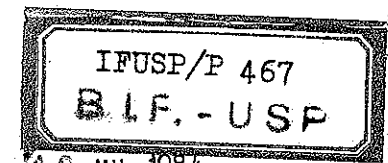


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EXCHANGE THREE-NUCLEON POTENTIAL

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ABSTRACT

We argue that the straightforward introduction of pion-nucleon form-factors into the s-wave component of the two-pion exchange three-body force derived by means of chiral symmetry leads to inconsistencies. These can be avoided by means of a redefinition of the potential which considers its physical content.

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I - Introduction

Accurate calculations of properties of three-nucleon systems have shown that they cannot be ascribed entirely to the nucleon-nucleon interaction. Indeed, various calculational techniques with realistic two-body forces yield results which consistently disagree with experiment. The present theoretical understanding of the problem suggests that at least part of this discrepancy may be explained by means of three-nucleon forces <sup>(1,2)</sup>.

The longest range three-nucleon potential is that due the exchange of two pions. It is denoted by  $\pi\pi E-3NP$  and corresponds to the process represented in fig.1. Since the pioneering work of Fujita and Miyazawa <sup>(3)</sup> it has been realized that this force contains terms originating from both s and p waves in the intermediate  $\pi N$  amplitude. The strength of the s wave component of the three-nucleon force ( $W_g$ ) was originally assumed to be proportional to the isospin even  $\pi N$  scattering length, which is rather small. This way of treating the problem inaugurated a tradition whereby the terms of the force due to p waves were considered to be largely dominant. This tradition lasted until the recent derivations of potentials based on chiral symmetry <sup>(4,5)</sup>, where the contributions of intermediate s and p waves were shown to be comparable. The use of chiral symmetry is important because it ensures that the pions are correctly described in all the relevant kinematic regions.

In the Tucson-Melbourne (TM) potential <sup>(4)</sup> chiral symmetry

has been implemented by means of current algebra in a model independent way, as pointed out in ref. (2). Unfortunately, this advantage is associated with an intrinsic lack of clarity regarding the dynamical implications of the soft pion limit<sup>(6)</sup>. The alternative approach to the implementation of chiral symmetry is based upon effective lagrangians and has recently been used by ourselves in a parallel derivation of the three-body potential<sup>(5)</sup>. These chiral lagrangians are not supposed to describe the fundamental hadronic interactions. Instead, they are just quick and efficient tools for producing results equivalent to those of current algebra. The advantage of the use of lagrangians is that it makes possible a clear understanding of the dynamical origins of the various contributions to the potential and hence is well suited for guiding one's intuition.

The most controversial aspect of the lagrangian approach concerns the description of the pion-nucleon  $\sigma$ -term. In the case of current algebra this contribution comes from the equal-time commutator of an axial current and its divergence. In the effective Lagrangian approach this contribution cannot be ascribed to the exchange of realistic particles or resonances, since no serious candidate for the sigma field seems to exist. Therefore the usual procedure consists in considering this contribution by means of a parametrized form<sup>(7)</sup>.

The  $\sigma$ -term contributes only to the function  $A^+$  of the relativistic amplitude for the process  $\pi^a(k) N(p) \rightarrow \pi^b(k') N(p')$ , denoted by  $T_{\pi N}^{ab}$  and whose general form is

$$T_{\pi N}^{ab} = \bar{u}(\vec{p}') \left[ \left( A^+ + \frac{K'+K}{2} B^+ \right) \delta_{ab} + \left( A^- + \frac{K'+K}{2} B^- \right) i \epsilon_{bac} \tau_c \right] u(\vec{p}). \quad (1)$$

In our derivation of the  $\pi\pi E-3NP$ <sup>(5)</sup> we have used the following form for the  $\sigma$ -contribution.

$$A_{\sigma}^+ = \alpha_{\sigma} + \beta_{\sigma} k \cdot k' \quad (2)$$

where  $\alpha_{\sigma}$  and  $\beta_{\sigma}$  are constants extracted from experiment. This form has been taken from ref. (7) and it is adequate for describing the scattering of free pions. When the pions are not free one has to include off-shell effects and the above form has to be modified, as it has been correctly pointed out in ref. (2). The parametrization adopted in the TM potential does not suffer from these difficulties and is consistent with the theoretical single and double soft pion limits of the intermediate  $\pi N$  amplitude. It is equivalent to the following form for  $A_{\sigma}^+$

$$\begin{aligned} A_{\sigma}^+ &= \frac{\sigma}{f_{\pi}^2} \left[ (1-\beta) \left( \frac{k^2+k'^2}{\mu^2} - 1 \right) + \beta \left( \frac{t}{\mu^2} - 1 \right) \right] \\ &= \frac{\sigma}{f_{\pi}^2} \left[ 1 - 2\beta \frac{k \cdot k'}{\mu^2} + \frac{1}{\mu^2} (k^2 - \mu^2) + \frac{1}{\mu^2} (k'^2 - \mu^2) \right], \quad (3) \end{aligned}$$

where  $\sigma$  is the pion-nucleon  $\sigma$ -term, that can be extracted from experiment.

When we make the identifications  $\alpha_{\sigma} = (\sigma/f_{\pi}^2)$  and  $\beta_{\sigma} = -(2\beta/\mu^2) (\sigma/f_{\pi}^2)$  it is possible to see that eqs. (2) and

(3) differ only by terms proportional to  $(k^2 - \mu^2)$  and  $(k'^2 - \mu^2)$ , describing off shell effects. In the derivation of the three-nucleon force these terms cancel pion propagators. Thus eqs. (2) and (3) yield potentials in coordinate space that differ only by terms proportional to  $\delta$ -functions. In the absence of form factors the short range repulsion between nucleons allows us to expect that these contact interactions would have very little influence on the numerical results of the calculations. However, actual calculations do require the use of form factors, because the short distance behaviour of the 3NP is not determined by the use of chiral symmetry. As discussed in refs. (5,8), the singular behaviour of the s and p components of the potential without form factors is responsible for unphysical nodes in the trinucleon wave-function.

The discussion of the problems associated with the inclusion of form factors in the s-wave component of the 3NP is found in sect. II and constitutes the main subject of this paper. Our conclusions are presented in sect. III.

## II - The influence of form factors on $W_s$

The  $\pi\pi E$ -3NP derived in refs. (4) and (5) is based on the contribution of the exchange of two pions to the elastic scattering of three unbound nucleons. This process corresponds to permutations of the indices of the diagram of Fig.1, where the definitions of the kinematical variables can be found. In this figure  $T_{\pi N}$  is the amplitude for the process  $\pi N \rightarrow \pi N$ , whose

general form is given by eq. (1). Thus the formal expression for the relativistic amplitude  $T_{3N}$ , describing the contribution of the  $\pi\pi E$  to the three-nucleon interaction, is

$$T_{3N} = [\bar{u}(\vec{p}_2') \not{K} \gamma_5 \tau_a u(\vec{p}_2)] \frac{g/2m}{k^2 - \mu^2} \left\{ \bar{u}(\vec{p}_1') \left[ \left[ A^+ + \frac{K' + \not{K}}{2} B^+ \right] \delta_{ab} + \right. \right. \\ \left. \left. + \left[ A^- + \frac{K' + \not{K}}{2} B^- \right] i \epsilon_{bac} \tau_c \right] u(\vec{p}_1) \right\} \frac{g/2m}{k'^2 - \mu^2} \left[ u(\vec{p}_3') \not{K}' \gamma_5 \tau_b u(\vec{p}_3) \right] \quad (4)$$

where  $g$  denotes the  $\pi N$  coupling constant and  $\mu$  and  $m$  are, respectively, the pion and nucleon masses.

Among the various contributions to  $T_{\pi N}^{ab}$  is that of a nucleon propagating forward in time, which must not be included in the  $\pi\pi E$ -3NP, since it corresponds to an iteration of the two-body potential. The subtraction of this contribution is denoted by the symbol (-) on top of the appropriate quantities. The potential is a meaningful concept for non-relativistic nucleons. In the evaluation of  $t_{3N}$ , the non-relativistic limit of the amplitude given by eq. (4), the nucleon three-momenta are assumed to be typically of the order  $\mu$ . We have

$$\bar{t}_{3N} = \frac{g}{k^2 + \mu^2} \frac{g}{k'^2 + \mu^2} (\bar{\sigma}^{(2)} \cdot \vec{k}) (\bar{\sigma}^{(3)} \cdot \vec{k}') \tau_a^{(2)} \tau_b^{(3)} \\ \times \{ [2mf^+ + i\bar{\sigma}^{(1)} \cdot (\vec{k}' \times \vec{k}) \bar{b}^+] \delta_{ab} + [2mf^- + i\bar{\sigma}^{(1)} \cdot (\vec{k}' \times \vec{k}) \bar{b}^-] i \epsilon_{bac} \tau_c^{(1)} \} \quad (5)$$

where  $\bar{\sigma}^{(i)}$  and  $\bar{\tau}^{(i)}$  indicate expectation values,

$$\vec{f}^\pm \equiv \vec{a}^\pm + \frac{1}{4m} (\vec{p}_1 + \vec{p}'_1) \cdot (\vec{k} + \vec{k}') \vec{b}^\pm$$

and  $\vec{a}^\pm$  and  $\vec{b}^\pm$  are the non relativistic reductions of the sub amplitudes  $\vec{A}^\pm$  and  $\vec{B}^\pm$ .

The potential in momentum space is defined as the following function of  $t_{3N}$

$$\langle \vec{p}'_1 \vec{p}'_2 \vec{p}'_3 | W^{123} | \vec{p}_1 \vec{p}_2 \vec{p}_3 \rangle = -(2\pi)^3 \delta^3(\vec{p}_f - \vec{p}_i) \frac{1}{8m^3} t_{3N}. \quad (7)$$

The final form for the three-body potential in coordinate space is obtained by Fourier transforming eq. (7), as found in refs. (4) and (5). The mathematical structure of the potential in coordinate space is such that each of its terms can be seen as the product of a strength parameter (with dimension of energy), an isospin operator and a spin operator coupled to derivatives that act on Yukawa functions. The expression for  $W_g$ , for instance, is given in eq. (11) below. The Yukawa functions are proportional to the Fourier transform of the pion propagator and they become infinite when the internucleon distances tend to zero. Therefore they have to be regularized by means of form factors before being realistically applied. From a formal point of view, this can be done by allowing the coupling constant to become momentum dependent:  $g \rightarrow g(k^2) \equiv g\bar{G}(k^2)$ , where the function  $\bar{G}$  is such that  $\bar{G}(\mu^2)=1$ . It is worth pointing out that this way of introducing form factors is not prescribed by chiral symmetry. Rather, it corresponds to an ad hoc phenomenological correction to the results obtained through the use of this symmetry.

The introduction of form factors must be done in such a way as to influence only the short distance behaviour of the potential and, in no way whatsoever, its long distance proper. A striking fact about  $W_g$  is that it fails to bear out this important expectation.

The derivations of the  $\pi N$ -3NP produced in refs. (4) and (5) have shown that the only s-wave contribution from the intermediate  $\pi N$  amplitude that survive in  $W_g$  is that due to the  $\sigma$  term. The s-wave contribution to the  $\sigma$  term is obtained from eq. (3) by setting  $\beta$  equal to zero.

The details of the calculation of  $W_g$  can be found in refs. (4) and (5) and will not be reproduced here. We just quote the result in momentum space for the diagram corresponding to fig.1.

$$W_g^{23} = - \frac{1}{4m^2} \frac{\sigma}{f_\pi^2} (\vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) (\vec{\sigma}^{(2)} \cdot \vec{k}) (\vec{\sigma}^{(3)} \cdot \vec{k}') \left[ -1 - \frac{\vec{k}^2 + \vec{k}'^2}{\mu^2} \right] \\ \times \frac{g}{k^2 + \mu^2} \frac{g}{k'^2 + \mu^2}. \quad (9)$$

We should notice that the above equation differs from the corresponding one in ref. (5) since a different expression for  $A_\sigma^+$  (see eq. (3)) has been considered, as the results of the discussion contained in the introduction of this work. This potential can also be rewritten as

$$W_S^{23} = -\frac{1}{4m^2} \frac{\sigma}{f_\pi^2} (\vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) (\vec{\sigma}^{(2)} \cdot \vec{k}) (\vec{\sigma}^{(3)} \cdot \vec{k}') \times \left[ \frac{g}{\vec{k}^2 + \mu^2} \frac{g}{\vec{k}'^2 + \mu^2} - \frac{1}{\mu^2} \left( g \frac{g}{\vec{k}^2 + \mu^2} + \frac{g}{\vec{k}'^2 + \mu^2} g \right) \right] \quad (10)$$

Introducing form factors and performing the Fourier transform we have

$$W_S^{23} = \left[ \frac{g}{2m} \right]^2 \left[ \frac{1}{4\pi} \right]^2 \frac{\sigma}{f_\pi^2} (\vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) (\vec{\sigma}^{(2)} \cdot \vec{v}_{12}) (\vec{\sigma}^{(3)} \cdot \vec{v}_{31}) \times \left\{ U(r_{12}) U(r_{31}) - [G(r_{12}) U(r_{31}) + U(r_{12}) G(r_{31})] \right\} \quad (11)$$

where  $r_{ij}$  represents the distance between nucleons  $i$  and  $j$ , and the functions  $U(r)$  and  $G(r)$  are given by

$$U(r) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i \vec{k} \cdot \vec{r}}}{\vec{k}^2 + \mu^2} \bar{G}(k^2) \quad (12)$$

$$G(r) = \frac{4\pi}{\mu^3} \int \frac{d\vec{k}}{(2\pi)^3} e^{-i \vec{k} \cdot \vec{r}} \bar{G}(k^2) \quad (13)$$

The expression for  $U(r)$  represents a Yukawa function regularized at the origin whereas  $G(r)$  is related to the distribution of hadronic matter within the nucleon. In the remainder of this work we will be mostly concerned with the role of  $G(r)$  in the study of trinucleon systems. For the purpose of this discussion we consider the following dipole

form for  $\bar{G}(k^2)$

$$G(k^2) = \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 - k^2} \right)^2 \quad (14)$$

The motivation for this choice is that this form has been used in a large number of works<sup>(4,5,9,10)</sup>. The discussion presented below is, however, general enough to be easily extended to other parametrizations of  $\bar{G}(k^2)$ .

In order to illustrate the importance of  $G(r)$  for the trinucleon system, we display in fig. 2 the equipotential plots for the expectation value of  $W_S$  between totally antisymmetric spin and isospin states, that is given by<sup>(5)</sup>

$$\langle W_S \rangle = - \left[ \frac{g\mu}{2m} \right]^2 \left[ \frac{1}{4\pi} \right]^2 \mu^2 \frac{\sigma}{f_\pi^2} (\vec{r}_{12} \cdot \vec{r}_{31})$$

$$\times \left\{ \frac{1}{\mu} \frac{\partial U(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial U(r_{31})}{\partial r_{31}} - \left[ \frac{1}{\mu} \frac{\partial G(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial U(r_{31})}{\partial r_{31}} + \frac{1}{\mu} \frac{\partial U(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial G(r_{31})}{\partial r_{31}} \right] \right\} + \text{cyclic permutations} \quad (15)$$

Following the work of Brandenburg and Glöckle<sup>(10)</sup>, we construct these diagrams by fixing the positions of two nucleons and using the third one as a probe. The fixed internucleon distance is taken to be  $x=0.88$  fm, corresponding to the minimum of the Reid two-body potential<sup>(11)</sup>. In order to be able to follow the dependence of eq. (15) on  $\Lambda$  we vary this parameter in six steps between infinity and the realistic value of  $4\text{fm}^{-1}$ . The first value corresponds to the elimination of the form factor

$$W_S^{23} = -\frac{1}{4m^2} \frac{\sigma}{f_\pi^2} (\vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) (\vec{\sigma}^{(2)} \cdot \vec{k}) (\vec{\sigma}^{(3)} \cdot \vec{k}') \\ \times \left[ \frac{g}{\vec{k}^2 + \mu^2} \frac{g}{\vec{k}'^2 + \mu^2} - \frac{1}{\mu^2} \left( g \frac{g}{\vec{k}^2 + \mu^2} + \frac{g}{\vec{k}'^2 + \mu^2} g \right) \right] \quad (10)$$

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$$U(r) = \frac{4\pi}{\mu} \int \frac{d\vec{k}}{(2\pi)^3} \frac{e^{-i\vec{k} \cdot \vec{r}}}{\vec{k}^2 + \mu^2} \bar{G}(k^2) \quad (12)$$

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$$\times \left\{ \frac{1}{\mu} \frac{\partial U(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial U(r_{31})}{\partial r_{31}} - \left[ \frac{1}{\mu} \frac{\partial G(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial U(r_{31})}{\partial r_{31}} + \frac{1}{\mu} \frac{\partial U(r_{12})}{\partial r_{12}} \frac{1}{\mu} \frac{\partial G(r_{31})}{\partial r_{31}} \right] \right. \\ \left. + \text{cyclic permutations} \right\} \quad (15)$$

Following the work of Brandenburg and Glöckle<sup>(10)</sup>, we construct these diagrams by fixing the positions of two nucleons and using the third one as a probe. The fixed internucleon distance is taken to be  $x=0.88$  fm, corresponding to the minimum of the Reid two-body potential<sup>(11)</sup>. In order to be able to follow the dependence of eq. (15) on  $\Lambda$  we vary this parameter in six steps between infinity and the realistic value of  $4\text{fm}^{-1}$ . The first value corresponds to the elimination of the form factor

and yields

$$G(r) = \frac{4\pi}{\mu^3} \delta^3(\vec{r}) \quad (16)$$

In this case the terms proportional to  $G(r)$  in eq. (14) represent interactions of two nucleons at the same point in space.

The first and last plots of fig. 2 are strikingly different. This fact is unexpected, since the diagrams for realistic value of  $\Lambda$  should be just modified versions of the one without form factors, the modifications being confined to small internucleon distances. In the present case, the form factors that should just correct the potential, as a matter of fact, determine its most important features. These results are even more disturbing when we remind ourselves that form factors correspond to relativistic effects. For instance, in the parametrization of  $G(k^2)$  used in this work we have, for realistic values of  $\Lambda: \vec{k}^2/\Lambda^2 \approx \mu^2/m^2 \approx 1/50$ . On the other hand, the form of  $W_S^{2,3}$  given by eq. (10) has been derived under the assumption that the nucleons are non-relativistic. Thus the effects introduced by form factors are, in principle, of the same order of magnitude as other neglected throughout the calculation of  $W_S$ . That relativistic corrections dominate  $W_S$  is a clear indication of an inconsistency with the non-relativistic hypothesis that led to the final form of the potential.

The origin of this inconsistent behaviour can be traced back to the terms proportional to the function  $G(r)$  in eq. (10), which are within square brackets. When no form factor is present

these terms are proportional to  $\delta$ -functions and hence their spacial influence is a very restricted one. However, when we adopt realistic values for  $\Lambda$  this influence extends to a very large region and in fact dominates the contribution from the other term. This excessive influence of  $G(r)$  on  $W_S$  can be determined directly from eq. (10), since its contribution relative to that of  $U(r)$  can be studied by means of the function  $R(r) = \ln[|\partial G/\partial r| / (|\partial U/\partial r|)]$ . The plot of this function for  $\Lambda = 5\text{fm}^{-1}$  is shown in fig. 3, where we note that the influence of the form factor is not confined to small internucleon distances.

The physical meaning of the terms proportional to  $G(r)$  in eq. (10) is given by the dynamical content of the  $\pi N$  form factor. When no form factors are present,  $G(r)$  is given by eq. (16) and the terms of  $W_S$  proportional to this function describe contact interactions between two nucleons, corresponding to permutations of the configuration space diagram of fig. 4a. In this figure the  $\sigma$  has been represented as a propagating particle for the sake of clarity but, in fact, it corresponds to a contact interaction, that can be formally obtained by ascribing a very large mass to the  $\sigma$ . The broken lines represent the pion propagation between two different points in space and are associated to the Yukawa functions.

When we consider form factors, the function  $\bar{G}(\vec{k}^2)$  is not equal to one, the nucleons are no longer point-like, and we have "contact" interactions between extended objects. In order to make this statement more precise, we consider the dynamical content of the  $\pi N$  form factor. Within the context of the chiral  $SU(2) \times SU(2)$  group, it corresponds to diagrams such as those of



fig. 4b. So, by "contact" interactions between extended objects we mean the processes represented in fig. 4c.

The inclusion of the function  $G(r)$  into the potential means that we are considering forces whose dynamical content remains hidden behind a parametrization. This makes it difficult to understand which are the Feynman diagrams one is including in the potential. Of course, the diagrams depicted in

fig. 4c. should be evaluated at some stage of the research program on three-body forces. However, their inclusion should be the result of explicit calculations, using an appropriate dynamics such as chiral symmetry. Moreover, in this research program, the study of many other processes such as pion-rho, rho-rho, pion-omega, three-pion exchanges should precede those of fig. 4c., since they correspond to forces of longer range.

Thus, we see that the three nucleon potential given by eq. (10) is composed of two types of terms. One of them, containing the Fourier transform of the form factor, describes a "contact" interaction between two extended nucleons and is associated with the exchange of several different particles. The other one contains only functions  $U(r)$ , that correspond to the propagation of pions between different points in space. The considerations produced in this paper show that only the latter deserves the name of  $\pi\pi E-3NP$  since it can be interpreted as describing the exchange of pions between different points in space. The former term, on the other hand, describes a  $\pi?E-3NP$ , where (?) denotes all the particles included under the cover of the parameter  $\lambda$ .

All the problems mentioned above regarding  $W_S$  can be

to the actual different-point in space propagation of two pions. This corresponds to the elimination of  $G(r)$  from eq. (11) and we obtain

$$W_S^{23} = \left[ \frac{\sigma}{2m} \right]^2 \left[ \frac{1}{4\pi} \right]^2 \frac{\sigma}{f_\pi^2} (\vec{\tau}^{(2)} \cdot \vec{\tau}^{(3)}) (\vec{\sigma}^{(2)} \cdot \vec{\nabla}_{12}) (\vec{\sigma}^{(3)} \cdot \vec{\nabla}_{31}) U(r_{12}) U(r_{31}) \quad (17)$$

where the symbol ( $\sim$ ) indicates the above modification.

This exclusion of  $G(r)$  from eq. (10) amounts to saying that we should regularize the results of chiral symmetry by eliminating all the possible  $\delta$ -functions before the inclusion of the form-factors. In doing the opposite we would be using form-factors to regularize a  $\delta$ -function. The absence of  $\delta$ -functions in the parts of the potential due to p-waves in the intermediate  $\pi N$  system means that they do not suffer from the same difficulties as  $W_S$ .

The modifications induced into  $\hat{W}_S$  by the inclusion of form factors can be followed by inspecting fig. 5, the analog of fig. 2 for this redefined version of the potential. We note that now the difference between the various plots is much less pronounced and so the influence of the form factor tends to be confined, as it should, to small internucleon distances.

Inspecting figs. 2 and 5 we note that the potential corresponding to the former favours the triangular configuration, whereas the latter has much less structure and is mostly repulsive. These features have definite consequences for the trinucleon binding energy. In table 1 we display the values of the contribution of  $W_S$  to the binding energy of  ${}^3H$  and  ${}^3He^{(5)}$ , evaluated by means of the hyperspherical harmonic method <sup>(12)</sup>.

In this calculation we have considered only the fully symmetric S wave ground state, since we are mostly interested in the qualitative features of the problem.

### III- Conclusions

In this work we have shown that the straightforward inclusion of realistic dipole form factors into the s-wave component of the  $\pi\pi E$ -3NP derived by means of chiral symmetry is the cause of several undesired features. One of them is that the form factor does not just regularize the potential close to the origin. Instead, it determines the its form in a much larger region. This behaviour is not consistent with the hypothesis of non-relativistic nucleons which is basic for the derivation of the potential.

The origin of this peculiar quality is traced to a term in the potential that becomes a  $\delta$ -function in the absence of form factors and hence can be thought as a "contact" interactions between extended nucleons. An interaction of this kind is prevented by the repulsive core of the nucleon-nucleon force. Moreover, it has the same range as the exchange of various heavier bosons that have not been included dynamically into the calculation. These problems suggest that, in a conservative approach to the problem, we should redefine the  $\pi\pi E$ -3NP as the potential due to the actual propagation of pions between different points in space. The redefined version of the potential does not suffer from the

problems mentioned above. The results presented in this work show that the redefined s-wave component of the three-body potential due to the exchange of two pions is mostly repulsive.

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TABLE 1

Influence of  $W_S$  on the binding energies of trinucleons.

Nucleus	Form of $W_S$	$\Delta$ BE (MeV)
${}^3\text{H}$	eq. (11)	+0.939
	eq. (17)	-0.203
${}^3\text{He}$	eq. (11)	+0.906
	eq. (17)	-0.197

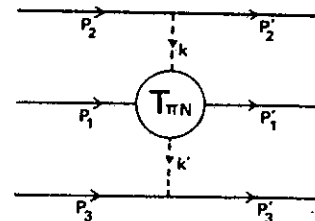


Fig.1 - Basic diagram contributing to the  $\pi N E-3NP$ .

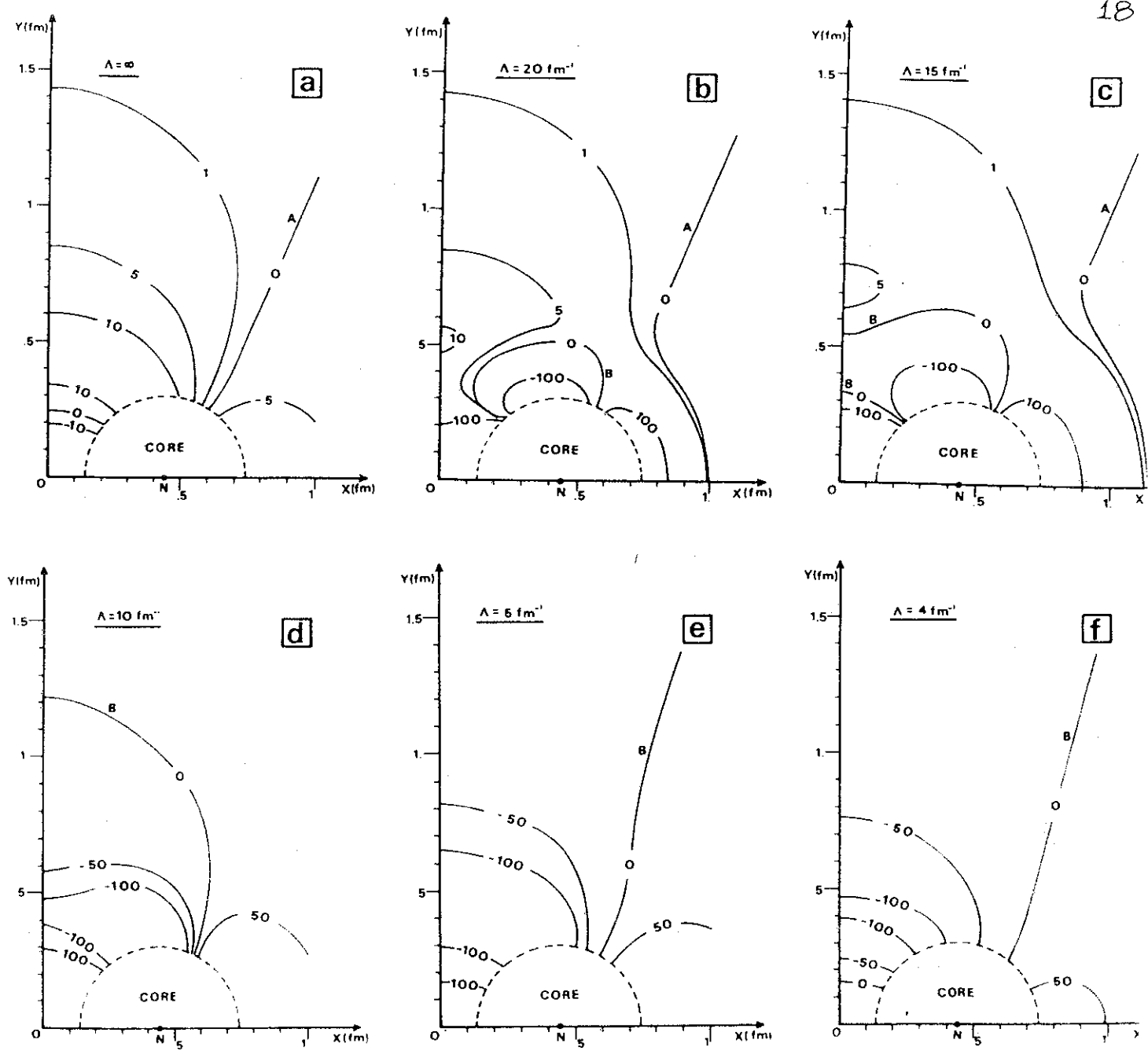


Fig.2 - The influence of form factors on the TM s-wave potential. The equipotential curves are symmetric about the X and Y axes. All energies are given in MeV. The point N indicates the position of one of the fixed nucleons; the other one is located symmetrically about the origin. The various values of  $\Lambda$  are shown in the figures;  $\Lambda = \infty$  corresponds to the absence of form factors. The understanding of these figures becomes easier when the evolution of the lines labelled with A and B is followed.

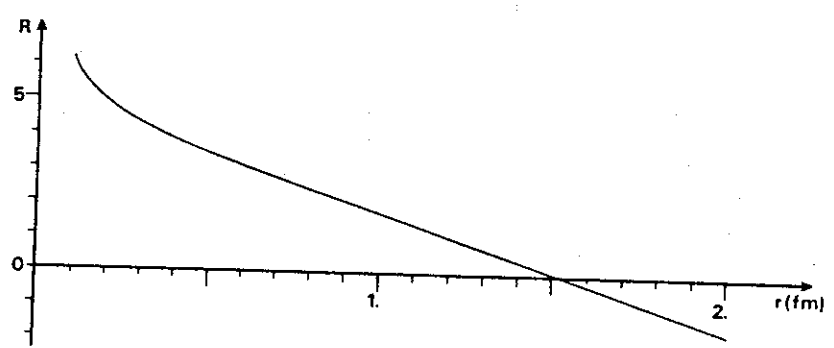


Fig.3 - Behaviour of the function  $R = \ln \left[ \frac{\partial G}{\partial r} / \left( \frac{\partial U}{\partial r} \right) \right]$  .

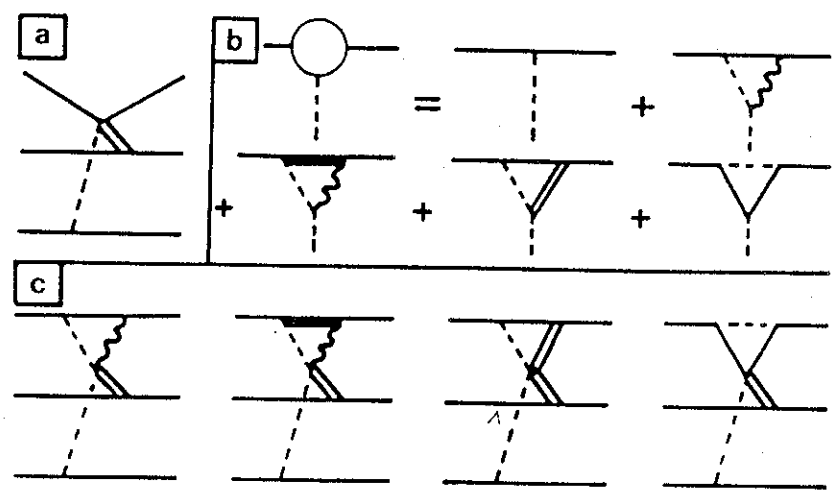


Fig.4 - a. Contact interaction between two point-like nucleons.  
 b. Some diagrams contributing to the pion-nucleon form factor.  
 c. "Contact" interaction between "extended" nucleons. In these figures continuous and thick lines represent nucleons and deltas, whereas broken, wavy and double lines represent pions, rhos and sigma.

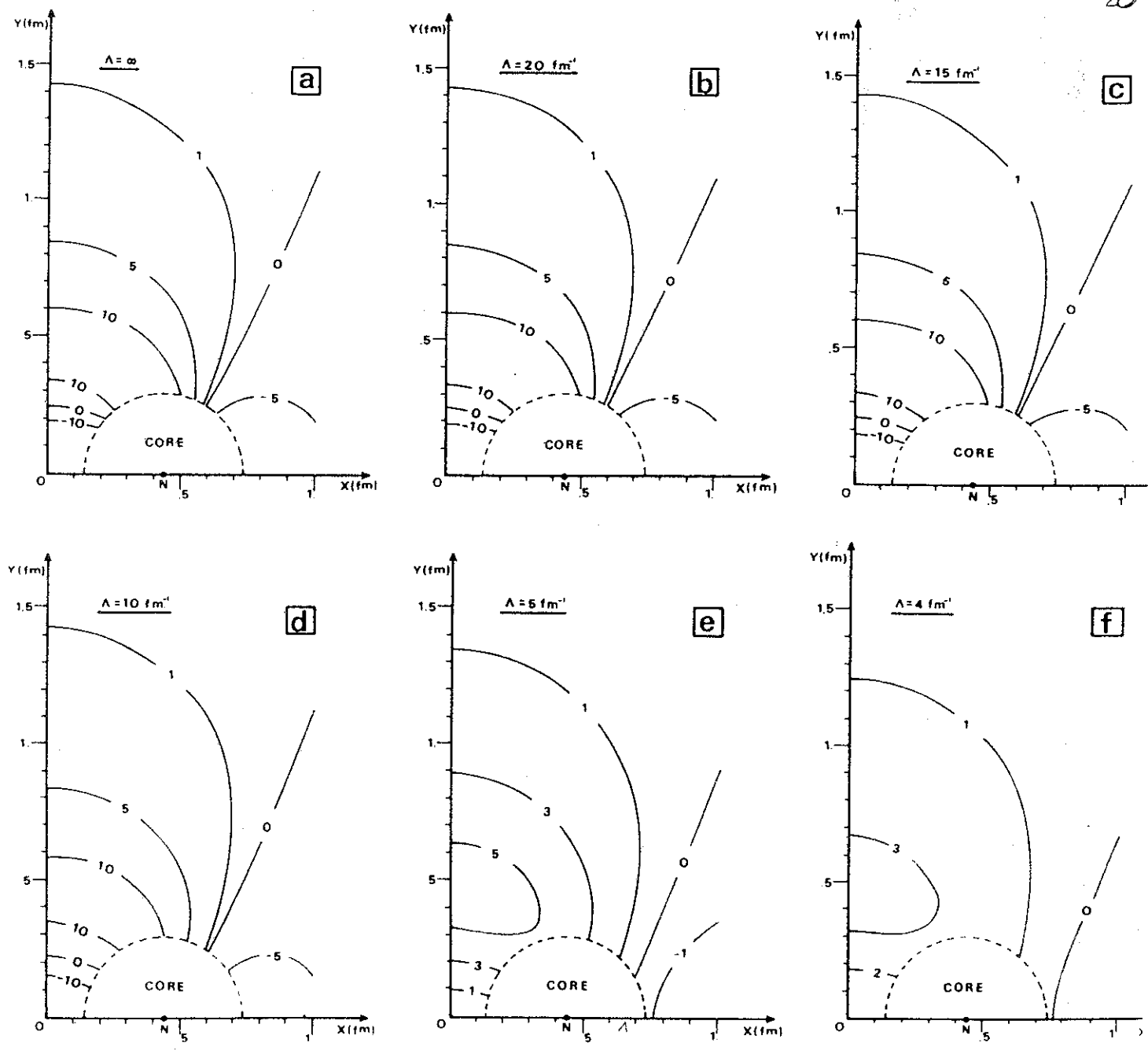


Fig.5 - The influence of form factors on the redefined s-wave potential. The equipotential curves are symmetric about the X and Y axes. All energies are given in MeV. The point N indicates the position of one of the fixed nucleons; the other one is located symmetrically about the origin. The various values of  $\Lambda$  are shown in the figures;  $\Lambda = \infty$  corresponds to the absence of form factors.