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DECAY RATE OF THE FALSE VACUUM AT HIGH TEMPERATURES

by

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ABSTRACT

We investigate, within the semiclassical approach, the high temperature behavior of the decay rate of the metastable vacuum in Field Theory. We have shown that, contrarily to what has been proposed in the literature, the pre-exponential factor exhibits a nontrivial dependence on the temperature. Furthermore, this dependence is such that at very high temperatures it is as important as the exponential factor and consequently it spoils many conclusions drawn up to now on Cosmological Phase Transitions.

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I. INTRODUCTION

It is well known that spontaneously broken symmetries can be restored at high temperatures(1).

In the very early universe the temperatures were so high that broken symmetries could be restored. We expect that phase transitions between the symmetric and broken-symmetry phases took place as the universe cooled off during its expansion.

Those phase transitions should have very rich physical consequences. For instance, if the phase transition occurred quickly, with only a negligible amount of supercooling, too many magnetic monopoles would be produced and this fact contradicts observational data(2).

Recently A. Guth(3) proposed an alternative scenario (the inflationary universe) for the Big Bang and how to solve the flatness and horizon problems if the phase transition is such that it allows the existence of supercooling.

An important element of the phase transition is the decay rate of the false vacuum Γ (i.e., tunnelling probability per unity time). Having in hand this quantity we can calculate, for instance, the fraction of the universe in the new phase, the density of primordial magnetic monopoles, or the time when the phase transition is over.

The study of the decay of the false vacuum at zero temperature has been carried off by Coleman(4). He used a semiclassical approximation and showed that the important classical solutions are those with symmetry $O(4)$. The first quantum corrections to the classical solution were also calculated by Callan and Coleman(4).

One can extend their result to finite temperatures by using a semiclassical approximation[5]. The result that one obtains for a single scalar field theory is:

$$\frac{\Gamma}{V} = -2\,T\,\mathrm{Im}\left[\frac{S^E(\phi_C)}{2\pi}\right]^{Z/2}\left[\frac{\det'\left[-\Box_E + V''(\phi_C)\right]}{\det\left[-\Box_E + V''(\phi_{VAC})\right]}\right]^{-1/2}\exp\left\{-S^E(\phi_C)\right\} \tag{1.1}$$

where Z is the number of zero eigenvalues of $\left[-\Box_E + V''(\phi_C)\right]$, ϕ_C is a solution to the classical equations of motion, and the Lagrangian density is $L_E = \frac{1}{2}\sum_{i=1}^{D}(\partial_i\phi)^2 + V(\phi)$.

We intend to study the dependence with T of $\frac{\Gamma}{V}$ in the limit of high temperatures. Contrary to the belief of many authors[10-13], we are going to show that in this limit, the determinant ratio appearing in (1.1) contains the main dependence with temperature of $\frac{\Gamma}{V}$.

The outline of this paper is as follows. In Section II we review briefly the semiclassical expression for $\frac{\Gamma}{V}$. Section III contains the derivation of a formal expression for $\frac{\Gamma}{V}$ in the high temperature limit and its consequences. In the following section we analyze some simple examples in 1+1 dimensions and show that the preexponential term is indeed very important at high temperatures. A 3+1 dimensional example is given in Section V. Finally Section VI summarizes the results and gives our conclusions.

II. SEMICLASSICAL APPROXIMATION FOR $\frac{\Gamma}{V}$

We are going to review briefly the functional integration formalism applied to QFT at finite temperature described by a single scalar field.

All the information about our system in equilibrium at a temperature[6] β^{-1} is contained in the partition function which is given by:

$$Z = \mathrm{tr}\, e^{-\beta H} \tag{2.1}$$

where H is the Hamiltonian of the system.

The Helmoltz free energy can be easily obtained from Z:

$$A = -\beta^{-1}\,\ell n\, Z \tag{2.2}$$

We can write a path integral representation for Z[7]:

$$Z = \int [D\phi]\; e^{-S_E(\phi)} \tag{2.3}$$

where

$$S_E(\phi) = \int d_E^D x \left\{\sum_{i=1}^{D}(\partial_i\phi)^2 + V(\phi)\right\} \tag{2.4}$$

and the integration is carried over periodic field configurations in the euclidean time with period β (i.e., $\phi(0,\vec{x}) = \phi(\beta,\vec{x})$).

Now we are going to perform the semiclassical approximation in order to obtain the expression (1.1) for $\frac{\Gamma}{V}$. In the semiclassical limit, the leading contributions to Z, given by (2.3), come from the field configurations which minimize the classical Euclidean action and therefore obey the Euler-Lagrange equation:

$$\sum_{i=1}^{D}\frac{\partial^2\phi_C}{\partial x_i^2} = V'(\phi_C) \tag{2.5}$$

where ϕ_C satisfies the boundary condition

$$\phi_C(0,\vec{x}) = \phi_C(\beta,\vec{x}) \tag{2.6}$$

It is easy to see that for high temperatures the relevant field configurations are those independent of the Euclidean time.

Now we make a functional Taylor expansion of S_E around ϕ_C and we keep only the quadratic terms in $\eta = \phi - \phi_C$:

$$z^{(1)} = e^{-S_E(\phi_C)} \int [D\eta] \exp - \left\{ \int d_E^D \vec{x} \left[\frac{1}{2} \sum_{i=1}^{D} (\partial_i \eta)^2 + \frac{1}{2} \eta V''(\phi_C) \eta \right] \right\} \tag{2.7}$$

The gaussian integral in (2.7) is easy to perform[4] and we get, formally

$$z^{(1)} = e^{-S_E(\phi_C)} \det^{-1/2} \left[- \sum_{i=1}^{D} \partial_i^2 + V''(\phi_C) \right] \tag{2.8}$$

The leading contribution to Z is given by constant ϕ_C associated to the vacuum of the theory:

$$z^{(0)} = e^{-S_E(\phi_{VAC})} \det^{-1/2} \left[- \sum_{i=1}^{D} \partial_i^2 + V''(\phi_{VAC}) \right] \tag{2.9}$$

Using the dilute gas approximation[4] we have:

$$Z = Z^0 + Z^1 = Z^0 \left(1 + \frac{Z^1}{Z^0} \right) \simeq Z^0 \exp \left(\frac{Z^1}{Z^0} \right)$$

defining the transition probability as[6] $\Gamma = -2 \operatorname{Im} A$ we obtain, by treating separately the zero eigenvalues:

$$\frac{\Gamma}{V} = -2T \operatorname{Im} \left[\frac{S_E(\phi_C)}{2\pi} \right]^{Z/2} \left[\frac{\det'(-\partial^2 + V''(\phi_C))}{\det(-\partial^2 + V''(\phi_{VAC}))} \right]^{-1/2} \exp - S_E(\phi_C) \tag{2.10}$$

where the prime indicates that the zero eigenvalues of $-\partial^2 + V''(\phi_C)$ must be omitted from the determinant and Z is the number of these eigenvalues.

After a little algebra (see appendix A) we can write $\frac{\Gamma}{V}$ as:

$$\frac{\Gamma}{V} = \frac{2\,T^{Z+1}}{\sin\left(\frac{\beta\omega}{2}\right)} \left(\frac{S_E(\phi_C)}{2\pi} \right)^{Z/2} \exp \left\{ - S_E(\phi_C) + \left[\frac{\beta}{2} \left(\sum_j \lambda_j^V - \sum_j{}'' \lambda_j^C \right] \right. \right.$$
$$\left. + \left[\sum_j \ell n \left(1 - e^{-\beta\lambda_j^V} \right) - \sum_j{}'' \ell n \left(1 - e^{-\beta\lambda_j^C} \right) \right] \right\} \tag{2.11}$$

where

$$\left[\sum_{\text{spatial}} - (\partial_i)^2 + V''(\phi_{VAC}) \right] \eta_j = \left(\lambda_j^V \right)^2 \eta_j \tag{2.12}$$

$$\left[\sum_{\text{spatial}} - (\partial_i)^2 + V''(\phi_C) \right] \eta_j = \left(\lambda_j^C \right)^2 \eta_j \quad , \tag{2.13}$$

the negative eigenvalue (which we assume to be unique) in (2.13) is written as

$$\left(\lambda_-^C \right)^2 = -\omega^2 \tag{2.14}$$

and the double prime indicates that the negative and zero eigenvalues must be omitted from the summation.

III. FORMAL HIGH-TEMPERATURE EXPANSION OF $\frac{\Gamma}{V}$

We shall develop a formal expansion for the ratio of determinants (R) which appears in (2.10) that will be useful in order to extract the dependence on T at high temperatures. R can be written as

$$R = \exp - \frac{1}{2} \left\{ \text{tr}\ \ell n \left[-\Box_E + V''(\phi_C) \right] - \text{tr}\ \ell n \left[-\Box_E + V''(\phi_{VAC}) \right] \right\} \tag{3.1}$$

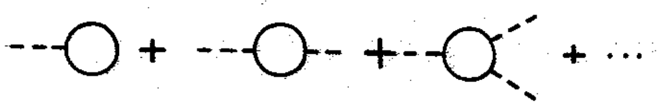
Then, from (3.1), it is easy to see that R can be written under the form

$$R = \exp - \frac{1}{2} \left\{ \text{tr}\ \ell n \left[1 + \frac{1}{-\Box_E + V''(\phi_{VAC})} \left(V''(\phi_C) - V''(\phi_{VAC}) \right) \right] \right\}$$

where $\frac{1}{-\Box_E + V''(\phi_{VAC})} = G_\beta$ is just the free propagator at finite temperature, with mass $\sqrt{V''(\phi_{VAC})}$.

If we expand the ℓn above in powers of $\frac{1}{-\Box_E + V''(\phi_{VAC})} \left[V''(\phi_C) - V''(\phi_{VAC}) \right]$, we get formally

$$\text{tr}\ \ell n \left\{ 1 + \frac{1}{-\Box_E + V''(\phi_{VAC})} \left[V''(\phi_C) - V''(\phi_{VAC}) \right] \right\} \equiv$$



(3.2)

where the dashed lines correspond to the "background field" $(V''(\phi_C) - V''(\phi_{VAC}))$, and the internal lines denote propagators G_β .

It is shown in appendix B that the first term of this series gives the leading contribution for β going to zero

when the space-time dimension is four. Then, we have:

$$R = \exp - \frac{1}{2}\ \text{tr}\ \frac{1}{-\Box_E + V''(\phi_{VAC})} \left[V''(\phi_C) - V''(\phi_{VAC}) \right] \tag{3.3}$$

for $\beta \to 0$ $(T \to 0)$.

We need to be careful when using (3.3). The formal manipulations that we made in order to get (3.3) work just for the eigenvalues belonging to the continuum. Then, negative and zero eigenvalues can be treated as we did in appendix A and the result for Im R is:

$$\text{Im}\ R = \frac{T^Z}{\sin \frac{\beta\omega}{2}} \exp - \frac{1}{2}\ \text{tr} \left\{ G_\beta \left[V''(\phi_C) - V''(\phi_{VAC}) \right] \right\} \tag{3.4}$$

We expect this expression to hold for high temperatures – that is, in the limit $\beta \to 0$. Lets find out the dependence on T of the exponent in (3.4) for the usual three dimensional space in this limit. We denote this exponent by σ – that is,

$$\sigma \equiv - \frac{1}{2}\ \text{tr} \left\{ G_\beta \left[V''(\phi_C) - V''(\phi_{VAC}) \right] \right\} \tag{3.5}$$

The reason why σ does not control the high temperature behavior of the preexponential factor for 1 and two spatial dimensions is given in appendix B.

III-A. (3+1) dimensional space

For (3+1) dimensional space we have, from (3.5):

$$\sigma = - \frac{1}{2} \sum_{n,k} \frac{1}{\left(\frac{2\pi n}{\beta}\right)^2 + k^2 + m^2} \frac{1}{\beta L} \int d^2x_E \left[V''(\phi_C) - V''(\phi_{VAC}) \right] \tag{3.6}$$

where $m^2 = V''(\phi_{VAC})$.

Performing the n summation and remembering that for high temperatures the relevant classical solution is independent of the euclidean time, we can further simplify (3.6):

$$\sigma = -\frac{1}{2}\int d^3x\left[V''(\phi_C(x)) - m^2\right]\int\frac{d^3k}{(2\pi)^3}\left\{\frac{\beta}{2\sqrt{k^2+m^2}} + \frac{\beta}{\sqrt{k^2+m^2}\left(e^{\beta\sqrt{k^2+m^2}}-1\right)}\right\} \tag{3.7}$$

The first integral in d^3k is infinity and must be renormalized. For a renormalizable theory, like those we will study in the subsequent sections, the way to get rid of these divergences is very simple. We just add to the Lagrangian the usual counterterms defined in perturbation theory[4]. These counterterms, for renormalizable theories, cancels the divergences which appear in the formal expansions and in particular, cancels the divergent piece in (3.7).

Therefore, in the high temperature limit, the main contribution to σ is given by:

$$\sigma = -\frac{1}{2}\int d^3x\left[V''(\phi_C) - m^2\right]\int\frac{d^3k}{(2\pi)^3}\frac{\beta}{\sqrt{k^2+m^2}\left(e^{\beta\sqrt{k^2+m^2}}-1\right)} \tag{3.8}$$

Then in the high temperature limit, σ behaves as:

$$\sigma = A \times T \tag{3.9}$$

where

$$A = -\frac{1}{2}\int d^3\vec{x}\,(V''(\phi_C) - m^2)\int\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{k(e^k-1)} \tag{3.10}$$

Therefore

$$\frac{\Gamma}{V} = \frac{2\,T^{Z+1}}{\sin\frac{\beta\omega}{2}}\left(\frac{S_E(\phi_C)}{2}\right)^{Z/2}\exp\left\{-\frac{B}{T} + AT\right\} \tag{3.11}$$

where $\frac{B}{T} = S_E(\phi_C)$.

Expression (3.11) is an achievement of this paper. It follows from our formal expansion developed in this section. In this expansion one easily realizes that, at the one loop level, the relevant contribution at high temperatures comes from the tadpole graph. The zero temperature part is a divergent one and such divergences are eliminated by adding to the Lagrangian the usual counterterms.

From expression (3.11) one can see that, as advanced earlier, at high temperatures quantum effects lead to contributions to the decay rate which are more important or comparable to the classical contributions (the exponential term).

IV. ONE DIMENSIONAL EXAMPLES

Now we are going to analyze some specific examples in order to get the asymptotic behaviour of the decay rate at high temperatures.

IV-A. An "Inverted" $\lambda\phi^4$ potential

The Lagrangian density for this first example is given by:

$$L_E = \frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\partial_x\phi)^2 + V(\phi) \tag{4.1}$$

where

$$V(\phi) = \frac{1}{2} m^2 \phi^2 - \frac{\lambda \phi^4}{4} \qquad (4.2)$$

with m^2 and λ positive.

Clearly, the state $\phi=0$ is metastable. Lets calculate its decay rate per unit volume at high temperatures.

First of all we have to obtain a static solution to

$$\partial_{xx}\phi - m^2\phi + \lambda\phi^3 = 0 \qquad (4.3)$$

A solution to (4.3) is(6):

$$\phi_C = \sqrt{\frac{2}{\lambda}}\, m\, \text{sech}(mx) \qquad (4.4)$$

The euclidean action of this solution is given by:

$$S_E(\phi_C) = \frac{4m^3}{3\lambda} \frac{1}{T} \qquad (4.5)$$

Now we need to find the eigenvalues of the operator $-\Box_E + V''(\phi_C)$:

$$\left[-\Box_E + V''(\phi_C)\right]\eta = \left[-\partial_{\tau\tau} - \partial_{xx} + m^2 - 6m^2\,\text{sech}^2(mx)\right]\eta = \varepsilon\eta \quad (4.6)$$

Then we have

$$\varepsilon = \left(\frac{2\pi n}{\beta}\right)^2 + \begin{cases} -3m^2 \\ 0 \\ k'^2 + m^2 \end{cases} \qquad (4.7)$$

where n is an integer.

Imposing periodic boundary conditions we get that:

$$k'L + \delta(k') = 2\pi n'$$

where n' is an integer and $\delta(k')$ is the phase-shift for k':

$$\delta(k) = -\frac{2}{m}\arctan\left[\frac{3km}{2m^2 - k^2}\right] \qquad (4.8)$$

In order to calculate $\text{Im}\,F$ given by (A.6), we assume that $\phi_{VAC} = 0$. Then we have:

$$\text{Im}\,R = \frac{T}{\sin\left(\frac{\sqrt{3}\,\beta m}{2}\right)} \exp\left\{\frac{\beta}{2}\left[\sum_k \sqrt{k^2+m^2} - \sum_{k'} \sqrt{k'^2+m'^2}\right] + \left[\sum_k \ell n\left(1-e^{-\beta\sqrt{k^2+m^2}}\right) - \sum_{k'} \ell n\left(1-e^{-\beta\sqrt{k'^2+m^2}}\right)\right]\right\} \qquad (4.9)$$

The above expression contains a divergent part given by

$$E1 = \frac{1}{2}\sum_k \sqrt{k^2+m^2} - \frac{1}{2}\sum_{k'} \sqrt{k'^2+m^2} \qquad (4.10)$$

For large L this expression becomes(8):

$$E1 = \frac{3\sqrt{2m^2}}{2\pi} + \int_{-\infty}^{+\infty} \frac{dk}{(2\pi)} \frac{d\delta(k)}{dk} \sqrt{k^2+m^2} \qquad (4.11)$$

$E1$ can be made finite by adding the counterterm

$$CT = \frac{1}{2}\int_{-\infty}^{+\infty} dx\left[\phi_C^2 - \frac{m^2}{\lambda}\right] \times 3\lambda \int_{-\infty}^{+\infty} \frac{dk}{(2\pi)} \frac{1}{\sqrt{k^2+m^2}} \qquad (4.12)$$

We are going to write:

$$\mathrm{Im}\,F = \frac{T}{\sin\left(\frac{\beta\sqrt{3}\,m}{2}\right)} \exp\left\{\beta E_1 + \beta C T + E_2\right\} \tag{4.13}$$

For L going to infinity we have:

$$E2 = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{d\delta}{dk} \ln\left(1 - e^{-\beta\sqrt{k^2+M^2}}\right) \tag{4.14}$$

Defining a new variable $\mu = \beta k$ we can write:

$$E2 = \beta m \int_{-\infty}^{+\infty} \frac{d\mu}{2\pi} \ln\left(1 - e^{-\sqrt{\mu^2+\beta^2 m^2}}\right) \left[\frac{4}{\mu^2 + 4\beta^2 m^2} + \frac{2}{\mu^2+\beta^2 m^2}\right] \tag{4.15}$$

The behaviour of $E2$ in the high temperature limit can be found in the literature[16] and is given by:

$$E2 \approx \text{constant} \times \beta \times \ln(\beta m) \tag{4.16}$$

Thus, for $\beta \to 0$ we have

$$\frac{\Gamma}{L} = \frac{2T^2}{\sin\left(\frac{\sqrt{3}\,m}{2T}\right)} \left(\frac{2m^3}{3\pi\lambda T}\right)^{1/2} \exp\left[\beta(\text{constant} + \text{constant}'\ \ln\beta m)\right] \tag{4.17}$$

In this case we see explicitly that in the high temperature limit the contribution from the determinant ratio is larger than the exponential factor.

IV-B. Spontaneously broken $\lambda\phi^4$ with a source term

Now we are going to consider $V(\phi)$, which appears in (4.1), of the form:

$$V(\phi) = -\frac{1}{2} m^2\phi^2 + \frac{\lambda\phi^4}{4} + \varepsilon\phi \tag{4.18}$$

where m^2, λ, and ε are positive.

We will consider the case $\varepsilon \ll 1$ - that is, we will perform a thin wall approximation.

The relative minima ϕ_- ($\cong \frac{m}{\sqrt{\lambda}} + \frac{\varepsilon}{m^2}$) is metastable and it decays to ϕ_+ ($= -\frac{m}{\sqrt{\lambda}} + \frac{\varepsilon}{m^2}$) with a decay rate per unit length $\frac{\Gamma}{L}$. We will obtain $\frac{\Gamma}{L}$ to the leading order in ε.

For high temperatures the static classical solution must satisfy:

$$\partial_{xx}\,\phi_C = \left.\frac{\delta V}{\delta\phi}\right|_{\phi=\phi_C} = -\mu^2\phi_C + \lambda\phi_C^3 + \varepsilon \tag{4.19}$$

We can expand ϕ_C in powers of ε as follows

$$\phi_C = \sum_{n=0}^{\infty} \varepsilon^n\,\phi_n \tag{4.20}$$

Plugging (4.20) into (4.19) and solving the resulting equation for ϕ_0 we get:

$$\phi_0 = \frac{m}{\sqrt{\lambda}} \tanh\left(\frac{mx}{\sqrt{2}}\right) \tag{4.21}$$

Next we need to solve the following eigenvalue problem:

$$\left[-\partial^2 + V''(\phi_C)\right]\eta_j = \alpha_j\,\eta_j \tag{4.22}$$

Again we expand η_j and α_j in powers of ε:

$$\eta_j = \sum_{n=0}^{\infty} \varepsilon^n\,\eta_{j,n} \tag{4.23}$$

$$\alpha_j = \sum_{n=0}^{\infty} \varepsilon^n\,\alpha_{j,n} \tag{4.24}$$

We can calculate explicitly the eigenvalues to zero order in ε and the result is:

$$\alpha_0 = \left(\frac{2\pi n}{\beta}\right)^2 + \begin{cases} 0 \\ \frac{3}{2}\,m^2 \\ k^2 + 2m^2 \quad \text{(for the continuous spectrum)} \end{cases} \tag{4.25}$$

We are going to assume the existence of just one negative eigenvalue and that it is at least of order $\varepsilon(\alpha_{neg} = -\gamma^2)$. Having the eigenvalues, we can calculate the pre-exponential factor, given by (A.6), to the lowest order in ε:

$$\mathrm{Im}\,R = \frac{T}{\sin\left(\frac{\gamma}{2T}\right)} \exp\left\{\frac{\beta}{2}\left[\frac{L}{2\pi}\int d\vec{k}\,\sqrt{k^2+2m^2} - \sum_{k'}\sqrt{k'^2+2m^2} - \frac{3}{2}\,m^2\right]\right.$$

$$\left. + \left[\frac{L}{2\pi}\int dk\,\ell n\left(1 - e^{-\beta\sqrt{k^2+2m^2}}\right) - \sum_{k'}\ell n\left(1 - e^{-\beta\sqrt{k'^2+2m^2}}\right) - \ell n\left(1 - e^{-\beta\sqrt{\frac{3}{2}\,m^2}}\right)\right]\right\} \tag{4.26}$$

The two first factors appearing in the above exponential are divergent. In order to render these contributions finite we must renormalize them by adding the counterterm:

$$CT = \frac{1}{2}\int_{-\infty}^{+\infty} dx\left[\phi_C^2 - \frac{m^2}{\lambda}\right] \times 3\lambda\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\,\frac{1}{\sqrt{k^2+2m^2}} \tag{4.27}$$

In the limit of L going to infinity we have:

$$\mathrm{Im}\,R = \frac{T}{\sin\left(\frac{\beta\gamma}{2}\right)} \exp\left\{\frac{\beta}{2}\,\frac{1}{2\pi}\int_{-\infty}^{+\infty} dk\;\delta(k)\,\frac{d}{dk}\sqrt{k^2+2m^2} + \beta CT\right.$$

$$\left. - \int_{-\infty}^{+\infty}\frac{dk}{2\pi}\,\frac{d\delta}{dk}\,\ell n\left(1 - e^{-\beta\sqrt{k^2+2m^2}}\right) - \frac{3}{4}\,m^2\beta - \ell n\left(1 - e^{-\beta\sqrt{\frac{3}{2}\,m^2}}\right)\right\} \tag{4.28}$$

In the high temperature limit we have

$$\mathrm{Im}\,R = \frac{T}{\sin\left(\frac{\beta\gamma}{2}\right)} \exp\left\{+\int_{-\infty}^{+\infty}\frac{dk}{\pi}\,\sqrt{2}m\left(\frac{1}{k^2+2m^2} + \frac{1}{2k^2+m^2}\right)\ell n\left(1 - e^{-\beta\sqrt{k^2+2m^2}}\right)\right.$$

$$\left. - \ell n\left(1 - e^{-\beta\sqrt{\frac{3}{2}\,m^2}}\right) - \frac{3}{4}\,m^2\beta + \beta\,C.T.\right\} \tag{4.29}$$

The integral appearing in (4.29) behaves like $\ell n(\beta m)$ for $\beta \to 0$. In this example we also see that the most important contribution to $\frac{\Gamma}{L}$ for $\beta \to 0$ comes again from the pre-exponential facts mainly from the zero mode and the bound state. In this example we have shown that an extra power of T appears when one evaluates explicitly the pre-exponential factor and $\frac{\Gamma}{L}$ is much greater then what is expected if one uses the results claimed in the literature(11).

Our next example is a non renormalizable theory. However, this example will illustrate that the most important contribution to $\frac{\Gamma}{V}$ in the high temperature limit continues to come from the pre-exponential factor.

IV-C. The Birula Mycielski model(14)

Now we are going to repeat the calculation of IV-A-B for the Lagrangian density:

$$L_E = \frac{1}{2} (\partial_\tau \phi)^2 + \frac{1}{2} (\partial_x \phi) + V(\phi) \qquad (4.30)$$

where

$$V(\phi) = \frac{m^2 \phi^2}{2} \left[1 - \ell n \frac{\phi^2}{C^2}\right] \qquad (4.31)$$

For high temperatures $\left(T > \frac{m}{\pi\sqrt{2}}\right)$ the relevant static classical solution is given by:

$$\phi_C = C\sqrt{e} \exp\left(-\frac{m^2 x^2}{2}\right) \qquad (4.32)$$

The euclidean action associated to this field configuration is:

$$S_E(\phi_C) = \frac{e\sqrt{\pi}}{2T} C^2 m \qquad (4.33)$$

In order to calculate the determinant of the fluctuations we need to know the eigenvalues of $-\partial^2 + V''(\phi_C)$.

$$\left[-\partial^2 + V''(\phi_C)\right]\eta = \varepsilon\eta = (-\partial_{\tau\tau} + \partial_{xx} + m^4 x^2 - 3m^2)\eta \qquad (4.34)$$

It is easy to check that $\varepsilon_{n,\ell}$ is given by:

$$\varepsilon_{n,\ell} = \left(\frac{2\pi n}{\beta}\right)^2 + 2m^2(\ell - 1) \qquad (4.35)$$

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$\ell = 0, 1, 2, \ldots$

We are going to assume again that $\phi_{VAC} = \phi$ and that $V''(\phi_{VAC}) = m_0^2$. So the pre-exponential factor for this system is given by:

$$\text{Im}\, R = \frac{T}{\sin\left(\frac{m^2}{\sqrt{2}\, T}\right)} \times \exp\left\{\frac{\beta}{2}\left[\sum_k \sqrt{k^2 + m_0^2} - \sum_{\ell=2}^{\infty} \sqrt{2m^2(\ell-1)}\right] + \right.$$

$$\left. + \left[\sum_k \ell n\left(1 - e^{-\beta\sqrt{k^2+m_0^2}}\right) - \sum_{\ell=2}^{\infty} \ell n\left(1 - e^{-\beta\sqrt{2m^2(\ell-1)}}\right)\right]\right\} \qquad (4.36)$$

The first term between brackets in the exponential corresponds to the zero-point energy and due to that it will be neglected. Then

$$\text{Im}\, R = \frac{T}{\sin\left(\frac{m}{\sqrt{2}\, T}\right)} \exp\left\{\int_{-\infty}^{+\infty} dk\, \ell n\left(1 - e^{-\beta\sqrt{k^2+m_0^2}}\right) - \sum_{\ell=2}^{\infty} \ell n\left(1 - e^{-\beta\sqrt{2m^2(\ell-1)}}\right)\right\}$$

In the high temperature limit, we have:

$$\int_{-\infty}^{+\infty} dk\, \ell n\left(1 - e^{-\beta\sqrt{k^2+m_0^2}}\right) \cong 2\beta^{-1} \int_0^{\infty} d\mu\, \ell n\left(1 - e^{-\mu}\right)$$

and

$$\sum_{\ell=2}^{\infty} \ell n\left[1 - e^{-\beta\sqrt{2m^2(\ell-1)}}\right] \cong \frac{\beta^{-2}}{m^2}\int_0^{\infty} d\mu\ \mu\ \ell n\left[1 - e^{-\mu}\right]$$

Finally we obtain:

$$\frac{\Gamma}{L} \cong \left\{\frac{e}{4\sqrt{\pi}\ T}\ c^2 m\right\}^{1/2} \frac{2\ T^2}{\sin\left(\frac{m}{\sqrt{2}\ T}\right)} \exp\left\{-\ \frac{e\ \sqrt{\pi}}{2T}\ c^2 m\ + \right.$$

$$\left. +\ \frac{T^2}{m^2}\int_0^{\infty} d\mu\ \mu\ \ell n\left[1 - e^{-\mu}\right] + 2T\int d\mu\ \ell n\left[1 - e^{-\mu}\right]\right\} \qquad (4.37)$$

As can be seen from (4.37) the pre-exponential term exhibits terms which are comparable, in the high temperature limit to the so called exponential term.

V. (1+3) DIMENSIONAL EXAMPLE

The (1+3) dimensional system, that we are going to consider is described by the Euclidean Lagrangian density:

$$L_E = \frac{1}{2}\sum_{\ell=1}^{4} (\partial_i \phi)^2 + \epsilon\phi - \frac{m^2}{2}\ \phi^2 + \frac{\lambda\phi^4}{4} \qquad (5.1)$$

when ϵ, m^2, and λ are positive and ϵ is much less than 1.

We will proceed like we did in another example - that is, we are going to calculate $\frac{\Gamma}{V}$ to the lowest order in ϵ.

Expanding ϕ_C in powers of ϵ like in (4.20) and substituting into the classical equations of motion we obtain:

$$\phi_0 = \frac{m}{\sqrt{\lambda}}\ \tanh\left(\frac{mx}{\sqrt{2}}\right) \qquad (5.2)$$

ϕ_0 describes a domain wall (Bloch wall) in three spatial dimensions[15].

Although ϕ_0 given by (5.2) depends on just one spatial variable, one can show that it describes some important features of the bounce solution[17].

The eigenvalues of $-\partial^2 + V''(\phi_C)$ to the lowest order in ϵ are given by

$$\alpha_0 = \left(\frac{2\pi n}{\beta}\right)^2 + kz^2 + ky^2 + \begin{cases} 0 \\ \frac{3}{2}\ m^2 \\ k'^2 + 2m^2 \end{cases} \qquad (5.3)$$

It is possible to prove the existence[4] of a negative eigenvalue which we will denote by $-\omega^2$ and assume that it is unique.

After using (A.6) and renormalizing the result we obtain in the high temperature limit[15]:

$$\frac{\Gamma}{V} = 2\ T^4 \left(\frac{\sqrt{2}\ m^3}{2\pi 3\lambda T}\right)^{3/2} \frac{1}{\sin\left(\frac{\beta\omega}{2}\right)} \exp\left\{-\ \frac{2\sqrt{2}\ m^3\ A}{3\lambda T} + \frac{T\ A\ m}{2\sqrt{2}}\right\} \qquad (5.4)$$

where $A = V^{2/3}$, and V is the volume of the space.

This example only shows that our formal expression (3.11) works, as it should, in four dimensional problems.

VI. CONCLUSIONS

As pointed out in the introduction, it has been proposed that some aspects on the evolution of the early universe should be strongly dependent on the decay rate of the false vacuum. Phenomenological implications such as monopole density in the early universe and the Great Supercooling that the universe underwent are among the consequences of the vacuum decay process(3).

Some conclusions were drawn based on a fairly simple parametrization for the decay rate(10,11,13), namely

$$\frac{\Gamma}{V} = T^4 e^{-S_C/T} \left(\frac{S_C}{2\pi T}\right)^{3/2} \tag{6.1}$$

Other parametrizations for the decay rate are presented in ref. (12).

The behaviour proposed by expression (6.1) obviously do not take into account, within the semiclassical approximation, in a proper way the contribution coming from the determinant ratios in (1.11).

We have devised a method which allows us to infer the high temperature behaviour of the determinant ratios in (1.1) without solving the complete eigenvalue problem(15). The method relies on a simple graphical expansion which allows us to get the proper asymptotic behavior as well as to perform the renormalization of the determinant in a straightforward way.

As can be inferred from (3.11) and in contradiction to Ref. (11) our result changes drastically the expression for the decay rate at high temperatures. The correction we found coming

from the determinant ratio is more important, at high enough temperature than the classical one - the so called exponential term.

Obviously our results change the standard picture of the evolution of the early universe. The question is in which direction and by how much. We will be concerned with this problem in a future publication.

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APPENDIX A

Here we are going to obtain an expression for the imaginary part of the determinant ratio that appears in (1.1). We assume that $-\Delta + V''(\phi_C)$ has only one negative $(-\omega^2)$ eigenvalue and that there are z zero eigenvalues $[(-\Delta + V''(\phi_C))\eta_j = \lambda_j^2 \eta_j]$

$$R = \left[\frac{\prod'_{n,j} (\omega_n^2 + \lambda_j^{S^2})}{\prod_{n,j} (\omega_n^2 + \lambda_j^{V^2})} \right]^{-1/2} \qquad (A.1)$$

where $\omega_n^2 = \left(\frac{2\pi n}{\beta}\right)^2$.

It is easy to see that

$$R = \frac{\prod_j \lambda_j^V \prod_j \dfrac{\sinh \beta \lambda_j^V/2}{\beta \lambda_j^V/2}}{\prod'_j \lambda_j^S \prod_j \dfrac{\sinh \beta \lambda_j^S/2}{\beta \lambda_j^S/2}} \qquad (A.2)$$

where we have used the identity

$$\prod_{n=1}^{\infty} (1 + z^2/n^2) = \frac{\sinh \pi z}{\pi z} \qquad (A.3)$$

Now we have to notice that the negative eigenvalue makes R pure imaginary. Analyzing (A.2) with care we get that:

$$\mathrm{Im}\, R = \left(\frac{2}{\beta}\right)^z \frac{\prod_j \sinh(\beta \lambda_j^V/2)}{\sin \frac{\beta\omega}{2} \prod''_j \sinh(\beta \lambda_j^S/2)} \qquad (A.4)$$

where the double prime indicates that the negative and zero eigenvalue are excluded from the product.

We can further transform (A.4) using that

$$\ln \sinh(\beta z/2) = \frac{\beta z}{2} + \ln(1 - e^{-\beta z}) - \ln 2 \qquad (A.5)$$

in order to get

$$\mathrm{Im}\, R = \frac{T^z}{\sin\left(\frac{\beta\omega}{2}\right)} \exp\left\{ \frac{\beta}{2}\left[\sum_j \lambda_j^V - \sum_j{}'' \lambda_j^S\right] + \left[\sum_j \ln\left(1 - e^{-\beta\lambda_j^V}\right) - \sum_j{}'' \ln\left(1 - e^{-\beta\lambda_j^S}\right)\right]\right\} \qquad (A.6)$$

APPENDIX B

In this appendix we will analyze the temperature dependence of each term appearing in (3.2). First of all, we would like to point out that each graphic appearing in (3.2) have zero external momentum[18] - that is, for high temperatures:

$$(3.2) = \sum_{j=1}^{\infty} \frac{(-1)^j}{j} \int d^4xE\left[V''(\phi_C) - V''(\phi_{VAC})\right]^j \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^D\vec{k}}{(2\pi)^D} \frac{1}{\left[\left(\frac{2\pi n}{\beta}\right)^2 + V''(\phi_{VAC}) + \vec{k}^2\right]^j} \qquad (B.1)$$

where D is the number of spatial dimensions.

Lets obtain the dependence with β of

$$I_j = \frac{1}{\beta} \sum_n \int \frac{d^D\vec{k}}{(2\pi)^D} \frac{1}{\left[\left(\frac{2\pi n}{\beta}\right)^2 + k^2 + m^2\right]^j}, \qquad (B.2)$$

where $m^2 = V''(\phi_{VAC})$, when $\beta \to 0$.

Performing the scaling $\vec{t} = \beta\vec{k}$ we can write

$$I_j = \beta^{2j-(D+1)} \sum_n \int \frac{d^D\vec{t}}{(2\pi)^D} \frac{1}{\left[(2\pi n)^2 + t^2 + m^2\beta^2\right]^j} \tag{B.3}$$

Now it is easy to see that:

$$I_j = \frac{(-1)^{j-1}}{(j-1)!} \beta^{j-D} \left(\frac{1}{2m}\right)^{j-1} \frac{d^{j-1}}{dx^{j-1}} f(x) \bigg|_{x=m\beta} \tag{B.4}$$

where

$$f(x) = \sum_n \frac{d^D t}{(2\pi)^3} \frac{1}{\left[(2\pi n)^2 + t^2 + x^2\right]} . \tag{B.5}$$

For $D \geq 3$ we have that

$$\lim_{\beta \to 0} f(m\beta) = \text{constant} .$$

Then, for $D \geq 3$, the term $j=1$ is the most important term of (B.1) in the limit $\beta \to 0$.

If we have $D < 3$, $f(x)$ diverges as x goes to zero due to the infrared of the theory. For example for $D=1$

$$\lim_{\beta \to 0} f(m\beta) \cong \frac{C}{m\beta} .$$

Using (B.4) we get that is proportional to β^{-1}. Since all terms in the series (B.1) have the same temperature dependence with temperature, we have to sum the whole series then our formal expansion (3.2) (B.1) does not lead to a simple result to the determinant ratio.

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