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STUDY OF A FOUR-NUCLEON POTENTIAL DUE TO EXCHANGE OF PIONS

by

M.R. Robilotta
Instituto de Física, Universidade de São Paulo

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M.R. Robilotta

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brasil

ABSTRACT

A four-body force due to the exchange of pions has been derived by means of an effective Lagrangian which is approximately invariant under chiral and gauge transformations. It includes effects corresponding to pion-pion scattering, pion production and pion-nucleon rescattering. The strength parameters of this four-body potential are typically one order of magnitude smaller than those of the two-pion exchange three-body force.

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I. INTRODUCTION

It is well known nowadays that the nucleon-nucleon interaction does not suffice for a precise description of many important effects in nuclear physics. For instance, the study of the nuclei of ^3H and ^3He by means of different techniques has shown that realistic two-body forces underbind these tri-nucleon systems by about 1.5 MeV [1]. This situation has led researches on the field to look elsewhere for the explanation of this and other discrepancies. In this context, three-body forces have deserved much attention, particularly that due to pion exchange, whose effects have been shown to be important[2].

The study of the four-body system is in a much less advanced stage. Nevertheless, the present possibility of tackling the problem by means of various techniques and different nucleon-nucleon potentials allows one to foresee that the inclusion of many body forces will be performed soon[3]. This makes opportune a discussion of the role of four-body forces.

The many-body forces of longer range are those due to the exchange of pions. In the case of the alfa particle, these forces are the result of proper interactions among either three or four nucleons. By proper interactions one means processes in which there are no nucleons propagating forward in time. The two-pion exchange three-body force corresponds to diagrams in which a virtual pion, emitted by one of the nucleons, is scattered by another and absorbed by a third one. In the most accurate theoretical treatments of this force the intermediate pion-nucleon scattering amplitude is described by means of chiral symmetry[4,5], since the interactions of pions with other hadrons are approximately invariant under transformations of the group SU(2) × SU(2). The symmetry is a crucial ingredient in the calculation of the force because it produces a pion-nucleon amplitude which is consistent with on shell data and is suitable for off-shell extrapolation.

The dynamical content of the pion-exchange four-body force, on the other hand, is related to three different types of intermediate processes, namely pion-pion scattering, pion production and pion-nucleon rescattering, as depicted in fig. 1. The first of them corresponds to the interaction of the virtual pions exchanged between different pairs of nucleons. The amplitude for pion production contributes in the case where a virtual pion, emitted by a nucleon, interacts with another nucleon producing two pions, which are absorbed by the remaining nucleons. Finally, in the third type of process, a nucleon emits one pion, which is scattered in succession by other two nucleons and absorbed by a fourth one.

In this work the four-body force due to exchange of pions is obtained by means of an effective Lagrangian that is approximately invaraint under chiral and gauge transformations. In the next section this Lagrangian is employed in the calculation of the amplitudes for the intermediate processes. One derives the four-body potential in section III, by evaluating the contributions of proper interactions to the scattering of four non-relativistic nucleons. Finally, conclusions are presented in section IV.

II. INTERMEDIATE AMPLITUDES

The relationship between the four-body potential and the scattering amplitude of free nucleons is totally

analogous to that of the three-body case. As discussed in the introduction, the potential is based on proper diagrams describing the propagation of pions in tree approximation and containing the amplitudes for pion-pion scattering, pion production and pion-nucleon rescattering. Thus, the calculation begins with the evaluation of these subamplitudes.

In this evaluation one chooses to implement chiral and gauge symmetries by means of an effective Lagrangian, for this approach produces a clear dynamical picture of all the processes. This Lagrangian describes interactions among nucleons, deltas, pions, rhos and the axial-vector mesons A_1. The relevant terms to the present discussion are the following:

$$L_{\pi\pi\pi\pi} = \frac{1}{8f_\pi^2}\left[2(1-\xi)\phi^2\partial_\mu\vec{\phi}.\partial^\mu\vec{\phi} + (\tfrac{1}{2}-\xi)\partial_\mu\phi^2\partial^\mu\phi^2 - \mu^2(1-\tfrac{3}{2}\xi)\phi^4\right] \quad (1)$$

$$L_{\pi\pi\rho} = \gamma_0\,\vec{\rho}_\mu\,.\,(\vec{\phi}\times\partial^\mu\vec{\phi}) \quad (2)$$

$$L_{\pi\pi\pi A_1} = \frac{\gamma_0}{f_\pi}\left[\phi^2\partial^\mu\vec{\phi}.\vec{A}_\mu - \frac{1}{2}\partial^\mu\phi^2\,\vec{\phi}.\vec{A}_\mu\right] \quad (3)$$

$$L_{\pi\rho A_1} = \frac{1}{2f_\pi}\left[\vec{\phi}.\partial^\mu\vec{\rho}^\nu\times(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu) - \partial_\nu\vec{\phi}.(\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu)\times\vec{A}_\mu\right] \quad (4)$$

$$L_{\pi NN} = \frac{g}{2m}\,\overline{N}\,\vec{\tau}\,\gamma^\mu\gamma_5\,N\,.\,\partial_\mu\vec{\phi} \quad (5)$$

$$L_{\pi\pi\pi NN} = \frac{g}{2m}\,\overline{N}\,\vec{\tau}\,\gamma^\mu\gamma_5 N\,.\,\frac{1}{4f_\pi^2}\left[(1-\xi)\partial_\mu\phi^2\,\vec{\phi} - \xi\phi^2\partial_\mu\vec{\phi}\right] \quad (6)$$

$$L_{\rho NN} = \frac{\gamma_0}{2}\,\overline{N}\,\vec{\tau}\,.\left[\gamma^\mu\vec{\rho}_\mu + \frac{\mu_p-\mu_n}{2m}\,\sigma^{\mu\nu}\,(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\right]N \quad (7)$$

$$L_{\pi\rho NN} = \frac{g}{2m}\,\gamma_0\,\overline{N}\,\vec{\tau}\,\gamma^\mu\gamma_5 N\,.\,\vec{\rho}_\mu\times\vec{\phi} \quad (8)$$

$$L_{ANN} = \frac{g}{m} f_n \gamma_0 \; \bar{N} \vec{i} \gamma^{\mu}\gamma_5 N \,.\, \vec{A}_{\mu} \tag{9}$$

$$L_{\pi N\Delta} = g_{\Delta} \; \bar{\Delta}_{\mu} \; \vec{M} \; (g^{\mu\nu} - \frac{\eta}{4} \gamma^{\mu}\gamma^{\nu}) N \,.\, \partial_{\nu}\vec{\phi} + \text{h.c.} \tag{10}$$

$$L_{\rho N\Delta} = i\gamma_{\Delta} \bar{\Delta}_{\mu} \vec{M} (g^{\mu\nu} - \frac{\lambda}{4} \gamma^{\mu}\gamma^{\nu}) \; \gamma^{\theta}\gamma_5 \, N \,.\, (\partial_{\theta}\vec{\rho}_{\nu} - \partial_{\nu}\vec{\rho}_{\theta}) + \text{h.c.} \tag{11}$$

$$\begin{aligned} L_{\pi\Delta\Delta} = C_{\pi\Delta\Delta} \; \bar{\Delta}^{\sigma} \; \vec{T}\Big[& \gamma_{\lambda} g_{\sigma\theta} - (1 - \frac{1}{2}\delta) g_{\sigma\lambda}\gamma_{\theta} - (1 - \frac{\delta}{2}) \gamma_{\sigma} g_{\lambda\theta} \\ & + (1 - \delta + \frac{3}{8}\delta^2) \; \gamma_{\sigma}\gamma_{\lambda}\gamma_{\theta}\Big] \; \gamma_5 \Delta_{\theta} \,.\, \partial^{\lambda}\vec{\phi} \end{aligned} \tag{12}$$

In these expressions the symbols N, Δ_{μ}, $\vec{\phi}$, $\vec{\rho}_{\mu}$ and $\vec{A}_{\mu}$ denote, respectively, the nucleon, delta, pion, rho, and A_1 fields, whose masses are m, M_{Δ}, μ, m_{ρ}, m_A. The parameter ξ has been introduced by Olsson and Turner[6] and is related to the form of the symmetry breaking term in the Lagrangian. The universal vector coupling constant is γ_0, whereas μ_p and μ_n are the proton and neutron anomalous magnetic moments. The parameters η, λ and δ represent the possibility of spin $\frac{1}{2}$ components in the off-pole delta wave-function. The couplings of the delta correspond to the following form for its propagator

$$G_{\mu\nu}(p) = \frac{(\not{p}+M_{\Delta})}{p^2 - M_{\Delta}^2} \left\{ g_{\mu\nu} - \frac{1}{3} \gamma_{\mu}\gamma_{\nu} - \frac{\gamma_{\mu} p_{\nu}}{3M_{\Delta}} + \frac{p_{\mu}\gamma_{\nu}}{3M_{\Delta}} - \frac{2p_{\mu}p_{\nu}}{3M_{\Delta}^2} \right\} \;. \tag{13}$$

In the derivation of the four-body potential the momenta of the nucleons are consitently assumed to be of the order of the pion mass. The momentum p of a nucleon is written as

$$p \equiv (E,\vec{p}) \simeq (m + \frac{\vec{p}^2}{2m} , \vec{p}) \quad , \tag{14}$$

whereas the momentum k of a pion emitted by this nucleon is given by

$$k \equiv (\omega,\vec{k}) \simeq \left(\frac{\vec{p}^2 - \vec{p}'^2}{2m} \; , \; \vec{p} - \vec{p}' \right) \quad . \tag{15}$$

Therefore, the orders of magnitude of these kinematical variables are the following: $E - m$, $|\vec{p}| \sim |\vec{k}| \sim \mu$, $\omega \sim \mu^2/m$.

A. Intermediate pion-pion scattering

The process $\pi^a(k)\pi^b(q) \to \pi^c(k')\pi^d(q')$ is described by the diagrams of fig. 2, representing a contact term and three exchanges of rho-meson. The corresponding amplitude, denoted by $T^{(A)}$, is obtained from the Lagrangian elements displayed above and has the following form

$$\begin{aligned} T^{(A)} = \frac{1}{f_{\pi}^2} \Big\{ & \delta_{ac}\delta_{bd}\Big[-(1-\xi)(k-k').(q-q') - \xi(k.k'+q.q') - \mu^2(1-\frac{3}{2}\xi)\Big] \\ & + \delta_{ad}\delta_{bc}\Big[-(1-\xi)(k-q').(q-k') - \xi(k.q'+q.k') - \mu^2(1-\frac{3}{2}\xi)\Big] \\ & + \delta_{ab}\delta_{cd}\Big[(1-\xi)(k+q).(k'+q') + \xi(k.q+k'.q') - \mu^2(1-\frac{3}{2}\xi)\Big]\Big\} \;. \end{aligned} \tag{16}$$

In this derivation one has used the relation $\gamma_0^2/m_{\rho}^2 = 1/2f_{\pi}^2$. The corresponding expression for non-relativistic nucleons is

$$\begin{aligned} t^{(A)} = \frac{1}{f_{\pi}^2} \Big\{ & \delta_{ac}\delta_{bd}\Big[(1-\xi)(\vec{k}-\vec{k}').(\vec{q}-\vec{q}') + \xi(\vec{k}.\vec{k}'+\vec{q}.\vec{q}') - \mu^2(1-\frac{3}{2}\xi)\Big] \\ & + \delta_{ad}\delta_{bc}\Big[(1-\xi)(\vec{k}-\vec{q}').(\vec{q}-\vec{k}') + \xi(\vec{k}.\vec{q}'+\vec{q}.\vec{k}') - \mu^2(1-\frac{3}{2}\xi)\Big] \\ & + \delta_{ab}\delta_{cd}\Big[-(1-\xi)(\vec{k}+\vec{q}).(\vec{k}'+\vec{q}') - \xi(\vec{k}.\vec{q}+\vec{k}'.\vec{q}') - \mu^2(1-\frac{3}{2}\xi)\Big]\Big\} \end{aligned} \tag{17}$$

B. Intermediate pion production

The dynamical content of the reaction $\pi^a(k)N(p) \to \pi^c(k')\pi^d(q')N(p')$ for free particles at low energies is shown in fig. 3 . The first diagram represents the pion-pole amplitude and cannot be included in the four-body potential, since this would mean the double counting of the pion-pion process. One also must not include the diagrams within round brackets, because they contain nuclear propagators and hence correspond to iterations of two and three body potentials.

The square bracket contains a seagull term besides others describing the propagation of vector mesons. When the effective Langragian adopted in this work is used, the contribution of the diagrams including the $\pi\rho NN$ vertex is cancelled to leading order in μ^2/m_ρ^2 by that describing the propagation of the A_1 . Moreover, processes containing both ρ and A_1 propagators produce only corrections to the leading term. Thus, the most important contribution from the diagrams within square brackets comes from the seagull term. The corresponding amplitude, represented by $T^{(B,S)}$, is

$$T^{(B,S)} = i\,\frac{g}{2m}\,\frac{1}{2f_\pi^2}\,\bar{u}\,\gamma^\mu\gamma_5\,u\,\Big\{\delta_{ac}\tau_d\Big[-\xi q'_\mu + (\xi-1)(k_\mu-k'_\mu)\Big] + \delta_{ad}\tau_c\Big[-\xi k'_\mu + (\xi-1)(k_\mu-q'_\mu)\Big] + \delta_{cd}\tau_a\Big[\xi k_\mu - (\xi-1)(k'_\mu+q'_\mu)\Big]\Big\} \tag{18}$$

The non-relativistic limit of this expression is

$$t^{(B,S)} = -\,i\,2m\,\frac{g}{2m}\,\frac{1}{2f_\pi^2}\Big\{\delta_{ac}\tau_d\Big[-\xi\vec{\sigma}\,.\,\vec{q}\,' + (\xi-1)\vec{\sigma}\,.\,(\vec{k}-\vec{k}\,')\Big] + \delta_{ad}\tau_c\Big[-\xi\vec{\sigma}\,.\,\vec{k}\,' + (\xi-1)\vec{\sigma}\,.\,(\vec{k}-\vec{q}\,')\Big] + \delta_{cd}\tau_a\Big[\xi\vec{\sigma}\,.\,\vec{k} - (\xi-1)\vec{\sigma}\,.\,(\vec{k}\,'+\vec{q}\,')\Big]\Big\} \tag{19}$$

The diagrams within curly brackets represent two types of processes, namely those containing one and two delta propagators. They are referred to as single and double delta diagrams and correspond to the amplitudes $T^{(B,\Delta)}$ and $T^{(B,\Delta\Delta)}$. The single delta processes depicted in fig. 4a yield the following amplitude

$$(T^{(B,\Delta)}) = \gamma_0\Big[i\,\epsilon_{acd}(T^+_\mu)_\Delta + (\delta_{ac}\tau_d - \delta_{ad}\tau_c)(T^-_\mu)_\Delta\Big]\,\frac{g^{\mu\nu} - Q^\mu Q^\nu/m_\rho^2}{Q^2 - m_\rho^2}\,(q'_\nu - k'_\mu) \tag{20}$$

where $(T^\pm_\mu)_\Delta$ are the same subamplitudes that contribute to the pion-rho exchange three-body force (8). Their most important parts are those proportional to the poles of the delta and are given by

$$\begin{aligned}(T^+_\mu)_\Delta = &-i\,\frac{\gamma_\Delta\,g_\Delta}{9M_\Delta^2}\,\bar{u}\,\gamma_5\,\Bigg\{\Big(\frac{1}{s-M_\Delta^2} + \frac{1}{u-M_\Delta^2}\Big)\Big[m\alpha + (Q^2-2m^2)\beta + \\ &+ 3M_\Delta^2(k^2 - 2Q.k - 2m^2 - 2mM_\Delta)\Big]\,\frac{\gamma^\lambda\gamma^\nu}{2} \\ &+ \Big(\frac{1}{s-M_\Delta^2} - \frac{1}{u-M_\Delta^2}\Big)\,\alpha\emptyset\,\frac{\gamma^\lambda\gamma^\nu}{2} + \beta(m\gamma^\lambda - Q^\lambda)\Big[\frac{(P+k)^\nu}{s-M_\Delta^2} + \frac{(P-k)^\nu}{s-M_\Delta^2}\Big] \\ &+ \Big(\frac{1}{s-M_\Delta^2} + \frac{1}{u-M_\Delta^2}\Big)6M_\Delta^2 k^\lambda P^\nu + 3M_\Delta^2(m+M_\Delta)\gamma^\lambda\Big[\frac{(P-k)^\nu}{s-M_\Delta^2} + \frac{(P+k)^\nu}{u-M_\Delta^2}\Big]\Bigg\} \\ &\times (g_{\mu\lambda}\,Q_\nu - g_{\mu\nu}\,Q_\lambda)\,u\end{aligned} \tag{21}$$

$$\begin{aligned}(T^-_\mu)_\Delta = &-i\,\frac{\gamma_\Delta\,g_\Delta}{18M_\Delta^2}\,\bar{u}\,\gamma_5\,\Bigg\{\Big(\frac{1}{s-M_\Delta^2} - \frac{1}{u-M_\Delta^2}\Big)\Big[m\alpha + (Q^2-2m^2)\beta + \\ &+ 3M_\Delta^2(k^2 - 2Q.k - 2m^2 - 2mM_\Delta)\Big]\,\frac{\gamma^\lambda\gamma^\nu}{2} +\end{aligned}$$

$$+ \left(\frac{1}{s-M_\Delta^2} + \frac{1}{u-M_\Delta^2}\right) \alpha \not{Q} \; \frac{\gamma^\lambda \gamma^\nu}{2} + \beta (m\gamma^\lambda - Q^\lambda) \left[\frac{(P+k)^\nu}{s-M_\Delta^2} - \frac{(P-k)^\nu}{u-M_\Delta^2}\right]$$

$$+ \left(\frac{1}{s-M_\Delta^2} - \frac{1}{u-M_\Delta^2}\right) 6M_\Delta^2 k^\lambda P^\nu + 3M_\Delta^2 (m+M_\Delta) \gamma^\lambda \left[\frac{(P-k)^\nu}{s-M_\Delta^2} - \frac{(P+k)^\nu}{u-M_\Delta^2}\right]$$

$$\times (g_{\mu\lambda} Q_\nu - g_{\mu\nu} Q_\lambda) u \tag{22}$$

where

$$\alpha \equiv (m + M_\Delta)(M_\Delta^2 - m^2) + k^2 (m + 2M_\Delta) \tag{23}$$

$$\beta \equiv (2M_\Delta^2 + mM_\Delta - m^2 + k^2) \tag{24}$$

$$P = (p + p') \quad . \tag{25}$$

When the nucleons are assumed to be non-relativistic and the diagrams corresponding to the permutations of the pions are added, one obtains the following form for the single delta amplitude:

$$t^{(B,\Delta)} = i\, 2m \frac{4\, \gamma_0\, \gamma_\Delta\, g_\Delta}{9\, m_\rho^2\, (M_\Delta - m)} \Big[12\, \epsilon_{acd}\, \vec{k}.\vec{k}' \times \vec{q}'$$

$$+ \delta_{ad}\tau_c\, (\vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} - 2\vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' + \vec{k}.\vec{k}'\vec{\sigma}.\vec{q}')$$

$$+ \delta_{ac}\tau_d\, (\vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} + \vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' - 2\vec{k}.\vec{k}'\vec{\sigma}.q')$$

$$+ \delta_{cd}\tau_a\, (-2\vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} + \vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' + \vec{k}.\vec{k}'\vec{\sigma}.\vec{q}') \Big] \quad . \tag{26}$$

In the derivation of this expression one has neglected the difference between the nucleon and delta masses.

The double delta amplitude, shown in fig. 4b ,

receives its dominant contribution from the double pole term, corresponding to the following partial amplitude

$$(T^{(B,\Delta\Delta)}) = g_\Delta^2\, C_{\pi\Delta\Delta} \left(\frac{5}{6} i\, \epsilon_{acd} + \frac{2}{3}\, \delta_{ac}\tau_d - \frac{1}{6}\, \delta_{cd}\tau_a - \frac{1}{6}\, \delta_{ad}\tau_c\right)$$

$$\times \bar{u}\; \gamma_5 \frac{\not{Q}' - M_\Delta}{Q'^2 - M_\Delta^2} \left[2M_\Delta\, k.k' - \frac{4}{3M_\Delta} (Q.kQ.k' + Q'.kQ'.k') + \frac{8}{9M_\Delta^3} (M_\Delta^2 + Q.Q')Q.kQ'.k'\right]$$

$$+ \left[-\frac{2}{3} Q.k' - \frac{4}{9M_\Delta^2} (2M_\Delta^2 - Q.Q')Q'.k'\right]\not{k} + \left[\frac{2}{3} Q'.k + \frac{4}{9M_\Delta^2} (2M_\Delta^2 - Q.Q')Q.k\right]\not{k}'$$

$$- \frac{2}{9M_\Delta} (4M_\Delta^2 + Q.Q')\not{k}'\not{k}\Big\} \; \frac{\not{Q} + M_\Delta}{Q^2 - M_\Delta^2}\, u \quad . \tag{27}$$

Taking the non-relativistic limit of this expression and including the contributions of all the other permutations of the pion quantum numbers, one gets

$$t^{(B,\Delta\Delta)} = i\, 2m \frac{g_\Delta^2\, C_{\pi\Delta\Delta}}{3\,(M_\Delta - m)^2} \Big[\frac{25}{3}\, \epsilon_{acd}\, \vec{k}.\vec{k}' \times \vec{q}'$$

$$+ \delta_{ad}\tau_c\, \left(\frac{7}{9}\, \vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} - 2\vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' + \frac{7}{9}\, \vec{k}.\vec{k}'\vec{\sigma}.\vec{q}'\right)$$

$$+ \delta_{ac}\tau_d\, \left(\frac{7}{9}\, \vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} + \frac{7}{9}\, \vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' - 2\vec{k}.\vec{k}'\vec{\sigma}.\vec{q}'\right)$$

$$+ \delta_{cd}\tau_a\, \left(-2\vec{k}'.\vec{q}'\vec{\sigma}.\vec{k} + \frac{7}{9}\, \vec{k}.\vec{q}'\vec{\sigma}.\vec{k}' + \frac{7}{9}\, \vec{k}.\vec{k}'\vec{\sigma}.\vec{q}'\right)\Big] \tag{28}$$

C. <u>Intermediate pion-nucleon scattering</u>

The amplitude for the process $\pi^a(k)N(p) \to \pi^c(k')N(p')$, for nucleons on shell, can be parametrized as

$$T^{(c)} = \bar{u}\left[(A^+ + \frac{\not{k}' + \not{k}}{2} B^+)\delta_{ac} + (A^- + \frac{\not{k}' + \not{k}}{2} B^-)\, i\, \epsilon_{cae}\tau_e\right]u \tag{29}$$

When the nucleons are non-relativistic, this expression can be written as

$$t^{(c)} = 2m\left[(f^+ + i\,\frac{b^+}{2m}\,\vec{\sigma}.\vec{k}'\times\vec{k})\delta_{ac} + (f^- + i\,\frac{b^-}{2m}\,\vec{\sigma}.\vec{k}'\times\vec{k})\,i\,\epsilon_{cae}\tau_e\right] \qquad (30)$$

where

$$f^{\pm} = a^{\pm} + (p+p').(k+k')\,\frac{b^{\pm}}{4m} \qquad (31)$$

and $a^{\pm}$ and $b^{\pm}$ are the non-relativistic limits of $A^{\pm}$ and $B^{\pm}$. The dynamical content of the pion-nucleon scattering amplitude is shown in fig. 5 . The diagram describing the propagation of a nucleon represents an iteration of the two-body potential and must not be considered. The relativistic expression for the contribution of the delta-pole and rho and sigma exchanges to $A^{\pm}$ and $B^{\pm}$ can be found in ref. (5) and will not be reproduced here. Their non-relativistic limits produce the following non-vanishing terms

$$f_{\Delta}^{+} = \frac{8g_{\Delta}^{2}}{9(M_{\Delta}-m)}\,\vec{k}\,.\,\vec{k}' \qquad (32)$$

$$b_{\Delta}^{-} = \frac{4g_{\Delta}^{2}m}{9(M_{\Delta}-m)} \qquad (33)$$

$$f_{\rho}^{-} = \frac{1}{2f_{\pi}^{2}}\,\frac{1}{4m}\,(p+p')\,.\,(k+k') \qquad (34)$$

$$b_{\rho}^{-} = \frac{1}{2f_{\pi}^{2}}\,(1+\mu_p-\mu_n) \qquad (35)$$

$$f_{\sigma}^{+} = \alpha_{\sigma} - \beta_{\sigma}\,\vec{k}.\vec{k}' \quad . \qquad (36)$$

The rho contribution to f_{ρ}^{-} is velocity dependent and produces non-local terms in the potential. Hence it will be ignored in this work. The form of the sigma contribution is the consequence of the parametrization used in ref. (5), that has been criticized on the grounds that it fails to reproduce the Adler zeros[(2)]. This criticism although reasonable from a purely conceptual point of view, does not affect the practical consequences of the calculation[(9)]. This can be seen by considering the parametrization adopted in ref. (4) , which is consistent with the Adler zeros. It corresponds to an amplitude given by

$$\begin{aligned}\tilde{f}_{\sigma}^{+} &= \frac{\sigma}{f_{\pi}^{2}}\left[(1-\beta)\left(\frac{k^2+k'^2}{\mu^2}-1\right)+\beta\left(\frac{t}{\mu^2}-1\right)\right]\\ &= \frac{\sigma}{f_{\pi}^{2}}\left[1-2\beta\,\frac{k.k'}{\mu^2}+\frac{1}{\mu^2}\,(k^2-\mu^2)+\frac{1}{\mu^2}\,(k'^2-\mu^2)\right] \qquad (37)\end{aligned}$$

where σ is the pion-nucleon sigma-term. When one makes the identifications $\alpha_{\sigma} \equiv \sigma/f_{\pi}^{2}$ and $\beta_{\sigma} \equiv -2\beta\,\sigma/f_{\pi}^{2}\mu^2$, one notes that eqs. (36) and (37) differ only by terms proportional to $(k^2-\mu^2)$ and $(k'^2-\mu^2)$. These factors cancell pion propagators, leading to amplitudes that correspond to contact interactions in coordinate space, since they are proportional to δ-functions. The short distance repulsion between nucleons allows one to neglect such contact interactions in practical calculations.

III. THE FOUR-BODY POTENTIAL

The four-body potential in momentum space is defined as

$$\langle \vec{p}_1'\vec{p}_2'\vec{p}_3'\vec{p}_4' | W^{1234} | \vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_4 \rangle \equiv -(2\pi)^3\, \delta^3\, (\vec{p}_f-\vec{p}_i)\, \frac{1}{(2m)^4}\, t_{4N} \tag{38}$$

where t_{4N} is the amplitude for the elastic scattering of four non-relativistic nucleons, excluding the contribution of intermediate nucleons propagating forward in time.

In the evaluation of the potential one uses the following kinematical variables

$$k = p_1-p_1' \ , \ k' = p_2'-p_2 \ , \ q' = p_3'-p_3 \ , \ q = p_4-p_4' \ , \ Q = k-k' = q'-q \ . \tag{39}$$

Energy-momentum conservation means that

$$k + q \ = \ k' + q' \quad . \tag{40}$$

The amplitude for the process $4N \to 4N$ due to the intermediate scattering of pions, as represented in fig. 1(A), is given by

$$\begin{aligned} t_{4N}^{(A)} = {} & (2m)^4 \left(\frac{g}{2m}\right)^4 \frac{1}{2f_\pi^2}\, \frac{1}{\vec{k}^2+\mu^2}\, \frac{1}{\vec{k}'^2+\mu^2}\, \frac{1}{\vec{q}'^2+\mu^2}\, \frac{1}{q^2+\mu^2}\, \vec{\sigma}^{(1)}.\vec{k}\,\vec{\sigma}^{(2)}.\vec{k}'\vec{\sigma}^{(3)}.\vec{q}'\,\vec{\sigma}^{(4)}.\vec{q} \\ & \times \Big\{\vec{\tau}^{(1)}.\vec{\tau}^{(2)}\vec{\tau}^{(3)}.\vec{\tau}^{(4)} \Big[-(1-\xi)\,(\vec{k}^2+\vec{k}'^2+\vec{q}'^2+\vec{q}^2+4\mu^2) + 2(\vec{k}.\vec{k}'+\vec{q}.\vec{q}') + \mu^2(2-\xi)\Big] \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(3)}\vec{\tau}^{(2)}.\vec{\tau}^{(4)} \Big[-(1-\xi)\,(\vec{k}^2+\vec{k}'^2+\vec{q}'^2+\vec{q}^2+4\mu^2) + 2(\vec{k}.\vec{q}'+\vec{q}.\vec{k}') + \mu^2(2-\xi)\Big] \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(4)}\vec{\tau}^{(2)}.\vec{\tau}^{(3)} \Big[-(1-\xi)\,(\vec{k}^2+\vec{k}'^2+\vec{q}'^2+\vec{q}^2+4\mu^2) - 2(\vec{k}.\vec{q}+\vec{k}'.\vec{q}') + \mu^2(2-\xi)\Big] \ . \end{aligned} \tag{41}$$

In this expression $\sigma^{(i)}$ and $\tau^{(i)}$ indicate expectation values.

The contribution of the pion production process to t_{4N}, indicated in fig. 1(B), is composed of three terms, namely

the seagull, single delta and double delta. The first of them results in the following value for the amplitude

$$\begin{aligned} t_{4N}^{(B,S)} = {} & (2m)^4 \left(\frac{g}{2m}\right)^4 \frac{1}{2f_\pi^2}\, \frac{1}{\vec{k}^2+\mu^2}\, \frac{1}{\vec{k}'^2+\mu^2}\, \frac{1}{\vec{q}'^2+\mu^2}\, \vec{\sigma}^{(1)}.\vec{k}\,\vec{\sigma}^{(2)}.\vec{k}'\,\vec{\sigma}^{(3)}.\vec{q}' \\ & \times \Big\{\vec{\tau}^{(1)}\vec{\tau}^{(2)}\vec{\tau}^{(3)}.\vec{\tau}^{(4)} \Big[(1-\xi)\,\vec{\sigma}^{(4)}.\vec{q} - \vec{\sigma}^{(4)}.\vec{q}'\Big] \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(3)}\vec{\tau}^{(2)}.\vec{\tau}^{(4)} \Big[(1-\xi)\,\vec{\sigma}^{(4)}.\vec{q} - \vec{\sigma}^{(4)}.\vec{k}'\Big] \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(4)}\vec{\tau}^{(2)}.\vec{\tau}^{(3)} \Big[(1-\xi)\vec{\sigma}^{(4)}.\vec{q}+\vec{\sigma}^{(4)}.\vec{k}\Big]\Big\} + (1\leftrightarrow 4) + (2\leftrightarrow 4) + (3\leftrightarrow 4) \ . \end{aligned} \tag{42}$$

The single delta diagram leads to

$$\begin{aligned} t_{4N}^{(B,\Delta)} = {} & (2m)^4 \left(\frac{g}{2m}\right)^3 \frac{4\gamma_0\,\gamma_\Delta g_\Delta}{9m_\rho^2\,(M_\Delta-m)}\, \frac{1}{\vec{k}^2+\mu^2}\, \frac{1}{\vec{k}'^2+\mu^2}\, \frac{1}{\vec{q}'^2+\mu^2} \\ & \times \vec{\sigma}^{(1)}.\vec{k}\,\vec{\sigma}^{(2)}.\vec{k}'\,\vec{\sigma}^{(3)}.\vec{q}' \Big\{-12\,\vec{\tau}^{(1)}.\vec{\tau}^{(2)}\times\vec{\tau}^{(3)}\,\vec{k}.\vec{k}'\times\vec{q}' \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(2)}\vec{\tau}^{(3)}.\vec{\tau}^{(4)} \Big(-\vec{k}'.\vec{q}'\vec{\sigma}^{(4)}.\vec{k} - \vec{k}.\vec{q}'\sigma^{(4)}.\vec{k}' + 2\vec{k}.\vec{k}'\vec{\sigma}^{(4)}.\vec{q}'\Big) \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(3)}\vec{\tau}^{(2)}.\vec{\tau}^{(4)} \Big(-\vec{k}'.\vec{q}'\vec{\sigma}^{(4)}.\vec{k} + 2\vec{k}.\vec{q}'\vec{\sigma}^{(4)}.\vec{k}' - \vec{k}.\vec{k}'\vec{\sigma}^{(4)}.\vec{q}'\Big) \\ & + \vec{\tau}^{(1)}.\vec{\tau}^{(4)}\vec{\tau}^{(2)}.\vec{\tau}^{(3)} \Big(2\vec{k}'.\vec{q}'\vec{\sigma}^{(4)}.\vec{k} - \vec{k}.\vec{q}'\vec{\sigma}^{(4)}.\vec{k}' - \vec{k}.\vec{k}'\vec{\sigma}^{(4)}.\vec{q}'\Big)\Big\} \\ & + (1\leftrightarrow 4) + (2\leftrightarrow 4) + (3\leftrightarrow 4) \end{aligned} \tag{43}$$

The double delta contribution is

$$t_{4N}^{(B,\Delta\Delta)} = (2m)^4 \left(\frac{g}{2m}\right)^3 \frac{g_\Delta^2\, C_{\pi\Delta\Delta}}{3\,(M_\Delta-m)^2}\, \frac{1}{\vec{k}^2+\mu^2}\, \frac{1}{\vec{k}'^2+\mu^2}\, \frac{1}{\vec{q}'^2+\mu^2} \times$$

$$\times\ \vec\sigma^{(1)}.\vec k\ \vec\sigma^{(2)}.\vec k'\ \vec\sigma^{(3)}.\vec q'\Big[-\frac{25}{3}\,\vec\tau^{(1)}.\vec\tau^{(2)}\times\vec\tau^{(3)}\ \vec k.\vec k'\times\vec q'$$

$$+\ \vec\tau^{(1)}.\vec\tau^{(2)}\vec\tau^{(3)}.\vec\tau^{(4)}\left(-\frac{7}{9}\vec k'.\vec q'\vec\sigma^{(4)}.\vec k-\frac{7}{9}\vec k.\vec q'\vec\sigma^{(4)}.\vec k'+2\vec k.\vec k'\vec\sigma^{(4)}.\vec q'\right)$$

$$+\ \vec\tau^{(1)}.\vec\tau^{(3)}\vec\tau^{(2)}.\vec\tau^{(4)}\left(-\frac{7}{9}\vec k'.\vec q'\vec\sigma^{(4)}.\vec k+2\vec k.\vec q'\vec\sigma^{(4)}.\vec k'-\frac{7}{9}\vec k.\vec k'\vec\sigma^{(4)}.\vec q'\right)$$

$$+\ \vec\tau^{(1)}.\vec\tau^{(4)}\vec\tau^{(2)}.\vec\tau^{(3)}\left(2\vec k'.\vec q'\vec\sigma^{(4)}.\vec k-\frac{7}{9}\vec k.\vec q'\vec\sigma^{(4)}.\vec k'-\frac{7}{9}\vec k.\vec k'\vec\sigma^{(4)}.q'\right)\Big]$$

$$+\ (1\leftrightarrow 4)\ +\ (2\leftrightarrow 4)\ +\ (3\leftrightarrow 4)\ . \qquad (44)$$

The pion rescattering diagram of fig. 1(C) corresponds to the following form for the four-nucleon amplitude, when non-local contributions are neglected

$$t_{4N}^{(C)}=-\frac{1}{2}\,(2m)^4\left(\frac{g}{2m}\right)^2\frac{1}{\vec k^2+\mu^2}\ \frac{1}{\vec Q^2+\mu^2}\ \frac{1}{\vec q^2+\mu^2}\ \vec\sigma^{(1)}.\vec k\ \vec\sigma^{(4)}.\vec q$$

$$\times\Big\{\vec\tau^{(1)}.\tau^{(4)}\left[\alpha_\sigma+\left(\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right)\vec k.\vec Q\right]\left[\alpha_\sigma-\frac{8g_\Delta^2}{9(m_\Delta-m)}-\beta_\sigma\Big)\vec Q.\vec q\right]$$

$$-\ \vec\tau^{(2)}.\vec\tau^{(4)}\times\vec\tau^{(1)}\left[\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\vec\sigma^{(2)}.\vec Q\times\vec k\left[\alpha_\sigma-\left(\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right)\vec Q.\vec q\right]$$

$$+\ \vec\tau^{(3)}.\vec\tau^{(4)}\times\vec\tau^{(1)}\left[\alpha_\sigma+\left(\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right)\vec k.\vec Q\right]\vec\sigma^{(3)}.\vec q\times\vec Q\left[\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]$$

$$+\left[\vec\tau^{(1)}.\vec\tau^{(4)}\vec\tau^{(2)}.\vec\tau^{(3)}-\vec\tau^{(1)}.\vec\tau^{(3)}\vec\tau^{(2)}.\vec\tau^{(4)}\right]\left[\frac{1}{2f_\pi^2}\ \frac{(1+\mu_p-\mu_n)}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]$$

$$\times\ \vec\sigma^{(2)}.\vec Q\times\vec k\ \vec\sigma^{(3)}.\vec q\times\vec Q\ \Big\}+\ \text{(all nucleon permutations)}\ . \qquad (45)$$

The fator $\frac{1}{2}$ in front of the amplitude has been introduced because every independent diagram is double counted when one performs all possible permutations of the nucleon indices.

The potential in configuration space is given by

$$\langle\vec r_1'\vec r_2'\vec r_3'\vec r_4'|W^{1234}|\vec r_1\vec r_2\vec r_3\vec r_4\rangle=$$

$$=-\frac{(2\pi)^3}{(2m)^4}\int\frac{d\vec p_1}{(2\pi)^3}\cdots\frac{d\vec p_4'}{(2\pi)^3}\,\delta^3(\vec p_f-\vec p_i)\ e^{i\vec p_1.\vec r_1}\cdots e^{-i\vec p_4'.\vec r_4'}\ t_{4N}\ . \qquad (46)$$

The expressions for t_{4N} obtained above allow one to write

$$\langle\vec r_1'\vec r_2'\vec r_3'\vec r_4'|W^{1234}|\vec r_1\vec r_2\vec r_3\vec r_4\rangle\equiv\delta^3(\vec r_1'-\vec r_1)\ldots\delta^3(\vec r_4'-\vec r_4)\ W_{4B} \qquad (47)$$

where

$$W_{4B}=-\frac{(2\pi)^3}{(2m)^4}\int\frac{d\vec k}{(2\pi)^3}\ \frac{d\vec k'}{(2\pi)^3}\ \frac{d\vec q'}{(2\pi)^3}\ \frac{d\vec q}{(2\pi)^3}\ \delta(\vec k-\vec k'-\vec q'+\vec q)$$

$$\times\ e^{i(\vec k.\vec r_1-\vec k'.\vec r_2-\vec q'.\vec r_3+\vec q.\vec r_4)}\ t_{4B} \qquad (48)$$

The function W_{4B} is the four-body potential. It is made up of various terms, representing the partial contributions from pion-pion scattering, seagull, single delta, double delta and pion-nucleon rescattering. It is given by

$$W_{4B}=W_{4B}^{(A)}+W_{4B}^{(B,S)}+W_{4B}^{(B,\Delta)}+W_{4B}^{(B,\Delta\Delta)}+W_{4B}^{(C)}\ . \qquad (49)$$

The explicit form of the potential contains the Yukawa function U, defined as

$$U(\mu r)\equiv\frac{4\pi}{\mu}\int\frac{d\vec k}{(2\pi)^3}\ \frac{e^{-i\vec k.\vec r}}{\vec k^2+\mu^2}=\frac{e^{-\mu r}}{\mu r}\ . \qquad (50)$$

This function is not regular at the origin. Its regularization can be achieved by means of form factors and hard cores[5]. The former correspond to cutt offs in momentum space whereas the latter are cut offs in configuration space. These procedures are not mutually exclusive, since they are motivated by different physical causes. In this work the Yukawa function is assumed to be somehow regularized. However, one does not choose a specific method of regularization because applications of the potential are not being considered here.

The form of the functions W_{4B} becomes simpler when one uses the dimensionless variables $\vec{x}_i \equiv \mu\vec{r}_i$ and $\vec{x}_{ij} \equiv \vec{x}_i - \vec{x}_j$. These definitions result in the following expressions for the partial contributions to the four-body potential.

A. Intermediate pion-pion scattering

$$
\begin{aligned}
W_{4B}^{(A)} = {} & \left(\frac{1}{4\pi}\right)^3 \left(\frac{g}{2m}\right)^4 \frac{1}{2f_\pi^2}\,\mu^7\, (\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\ \vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\ \vec{\sigma}^{(3)}\cdot\vec{\nabla}_3\,\vec{\sigma}^{(4)}\cdot\vec{\nabla}_4) \\
& \times \Big\{-(1-\xi)\,(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)} + \vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)} + \vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}) \\
& \times \Big[U(x_{14})U(x_{24})U(x_{34}) + (1 \leftrightarrow 4) + (2 \leftrightarrow 4) + (3 \leftrightarrow 4)\Big] \\
& + 2\Big[(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)})(\vec{\nabla}_1\cdot\vec{\nabla}_2 + \vec{\nabla}_3\cdot\vec{\nabla}_4 + 1 - \tfrac{1}{2}\xi) + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)}) \\
& \times (\vec{\nabla}_1\cdot\vec{\nabla}_3 + \vec{\nabla}_2\cdot\vec{\nabla}_4 + 1 - \tfrac{1}{2}\xi) + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}) \\
& \times (\vec{\nabla}_1\cdot\vec{\nabla}_4 + \vec{\nabla}_2\cdot\vec{\nabla}_3 + 1 - \tfrac{1}{2}\xi)\Big]\, \frac{1}{4\pi}\int d\vec{x}\; U(|\vec{x}_1-\vec{x}|)U(|\vec{x}_2-\vec{x}|)U(|\vec{x}_3-\vec{x}|)U(|\vec{x}_4-\vec{x}|)\ . \qquad (51)
\end{aligned}
$$

B. Intermediate pion-production

Seagull

$$
\begin{aligned}
W_{4B}^{(B,S)} = {} & -\left(\frac{1}{4\pi}\right)^3 \left(\frac{g}{2m}\right)^4 \frac{1}{2f_\pi^2}\,\mu^7 \Big\{\Big[(1-\xi)\,(\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\ \vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\ \vec{\sigma}^{(3)}\cdot\vec{\nabla}_3\ \vec{\sigma}^{(4)}\cdot\vec{\nabla}_4) \\
& \times (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)} + \vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)} + \vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})\Big] \\
& + (\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\vec{\sigma}^{(3)}\cdot\vec{\nabla}_3)\Big[(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_3 \\
& + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_2 + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})\vec{\sigma}^{(4)}\cdot\vec{\nabla}_1\Big]\Big\}\, U(x_{14})U(x_{24})U(x_{34}) \\
& + (1 \leftrightarrow 4) + (2 \leftrightarrow 4) + (3 \leftrightarrow 4) \qquad (52)
\end{aligned}
$$

It is worth noting that the terms proportional to $(1-\xi)$ in eqs. (51) and (52) have opposite signs and cancell when both contributions are added together. This cancellation is motivated by chiral symmetry, as discussed in section IV.

Single delta

$$
\begin{aligned}
W_{4B}^{(B,\Delta)} = {} & -\left(\frac{1}{4\pi}\right)^3 \left(\frac{g}{2m}\right)^3 \frac{4\gamma_0\,\gamma_\Delta g_\Delta}{9m_\rho^2(M_\Delta - m)}\,\mu^9 \\
& \times (\vec{\sigma}^{(1)}\cdot\vec{\nabla}_1\ \vec{\sigma}^{(2)}\cdot\vec{\nabla}_2\ \vec{\sigma}^{(3)}\cdot\vec{\nabla}_3)\Big\{12(\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)} \times \vec{\tau}^{(3)})\,(\vec{\nabla}_1\cdot\vec{\nabla}_2\times\vec{\nabla}_3) \\
& + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}\vec{\tau}^{(3)}\cdot\vec{\tau}^{(4)})\Big[\vec{\nabla}_2\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_1 + \vec{\nabla}_1\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_2 - 2\vec{\nabla}_1\cdot\vec{\nabla}_2\vec{\sigma}^{(4)}\cdot\vec{\nabla}_3\Big] \\
& + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(3)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(4)})\Big[\vec{\nabla}_2\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_1 - 2\vec{\nabla}_1\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_2 + \vec{\nabla}_1\cdot\vec{\nabla}_2\vec{\sigma}^{(4)}\cdot\vec{\nabla}_3\Big] \\
& + (\vec{\tau}^{(1)}\cdot\vec{\tau}^{(4)}\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)})\Big[-2\vec{\nabla}_2\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_1 + \vec{\nabla}_1\cdot\vec{\nabla}_3\vec{\sigma}^{(4)}\cdot\vec{\nabla}_2 + \vec{\nabla}_1\cdot\vec{\nabla}_2\vec{\sigma}^{(4)}\cdot\vec{\nabla}_3\Big]\Big\} \\
& \times U(x_{14})U(x_{24})U(x_{34}) + (1 \leftrightarrow 4) + (2 \leftrightarrow 4) + (3 \leftrightarrow 4)\ . \qquad (53)
\end{aligned}
$$

Double delta

$$W_{4B}^{(B,\Delta\Delta)} = -\left(\frac{1}{4\pi}\right)^3 \left(\frac{g}{2m}\right)^3 \frac{g_\Delta^2\, C_{\pi\Delta\Delta}}{3(M_\Delta - m)^2}\, \mu^9$$

$$\times\ (\vec{\sigma}^{(1)}.\vec{\nabla}_1\ \vec{\sigma}^{(2)}.\vec{\nabla}_2\ \vec{\sigma}^{(3)}.\vec{\nabla}_3)\ \left\{\frac{25}{3}\ (\vec{\tau}^{(1)}.\vec{\tau}^{(2)}\times\vec{\tau}^{(3)})\ (\vec{\nabla}_1.\vec{\nabla}_2\times\vec{\nabla}_3)\right.$$

$$+\ (\vec{\tau}^{(1)}.\vec{\tau}^{(2)}\vec{\tau}^{(3)}.\vec{\tau}^{(4)})\left[\frac{7}{9}\vec{\nabla}_2.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_1 + \frac{7}{9}\vec{\nabla}_1.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_2 - 2\vec{\nabla}_1.\vec{\nabla}_2\vec{\sigma}^{(4)}.\vec{\nabla}_3\right]$$

$$+\ (\vec{\tau}^{(1)}.\vec{\tau}^{(3)}\vec{\tau}^{(2)}.\vec{\tau}^{(4)})\left[\frac{7}{9}\vec{\nabla}_2.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_1 - 2\vec{\nabla}_1.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_2 + \frac{7}{9}\vec{\nabla}_1.\vec{\nabla}_2\vec{\sigma}^{(4)}.\vec{\nabla}_3\right]$$

$$\left.+\ (\vec{\tau}^{(1)}.\vec{\tau}^{(4)}\vec{\tau}^{(2)}.\vec{\tau}^{(3)})\left[-2\vec{\nabla}_2.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_1 + \frac{7}{9}\vec{\nabla}_1.\vec{\nabla}_3\vec{\sigma}^{(4)}.\vec{\nabla}_2 + \frac{7}{9}\vec{\nabla}_1.\vec{\nabla}_2\vec{\sigma}^{(4)}.\vec{\nabla}_3\right]\right\}$$

$$\times\ U(x_{14})U(x_{24})U(x_{34})\ +\ (1 \leftrightarrow 4) + (2 \leftrightarrow 4) + (3 \leftrightarrow 4) \qquad (54)$$

C. Intermediate pion-nucleon scattering

$$W_{4B}^{(C)} = -\frac{1}{2}\left(\frac{1}{4\pi}\right)^3 \left(\frac{g}{2m}\right)^2 \mu^9\ (\vec{\sigma}^{(1)}.\vec{\nabla}_{12}\ \vec{\sigma}^{(4)}.\vec{\nabla}_{34})$$

$$\times\left\{\vec{\tau}^{(1)}.\vec{\tau}^{(4)}\left[-\left(\frac{\alpha_\sigma}{\mu^2}\right)^2 + \frac{\alpha_\sigma}{\mu^2}\left(\frac{8g_\Delta^2}{9(M_\Delta - m)} - \beta_\sigma\right)(\vec{\nabla}_{23}.\vec{\nabla}_{34}+\vec{\nabla}_{12}.\vec{\nabla}_{23}) - \left(\frac{8g_\Delta^2}{9(M_\Delta - m)} - \beta_\sigma\right)^2 (\vec{\nabla}_{12}.\vec{\nabla}_{23}\vec{\nabla}_{23}.\vec{\nabla}_{34})\right]\right.$$

$$-\ \frac{\alpha_\sigma}{\mu^2}\left(\frac{1}{2f_\mu^2}\ \frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta - m)}\right)\left[(\vec{\tau}^{(2)}.\vec{\tau}^{(1)}\times\vec{\tau}^{(4)})(\vec{\nabla}_{12}\times\vec{\nabla}_{23}) + (\vec{\tau}^{(3)}.\vec{\tau}^{(1)}\times\vec{\tau}^{(4)})(\vec{\nabla}_{23}\times\vec{\nabla}_{34})\right]$$

$$+\ \left(\frac{8g_\Delta^2}{9(M_\Delta - m)} - \beta_\sigma\right)\left(\frac{1}{2f_\pi^2}\ \frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta - m)}\right)\left[(\vec{\tau}^{(2)}.\vec{\tau}^{(1)}\times\vec{\tau}^{(4)})(\vec{\nabla}_{23}.\vec{\nabla}_{34}\vec{\sigma}^{(2)}.\vec{\nabla}_{12}\times\vec{\nabla}_{23})\right.$$

$$\left.+\ (\vec{\tau}^{(3)}.\vec{\tau}^{(1)}\times\vec{\tau}^{(4)})\ (\vec{\nabla}_{12}.\vec{\nabla}_{23}\ \vec{\sigma}^{(3)}.\vec{\nabla}_{23}\times\vec{\nabla}_{34})\right]$$

$$+\ \left(\frac{1}{2f_\pi^2}\ \frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta - m)}\right)^2 (\vec{\tau}^{(1)}.\vec{\tau}^{(4)}\vec{\tau}^{(2)}.\vec{\tau}^{(3)} - \vec{\tau}^{(1)}.\vec{\tau}^{(3)}\vec{\tau}^{(2)}.\vec{\tau}^{(4)})$$

$$\left.\times\ (\sigma^{(2)}.\vec{\nabla}_{12}\times\vec{\nabla}_{23}\ \sigma^{(3)}.\vec{\nabla}_{23}\times\vec{\nabla}_{34})\right\}\ U(x_{12})\ U(x_{23})\ U(x_{34})$$

$$+\ (\text{all nucleon permutations}) \quad . \qquad (55)$$

The final form for the potential derived in this work is that shown in eq. (49), where the partial contributions are those given by eqs. (51-55). Here, as in the case of three-body forces, each term of the potential is written as the product of four kinds of terms, namely a strength parameter with dimension of energy, an isospin operator and a spin operator coupled to derivatives, acting on Yukawa functions. There would be, of course, other ways of writing the potential. For instance, the explicit evaluation of the derivatives of the functions U would produce a result in terms of the functions U_0 , U_1 and U_2 used in ref. (5) . The main advantage of the form adopted above is that the expressions tend to be more compact than the alternative ones.

The relative importance of the various partial contributions can be estimated by comparing their strengthes. In their numerical evaluation one adopts the following values for the "experimental" masses and coupling constants: μ = 139.57 MeV, m_ρ = 770 MeV , m = 938.28 MeV , M_Δ = 1220 MeV [10] ; g = 13.39 , $g_\Delta = 1.84\ \mu^{-1}$ [10] , f_π = 93 MeV [11] , $\mu_p - \mu_n = 3.7$, $\gamma_0 = 6.0$, $\gamma_\Delta = 2.0\ \mu^{-1}$. The value of γ_0 has been derived from the relation $\gamma_0 = m_\rho/\sqrt{2}\ f_\pi$, whereas γ_Δ is linked to the $\gamma N\Delta$ form factor C by $\gamma_\Delta = C\,\gamma_0$. The value of C can be extracted from electroproduction and here one adopts $C = 0.34\ \mu^{-1}$ [12]. The sigma parameters are $\alpha_\sigma = 1.05\ \mu^{-1}$ [11] and $\beta_\sigma = -0.80\ \mu^{-3}$ [10]. Finally, SU(4) symmetry is used to produce the result: $C_{\pi\Delta\Delta} = \frac{6}{5}\ g/2m$ [15].

These "experimental" parameters produce the following values for the strength constants of the various partial contributions

$$C_{(A)} = C_{(B,S)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^4 \frac{1}{2f_\pi^2}\, \mu^2 = 0.0779 \text{ MeV}$$

$$C_{(B,\Delta)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^3 \frac{4\gamma_0\gamma_\Delta g_\Delta}{9m_\rho^2(M_\Delta-m)}\, \mu^6 = 0.0111 \text{ MeV}$$

$$C_{(B,\Delta\Delta)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^3 \frac{g_\Delta^2\, C_{\pi\Delta\Delta}}{3(M_\Delta-m)^2}\, \mu^6 = 0.0230 \text{ MeV}$$

$$C_{(C,\sigma-\sigma)} \equiv \left(\frac{1}{4}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{\alpha_\sigma}{\mu^2}\right)^2 \mu^7 = 0{,}0769 \text{ MeV}$$

$$C_{(C,\sigma-\Delta\sigma)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{\alpha_\sigma}{\mu^2}\right) \left(\frac{8g_\Delta^2}{9(M_\Delta-m)} - \beta_\sigma\right) \mu^7 = 0.1677 \text{ MeV}$$

$$C_{(C,\Delta\sigma-\Delta\sigma)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{8g_\Delta^2}{9(M_\Delta-m)} - \beta_\sigma\right)^2 \mu^7 = 0.3661 \text{ MeV}$$

$$C_{(C,\sigma-\rho\Delta)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{\alpha_\sigma}{\mu^2}\right) \left(\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta-m)}\right) \mu^7 = 0.0561 \text{ MeV}$$

$$C_{(C,\Delta\sigma-\Delta\sigma)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{8g_\Delta^2}{9(M_\Delta-m)} - \beta_\sigma\right) \left(\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta-m)}\right) \mu^7$$

$$= 0.1225 \text{ MeV}$$

$$C_{(C,\rho\Delta-\rho\Delta)} \equiv \left(\frac{1}{4\pi}\right)^3 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta-m)}\right)^2 \mu^7 = 0.0410 \text{ MeV} .$$

The meaning of these values is discussed in the next section.

IV. CONCLUSIONS

The derivation of the pion-exchange four-body potential presented in this work is based on the assumption that the nucleon momenta are comparable to the pion mass. The spin and isospin structures of the potential are somewhat complex and a precise assessment of the relative importance of its various contributions can only be done in specific applications. Nevertheless, several semi-quantitative conclusions can be drawn by inspecting the strength parameters displayed above.

First, one notes that the seagull term (B,S) dominates the contribution of the intermediate pion-production amplitude, relatively to the single delta (B,Δ) and double delta (B,ΔΔ) terms. This result is the direct consequence of chiral and gauge symmetries and hence is similar to the case of the pion-rho exchange three-body force, where a seagull diagram has been shown to be about ten times more important than that of the delta[8].

The contributions from the intermediate pion-pion scattering (A) and seagull in pion production (B,S) have the same strength and cancell partially when they are added together. This behaviour can also be ascribed to the symmetries and is analogous to that observed on the exchange current contribution to the elastic pion-deuteron scattering[14].

The strength parameters associated with the intermediate pion-nucleon rescattering show that this process yields the most important contributions to the four-body potential. The largest term comes from p-waves in the isospin even amplitude and is due to the diagrams describing the delta-pole and sigma exchange in fig. 5.

A crude assessment of the relative importance between three and four body forces due to the exchange of pions can be made by comparing their strengthes. The three-body force derived in ref. (5) is characterized by the following parameters

$$C_s = \left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{\alpha_\sigma}{\mu^2}\right) \mu^4 = 0.92 \text{ MeV}$$

$$C_p = -\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{8g_\Delta^2}{9(M_\Delta - m)} - \beta_\sigma\right) \mu^4 = -2.01 \text{ MeV}$$

$$C'_p = -\left(\frac{1}{4\pi}\right)^2 \left(\frac{g\mu}{2m}\right)^2 \left(\frac{1}{2f_\pi^2}\,\frac{(1+\mu_p-\mu_n)}{2m} + \frac{2g_\Delta^2}{9(M_\Delta - m)}\right) \mu^4 = -0.67 \text{ MeV}$$

These values are typically one order of magnitude greater than those of the four body force. The three-body parameters are, in turn, one order of magnitude smaller than that of the one pion exchange nucleon-nucleon potential, which is given by

$$C_{OPEP} = \left(\frac{1}{4\pi}\right) \left(\frac{g\mu}{2m}\right)^2 \mu = 11.02 \text{ MeV} \quad .$$

The comparison among these various strength parameters shed some light on the hierarchy of many body forces due to pion-exchange. These forces correspond, in general, to a succession of vertices, describing interactions, and pion propagators. The latter are represented, in configuration space, by a factor $\left[\frac{1}{4\pi} U(x)\right]$ for each pion, where $U(x)$ is an Yukawa function. The vertices, on the other hand, produce the remaining factors of the strength parameters.

In general, the strength of a many body potential should depend on the number of its vertices and propagators. However, inspection of the strength parameters of two, three

and four-body potentials allows one to conclude that the contribution of the vertices are roughly independent of their number, provided that the pion mass is adopted as a unit for the momenta. This means that the propagation of pions is the dominant factor in determining the strength of the potential. This influence is felt both through the radial variation of the Yukawa function and the factors $\left(\frac{1}{4\pi}\right)$. The various powers of the latter determine the different orders of magnitude of the strength parameters of two, three and four-body potentials.

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REFERENCES

(1) A. Laverne and C. Gignoux, Nucl. Phys. A203 (1973) 597; R.A. Brandenburg, Y.E. Kim and A. Tubis, Phys. Rev. C12 (1975) 1368; G.L. Payne, J.L. Friar, B.F. Gibson and I.R. Afnan, Phys. Rev. C22 (1980) 823; I.R. Afnan and N.D. Birrell, Phys. Rev. C16 (1977) 823; R.A. Brandenburg, P.U. Sauer and R. Machleidt, Z. Phys. A280 (1977) 93; N.D. Birrell and I.R. Afnan, Phys. Rev. C17 (1978) 326.

(2) B.H.J. McKellar, to appear in Nuclear Physics and references contained therein.

(3) J.L. Ballot, Z. Phys. A302 (1981) 347; Phys. Lett. 127B (1983) 399 and references contained therein.

(4) S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt and B.H.J. McKellar, Nucl. Phys. A317 (1979) 242.

(5) H.T. Coelho, T.K. Das and M.R. Robilotta, Phys. Rev. C28 (1983) 1812.

(6) M.G. Olsson and L. Turner, Phys. Rev. Lett. 20 (1968) 1127.

(7) J. Wess and B. Zumino, Phys. Rev. 163 (1967) 1727.

(8) M.R. Robilotta and M.P. Isidro Filho, to appear in Nucl. Phys.

(9) M.R. Robilotta, M.P. Isidro Filho, H.T. Coelho and T.K. Das, preprint IFUSP/P-407, submitted for publication.

(10) M.G. Olsson and E.T. Osypowski, Nucl. Phys. B101 (1975) 136.

(11) G. Höhler, F. Kaiser, R. Koch and E. Pietarinen, Physics Data - Karlsruhe Report, 1979.

(12) M.G. Olsson and E.T. Osypowski, Phys. Rev. D17 (1978) 174.

(13) M.R. Robilotta, unpublished.

(14) M.R. Robilotta and C. Wilkin, J. Phys. G4 (1978) L115; M.R. Robilotta, Phys. Lett. 92B (1980) 26.

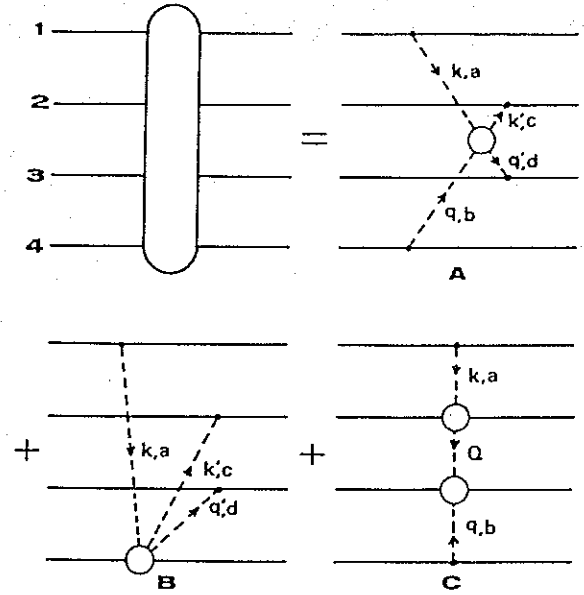

Fig.1 - Contributions to the pion-exchange four-body force: pion-pion scattering (A), pion production (B) and pion--nucleon rescattering (C).

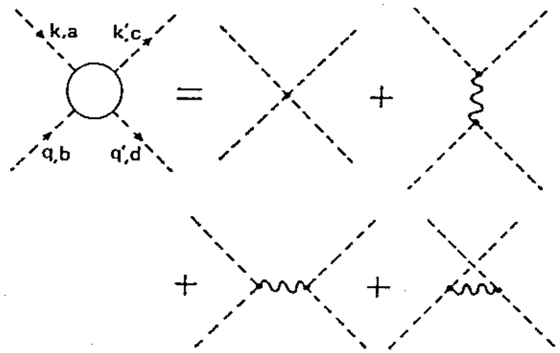

Fig.2 - Low-energy pion-pion scattering amplitude. Pions and rhos are represented by broken and wavy lines.

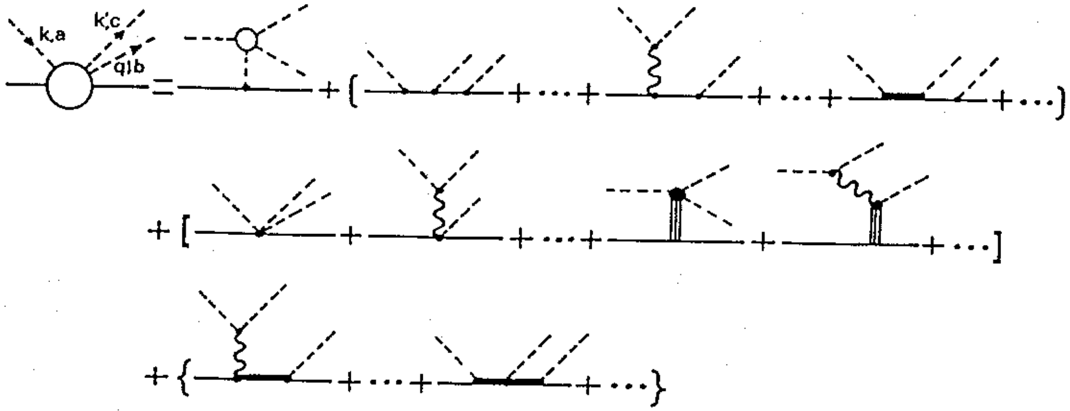

Fig.3 - Low-energy pion production amplitude. Pions, rhos and axial-vector mesons are represented by broken, wavy and triple lines. Full and thick lines denote nucleons and deltas.The symbol (...) indicates permutations of the pions.

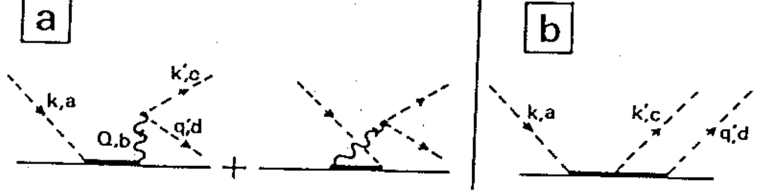

Fig.4 - Single (a) and double (b) delta diagrams.

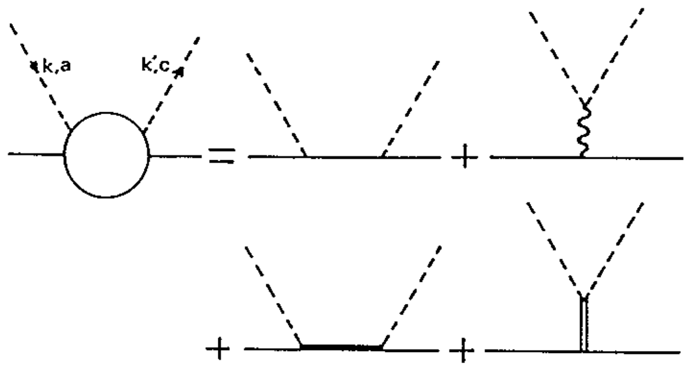

Fig.5 - Low-energy pion-nucleon amplitude. Pions, rhos and sigma are denoted by broken, wavy and double lines. Full and thick lines represent nucleons and deltas.