\sqrt{33}}{2}\,\kappa\,\gamma_5\,\gamma_d\,\lambda^d\,\delta_{\mu\nu} \qquad (10)$$

If we substitute e^a_μ by its flat-space background tetrad δ^a_μ in the effective action (9) we obtain:

$$S_{eff}\Big|_{e^a_\mu \sim \delta^a_\mu} = -\frac{i}{2}\,\mathrm{Tr}\,\ell n\,\Big\{i\,\varepsilon_{acdb}\,\gamma_5\,\gamma^c\,\partial^d - i\sqrt{11}\,\kappa\,\sigma\,\delta_{ab} +$$

$$+ \sqrt{11}\,\kappa\,\gamma_5\,\pi\,\delta_{ab} + \frac{\sqrt{33}}{2}\,\kappa\,\gamma_5\,\gamma_d\,\gamma^d\,\delta_{ab}\Big\} + \int dx^4\big[\sigma^2 + \pi^2 + \lambda d^2\big] \qquad (11)$$

In order to avoid infrared divergent expressions, due to the masslessness of the fermion field, the σ field has to develop a non-vanishing v.e.v. [8], that is, we proceed to the following shift on σ :

$$\sigma(x) = \sigma'(x) + \sigma_0 \qquad (12)$$

where σ_0 is constant and $\langle\sigma'\rangle_0 = 0$. Substituting (12) in (11) gives us a mass term for the gravitino (M). Using our gauge (4) (so that δ_{ab} turns into $-2\,\sigma_{ab}$) [‡2], we conclude that the gravitino mass is given by

$$M = 2\sqrt{11}\,\kappa\,\sigma_0 \qquad (13)$$

Next we calculate the linear terms of $\sigma(\sigma')$, π and λd . Using for the gravitino propagator, in momentum space, the expression [11] [‡3]

$$\tilde P_{ab}(p) = -\Big[\Big[\delta_{ab} + \frac{p_a p_b}{M^2}\Big](i\not p - M) + \frac{1}{3}\Big[(\gamma_a - i\,\frac{p_a}{M})\,(i\not p + M)\,(\gamma_b - i\,\frac{p_b}{M})\Big]\Big]\;\frac{1}{p^2 + M^2} \qquad (14)$$

we can show that the linear terms of π

$$\Gamma^\pi = -\frac{i}{2}\sqrt{11}\,\kappa\,\mathrm{Tr}\big[P_{ab}\,\gamma_5\,\pi\,\delta_{ab}\big] \qquad (15)$$

and of λd

$$\Gamma^{\lambda d} = -\frac{i}{4}\sqrt{33}\,\kappa\,\mathrm{Tr}\big[P_{ab}\,\gamma_5\,\gamma_d\,\lambda^d\,\delta_{ab}\big] \qquad (16)$$

vanish.

‡2 $\sigma_{ab} = \frac{1}{4}\,[\gamma_a\,,\gamma_b]$.

‡3 We represent the Fourier transform of $\sigma'(x)$ and $P_{ab}(x)$ by $\tilde\sigma'(p) = \int dx^4\, e^{-ipx}\,\sigma'(x)$ and $\tilde P_{ab}(p) = \frac{1}{(2\pi)^4}\int dx^4\, e^{-ipx}\,P_{ab}(x)$.

On the other hand, for the linear term of σ' we have

$$\Gamma^{\sigma'} = -\frac{\sqrt{11}}{2}\kappa\,\mathrm{Tr}\left[P_{ab}\,\sigma'\right] + \frac{M}{\sqrt{11}\,\kappa}\int dx^4\,\sigma'(x) \tag{17}$$

which becomes, in momentum space

$$\Gamma^{\tilde{\sigma}'} = \left\{\left[-\frac{4\sqrt{11}\,\pi\kappa}{3}\int dk^4\left[4+\frac{k^2}{M^2}\right]\frac{1}{k^2+M^2} + \frac{1}{\sqrt{11}\,\kappa}\right]M\tilde{\sigma}(0)\right\} \tag{18}$$

Imposing the elimination of $\Gamma^{\tilde{\sigma}'}$, which corresponds to the vanishing of the σ' tadpole, leads us to the following relation

$$\frac{3}{44\kappa^2} = \int dk^4\left[4+\frac{k^2}{M^2}\right]\frac{1}{k^2+M^2} \tag{19}$$

This equation resambles the energy-gap equation for the coupling constant g_0 in the Jona-Lasinio-Numbu model, related to superconductivity. It is very divergent in the ultraviolet region, so that we regularize it by the introduction of the cut-off Λ, thus breaking supersymmetry. So we obtain

$$(19)_{reg} \Rightarrow \frac{3}{44\kappa^2} = \pi^2\left\{\frac{\Lambda^4}{2M^2} + 3\Lambda^2 - 3M^2\,\ell og\left(\frac{\Lambda^2}{M^2}+1\right)\right\} \tag{20}$$

If we identify Λ as the supersymmetry breaking scale M_{SSB}, we have for its leading term the relation

$$M^4_{SSB} = \frac{3}{22\pi^2}\,\frac{M^2}{\kappa^2} \tag{21}$$

Wess and Samuel [4] obtained a similar relation, in the context

of non-linear supersymmetry, with the presence of a cosmological term, given by

$$M^4_{SSB} = \frac{3}{8}\,\frac{M^2}{\kappa^2} \tag{22}$$

In spite of the different constants appearing in (21) and (22) we can afirm that they are very similar expressions. In this sense, we conjecture that expression (20) (substituting M_{SSB} in the place of Λ) is a generalization of expression (22).

The quadratic functions of the fields σ', π and λd, can be shown, to develop kinetic terms [8,12], which is essentially a quantum mechanical property.

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