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PERCOLATION TEMPERATURE AND THE "INSTABILITY" OF THE EFFECTIVE POTENTIAL*

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ABSTRACT

We show that in spontaneously broken $\lambda\phi^4$ theory the percolation temperature coincides with the temperature at which the semiclassical (loop) expansion of the effective potential (free energy) of the system around a uniform field configuration fails. This allows us to extract the percolation temperature directly from the effective potential. The addition of fermions or gauge fields does not alter the result as long as they are weakly coupled to the scalars. The coincidence holds in the high temperature limit at every order in the loop expansion.

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I - INTRODUCTION

The question of whether a field theoretical model at finite temperature exhibits spontaneous symmetry violation may be studied with the help of the finite temperature effective action, $\Gamma(\beta,M_J(x))$. This quantity is the Gibbs free energy of the system and may be defined through equations [1]:

$$Z(\beta,J(x)) \equiv N^{-1} \oint [D\phi] \exp\left\{-\int_0^\beta d\tau \int d\overset{\nu}{x} \left[\mathcal{L}_E^{eff} - J(x)\phi(x)\right]\right\} \quad (1.1)$$

$$F(\beta,J(x)) \equiv -\beta^{-1} \ln Z(\beta,J(x)) \quad (1.2)$$

$$M(x,J(x)) \equiv M_J(x) \equiv -\frac{\delta(\beta F)}{\delta J(x)} \quad (1.3)$$

$$\Gamma(\beta,M_J(x)) \equiv F(\beta,J(x)) + \beta^{-1}\int_0^\beta d\tau \int d\overset{\nu}{x}\, M_J(x)\phi(x) \quad (1.4)$$

$$\frac{\delta\Gamma}{\delta M_J(x)} = J(x) \quad (1.5)$$

$\Gamma(\beta,M_J(x))$ is the generating functional for one-particle irreducible Green's functions. In the case of $\lambda\phi^4$ theory, whose Hamiltonian processes the Z(2) - symmetry ($\phi\to-\phi$) one should expect $M=0$ at $J=0$ if the symmetry is realized in the space of states. Spontaneous symmetry violation is characterized by a nonvanishing value of $M(x)$ at $J=0$, ie, for which (1.5) is satisfied with J=0. We may take $M_J(x)$ to be x-independent or, equivalently, restrict the analysis to x-independent external currents, $J \neq J(x)$, as translational invariance is not expected to be broken. Thus, one looks for solutions of:

$$\left. \frac{\partial V(\beta,\bar{M})}{\partial \bar{M}} \right|_{\bar{M} = \bar{M}_V(\beta)} = 0 \qquad (1.6)$$

where $V(\beta,\bar{M}) \equiv [\beta*(\text{spatial volume})]^{-1}\, \Gamma(\beta,\bar{M})$ and the bar denotes x-independence.

Theories whose symmetry is spontaneously broken at zero temperature can have this symmetry restored at higher temperatures. One should then expect:

$$\bar{M}_V(\beta_1) < \bar{M}_V(\beta_2) \text{ for } \beta_1 < \beta_2 \qquad (1.7)$$

so that there may be a critical temperature at which:

$$\bar{M}_V(\beta_c) = 0 \qquad (1.8)$$

The situation described by (1.7) is the one that emerges when one analyzes the behavior of the minima of the effective potential as a function of temperature. Property (1.8), which determines the critical temperature, has been claimed to be obtainable from a one-loop analysis of the effective potential.

In this paper we make a detailed analysis of the remark made in reference [2] that there exists a limiting

temperature, T_L, beyond which the semiclassical (loopwise) evaluation, based on uniform background fields, of the minima of the effective potential becomes unreliable. Unfortunately, estimates of the critical temperature obtained from one-loop calculations of that type yield values greater than T_L and, therefore, lie outside their expected domain of validity.

The shortcomings of the loop expansion of the effective potential in the broken phase around uniform backgrounds had already been noticed by several authors who investigated the T=0 problem[3 - 6]. Recent suggestions to overcome them may be found in references [4 - 6]. This point was also made in our analysis of the finite temperature case[2] where we presented an alternative way of calculating transition temperatures within the semiclassical framework. Most of the attempts to remedy this situation, ours included, make appeal to nontrivial extrema of the action as background fields for the semiclassical expansion. In our case, a spatially-dependent kink solution describing a domain wall was used to estimate a "percolation" temperature, through a Peierls-type argument, in a calculation analogous to the one done by Ventura in ref. [7].

Our basic claim is as follows: if one computes the free energy difference per unit area of a domain wall with respect to that of a uniform background, by calculating the effective action in a semiclassical expansion around this non-uniform background, one finds that it is positive at low temperatures but vanishes at the percolation temperature, T_p; this temperature <u>coincides</u> with the temperature T_L, defined before, at which the minima of the effective potential reach the boundary of the region where the semiclassical result is complex (this occurs because the zero-loop potential is nonconvex). The coincidence survives the addition of fermions (Yukawa coupled) and gauge fields (minimally coupled) as long as they are weakly coupled to the scalar bosons.

The paper is organized in five sections. Section II shows how a singularity in the effective potential emerges in a one-loop calculation and introduces T_L. In Section III the free energy per unit area of a domain wall is used to obtain T_p. Section IV analyzes the coincidence of T_L and T_p and its dependence on the high-temperature limit, the presence of other fields, the nature of the background field and the order in the loop-expansion. Our conclusions are presented in Section V.

II - THE "INSTABILITY" OF THE UNIFORM BACKGROUND

Let $V(\beta,\bar{M}_J)$ be the free energy per unit volume corresponding to the magnetization $\bar{M}_J$,ie, the average value of $\phi(x)$ in the presence of an x-independent external current, J. If we subtract from this quantity the value of its

zero-temperature, zero-current minimum, $V(\infty,\bar{M}_V)$ we obtain the finite temperature effective potential:

$$V_{eff}(\beta,\bar{M}_J) \equiv V(\beta,\bar{M}_J) - V(\infty,\bar{M}_V) \tag{2.1}$$

We shall analyze the behavior of this potential at the one-loop level for a $\lambda\phi^4$ model of a real boson field coupled to a massless fermion field ψ through a Yukawa coupling. The Euclidean Lagrangian density is thus:

$$\mathcal{L} = \frac{1}{2}(\partial\mu\phi)^2 + \frac{\lambda}{4!}(\phi^2-\phi_V^2)^2 + \bar{\psi}\,\gamma_\mu\partial_\mu\psi + ig\phi\,\bar{\psi}\psi \tag{2.2}$$

with $\phi_V^2 \equiv 6m^2/\lambda$. In three spatial dimensions the semiclassical approximation to the effective potential is (up to one-loop and for J=0):

$$V_{eff}(\beta,\bar{\phi}) = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_V^2)^2 + T\int\frac{d^3k}{(2\pi)^3}\,\ell n(1-e^{-\beta\sqrt{\vec{k}^2+m_B^2}}) - 2T\int\frac{d^3k}{(2\pi)^3}\,\ell n(1+e^{-\beta\sqrt{\vec{k}^2+m_F^2}}) + (\text{zero pt energies}) \tag{2.3}$$

The first term represents the classical (tree) approximation, the second is the temperature dependent part of the boson loops, while the third is the temperature dependent part of the fermion loop. $m_B(\bar{\phi})$ and $m_F(\bar{\phi})$ are, respectively, the boson and fermion effective masses in the background field $\bar{\phi}$ (which coincides with $\bar{M}$ in zero-loop)[1].

$$m_B^2(\bar\phi) = 2m^2 + \frac{\lambda}{2}(\bar\phi^2 - \phi_V^2) = \frac{\lambda}{2}\bar\phi^2 - m^2 \qquad (2.4.a)$$

$$m_F^2(\bar\phi) = g^2\bar\phi^2 \qquad (2.4.b)$$

The zero-point contributions can be found, for example, in reference [2]. As they do not depend on temperature, they may be discarded in the so-called high-temperature limit, $T >> m$, that is often used to simplify the analysis.

One immediately notes that, although m_F is always nonnegative, the same is not true for m_B^2. In the interval $|\bar\phi| < (2m^2/\lambda)^{1/2}$ the squared boson mass is negative, and, as a consequence, the effective potential develops an imaginary part.

Imaginary contributions to the free energy are normally associated to instabilities[8]. Here, however, what we have is an instability of the uniform background as the basis for the loop expansion. As we shall argue in Section IV, the semi-classical approximation to the effective potential will be complex for $|\bar\phi| < (2m^2/\lambda)^{1/2}$ in every order of the expansion. This does not mean that the effective potential, calculated in an entirely nonperturbative way (ie, through MonteCarlo calculations on a lattice), should be complex. Nevertheless, the use of uniform backgrounds in a semiclassical

approximation to the potential does lead to complex values and, in this sense, we may say that such backgrounds become semiclassicaly unstable for $|\bar\phi| < (2m^2/\lambda)^{1/2}$ - independently of temperature!

As discussed before, the minima of the effective potential, $\pm\,\phi_V(\beta)$, are temperature dependent. Furthermore, this semiclassical approximation has been shown[5] to yield very good results as long as we only consider the region outside such minima $(|\bar\phi| > \phi_V(\beta))$. For T=0, $\phi_V^2(\infty) = \phi_V^2 = (6m^2/\lambda)$ which is greater than the value $2m^2/\lambda$ where the potential becomes complex. However, there will be a temperature T_L for which the minima coincide with that value:

$$\phi_V^2(T_L) = 2m^2/\lambda \qquad (2.5)$$

Beyond this temperature $(T > T_L)$ the effective potential minima would fall inside the forbidden region $|\bar\phi| < (2m^2/\lambda)^{1/2}$ Thus, for temperatures in this range the use of uniform backgrounds in a loop-expansion of the effective potential should certainly be avoided.

In the high-temperature limit $(T>>m)$, the leading contribution to $V_{eff}(\beta,\bar\phi)$ is:

$$V_{eff}(\beta,\bar\phi) \underset{\substack{T>>m \\ \lambda<<1}}{=} \frac{\lambda}{4!}(\bar\phi^2 - \phi_V^2)^2 + \frac{1}{18}\left(\frac{T^2}{2}\right)\left[1 + 3\frac{g^2}{\lambda}\right]\lambda(\bar\phi^2 - \phi_V^2) \qquad (2.6)$$

In the broken phase, V_{eff} will attain its minimum values at:

$$\phi_V^2(T) = \phi_V^2 - \left[1 + 3\frac{g^2}{\lambda}\right]\frac{T^2}{3} = \frac{6m^2}{\lambda} - \left[1 + 3\frac{g^2}{\lambda}\right]\frac{T^2}{3} \tag{2.7}$$

From equations (2.4a), (2.5) and (2.7) we may conclude that m_B^2 results negative leading to V_{eff} complex for $T>T_L$ where:

$$T_L^2 = \frac{1}{\left[1+3\frac{g^2}{\lambda}\right]}\left(\frac{12m^2}{\lambda}\right) \tag{2.8}$$

III - NONUNIFORM BACKGROUND - THE PERCOLATION TEMPERATURE

We have seen that uniform backgrounds cannot be used for temperatures higher than T_L. Therefore we shall analyze at which temperature nonuniform backgrounds might play a relevant role as far as thermodynamics is concerned.

Within the semiclassical approach we shall therefore use as a background the nonuniform (kink) solution of the equations of motion given by [9]:

$$\phi_K(x_L) = \sqrt{6}\,\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{mx_L}{\sqrt{2}}\right) \tag{3.1}$$

where x_L is a longitudinal coordinate. Just like a Block wall, ϕ_k divides the system into (+) and (-) domains. These domain walls will be relevant as backgrounds for the semi-classical expansion at a temperature for which their free energy difference per unit area, with respect to uniform backgrounds, changes sign. Peierls' argument [10] of Statistical Mehcanics is the basic ingredient being used here.

In order to get the free energy per unit area of the walls one compares the (Gibbs) free energies with respect to two different backgrounds: i) the nonuniform kink solution, $\phi_k(x_L)$; ii) the uniform background $\phi_v = \pm\sqrt{6}(m/\sqrt{\lambda})$. The free energy per unit area is then:

$$f(T) = \frac{1}{A}\left\{\Gamma(T,\phi_k(x_L)) - \Gamma(T,\phi_v)\right\} \tag{3.2}$$

where A is the area of the wall, a cross section of the volume of the system.

The computation of $f(T)$ in the one-loop approximation reduces to evaluating $Z(\beta)$ semiclassically for both back-grounds. In both cases we shall make use of the identity (which follows form charge conjugation invariance):

$$\{\det[\gamma_\mu\partial_\mu + ig\phi]\}^2 = \det[\gamma_\mu\partial_\mu + ig\phi]\det[\gamma_\mu\partial_\mu - ig\phi] = \det[(-\Box + g^2\phi^2)\mathbb{1} - ig\gamma_\mu(\partial_\mu\phi)] \tag{3.3}$$

Using ϕ_V as background one has:

$$Z_V(\beta) = C\,\det{}^{-1/2}\left[-\Box + 2m^2\right]_\beta \det\left[\gamma_\mu\partial_\mu + ig\phi_V\right]_\beta\, e^{-\beta S(\phi_V)} \tag{3.4}$$

where one is supposed to use periodic boundary conditions in the first determinant and antiperiodic ones in the second. Making use of identity (3.3) we arrive at:

$$Z_V(\beta) = C\,\det{}^{-1/2}\left[-\Box + 2m^2\right]_\beta \det{}^2\left[-\Box + g^2\phi_V^2\right] e^{-\beta S(\phi_V)} \tag{3.5}$$

We finally obtain $\Gamma(T,\phi_v)$ by simply taking the log and dividing by $(-\beta)$:

$$\Gamma(T,\phi_v) = \frac{S}{\beta}(\phi_v) + \frac{1}{2\beta}\,\mathrm{tr}\,\ell n\left[-\Box + 2m^2\right] - \frac{2}{\beta}\,\mathrm{tr}\,\ell n\left[-\Box + g^2\phi_v^2\right] \quad (3.6)$$

Going through the same steps, using $\phi_k(x_L)$ as background, we obtain:

$$\Gamma(T,\phi_k(x_L)) = \frac{S}{\beta}(\phi_k) + \frac{1}{2\beta}\,\mathrm{tr}\,\ell n\left[-\Box + 2m^2 + \frac{\lambda}{2}(\phi_k^2 - \phi_v^2)\right] - \frac{1}{\beta}\,\mathrm{tr}\,\ell n\left[-\Box + \frac{s^2m^2}{2} - \frac{s(s-1)}{2} m^2 \mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right] - \frac{1}{\beta}\,\mathrm{tr}\,\ell n\left[-\Box + \frac{s^2m^2}{2} - \frac{s(s+1)}{2} m^2 \mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right] \quad (3.7)$$

where we have used $\gamma_L \equiv i\begin{pmatrix} 1 & \\ & -1\end{pmatrix}$ and $s \equiv \sqrt{\frac{12}{\lambda}}\,g$. Putting together (3.2), (3.6) and (3.7) we end up with:

$$f(T) = \frac{T}{A}\left\{\left[S(\phi_k) - S(\phi_v)\right] + \frac{T}{2}\,\mathrm{tr}\,\ell n\left[1\!\!1 + (-\Box\ 2m^2)^{-1}\frac{\lambda}{2}(\phi_k^2 - \phi_v^2)\right] - \right.$$

$$- T\,\mathrm{tr}\,\ell n\left[1\!\!1 - \left(-\Box + \frac{s^2m^2}{2}\right)^{-1}\frac{s(s-1)}{2} m^2\,\mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right] -$$

$$\left. - T\,\mathrm{tr}\,\ell n\left[1\!\!1 - \left(-\Box + \frac{s^2m^2}{2}\right)^{-1}\frac{s(s+1)}{2} m^2\,\mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right] \right. \quad (3.8)$$

If we restrict our attention to the high-temperature limit, the results of reference [2] allow us to obtain the leading behavior from:

$$f(T) \underset{(T>>m)}{\simeq} \frac{T}{A}\left\{\left[S(\phi_k) - S(\phi_v)\right] + \frac{T}{2}\,\mathrm{tr}\left[(-\Box + 2m^2)^{-1}\frac{\lambda}{2}(\phi_k^2 - \phi_v^2)\right] - \right.$$

$$- T\,\mathrm{tr}\left[(-\Box + 2m^2)^{-1}\frac{s(s-1)}{2} m^2\mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right] -$$

$$\left. - T\,\mathrm{tr}\left[(-\Box + 2m^2)^{-1}\frac{s(s+1)}{2} m^2\mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right)\right]\right\} \quad (3.9)$$

This expression becomes:

$$f(T) \cong \frac{T}{A}\left\{\left[S(\phi_k) - S(\phi_v)\right] + \frac{A}{2}\int dx_L\,\frac{\lambda}{2}\left[\phi_k^2(x_L) - \phi_v^2\right] T\sum_n \int \frac{d^3k}{(2\pi)^3} *\right.$$

$$\left.\frac{1}{(2\pi nT)^2 + \vec{k}^2 + 2m^2} + A\int dx_L\, s^2m^2\mathrm{sech}^2\left(\frac{mx_L}{\sqrt{2}}\right) T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{((2n+1)\pi T)^2 + \vec{k}^2 + \frac{s^2m^2}{2}}\right\}$$

(3.10)

Taking T>>m we arrive at:

$$f(T) \cong 4\sqrt{2}\,\frac{m^3}{\lambda} - \frac{\sqrt{2}\,m\,T^2}{3}\left(1 + 3\frac{g^2}{\lambda}\right) \quad (3.11)$$

The percolation temperature is defined as the one that makes (3.11) vanish. We immediately see that its value coincides with that of T_L given by (2.8).

IV - THE COINCIDENCE OF T_L AND T_P

The results of the two previous sections show that, in the high-temperature limit, T_L and T_P coincide for a model of scalar bosons interacting with fermions via a Yukawa coupling. In reference [2] such a coincidence had already been observed for the model without fermions. The persistence of this intriguing equality has led us to analyze in detail the several ingredients used to derive it.

Let us consider the Lagrangian density (2.2). We may integrate over the fermions and obtain an effective Lagrangian for the bosons. Following the usual steps we may arrive at $\Gamma(\beta,M(x))$, the generating functional for one-particle irreducible Green's functions, which admits the expansion:

$$\Gamma(\beta,M(x)) = \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int_0^\beta d\tau_j \int d^\nu x_j \left[M(x_j)-M_V\right]^n \Gamma^{(n)}(\tau_1\vec{x}_1\ldots\tau_n\vec{x}_n) \tag{4.1}$$

We may trade $M(x)$ for $B(x) \equiv \frac{\lambda}{2} M^2(x)$ and write:

$$\Gamma(\beta,B(x)) = \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int_0^\beta d\tau_j \int d^\nu x_j \left[B(x_j)-B_V\right]^n \mathcal{G}^{(n)}(\tau_1\vec{x}_1\ldots\tau_n\vec{x}_n) \tag{4.2}$$

It is straight forward to relate $\Gamma^{(n)}$ and $\mathcal{G}^{(n)}$ and we refer the reader to reference [2].

The effective potential may be obtained from (4.2) by simply taking a $\bar{B}$ independent of the coordinates $(\tau_j,\vec{x}_j)$:

$$\Gamma(\beta,\bar{B}) = \sum_{n=1}^{\infty} \frac{1}{n!} \left\{ \prod_{j=1}^{n} \int_0^\beta d\tau_j \int d^\nu x_j \; \mathcal{G}^{(n)}(\tau_1\vec{x}_1\ldots\tau_n\vec{x}_n) \right\} (\bar{B}-B_V)^n \tag{4.3}$$

If we use the Fourier transform of $\mathcal{G}^{(n)}$, given by:

$$\mathcal{G}^{(n)}(\tau_1\vec{x}_1\ldots\tau_n\vec{x}_n) =$$

$$= \beta^{-n} \prod_{j=1}^{n} \sum_{N_j=-\infty}^{\infty} \int \frac{d^\nu k_j}{(2\pi)^\nu} \tilde{\mathcal{G}}^{(n)}(\omega_1\vec{k}_1\ldots\omega_n\vec{k}_n) \exp\left\{-i \sum_{\ell=1}^{n} \omega_\ell\tau_\ell + \vec{k}_\ell\cdot\vec{x}_\ell\right\} \tag{4.4}$$

where $\omega_\ell \equiv 2\pi N_\ell \beta^{-1}$, we may rewrite (4.3) in terms of zero-momentum transforms:

$$\Gamma(\beta,\bar{B}) = \sum_{n=1}^{\infty} \frac{1}{n!} \tilde{\mathcal{G}}^{(n)}(\{0\})(\bar{B}-B_V)^n \tag{4.5}$$

Translational symmetry allows us to put:

$$\tilde{\mathcal{G}}^{(n)}(\{\omega_i,\vec{k}_i\}) = \delta(\sum_i \omega_i)\,\delta^4(\sum_i \vec{k}_i)\,\bar{\mathcal{G}}^{(n)}(\{\omega_i,\vec{k}_i\}) \tag{4.6}$$

If we now use the fact that $B(x)$ in (4.2) is τ-independent, (4.2) and (4.5) become, respectively:

$$\Omega(\beta,B(k)) \equiv (\beta A)^{-1}\Gamma(\beta,B(k)) = \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int \frac{d^\nu k_j}{(2\pi)^\nu} B(-\vec{k}_j)\bar{\mathcal{G}}^{(n)}(\{\vec{k}_j;\omega_j=0\})A^{-1}\delta^\nu(\sum_j \vec{k}_j) \tag{4.7.a}$$

$$\Omega(\beta,\bar{B}) \equiv (\beta V)^{-1}\,\Gamma(\beta,\bar{B}) = \sum_{n=1}^{\infty} \frac{1}{n!} \bar{\mathcal{G}}^{(n)}(\{0\})(\bar{B}-B_V)^n \tag{4.7.b}$$

The graphs that contribute to $\bar{\mathcal{G}}^{(n)}$ will involve sums over the discrete ω_j which, once performed, yield a term independent of temperature plus one which has the full T-dependence. An example is the identity:

$$\beta^{-1} \sum_{n=-\infty}^{\infty} \frac{1}{\left(\frac{2\pi n}{\beta}\right)^2 + z^2} = \frac{1}{2z} + \frac{1}{z(e^{\beta z}-1)} \tag{4.8}$$

One may then split $\bar{\mathcal{G}}^{(n)}$ into two parts:

$$\bar{\mathcal{G}}^{(n)}(\{\vec{k}_i,\omega_i=0\}) = \bar{\mathcal{G}}_0^{(n)}(\{\vec{k}_i\}) + \bar{\mathcal{G}}_T^{(n)}(\{\vec{k}_i,\omega_i=0\}) \tag{4.9}$$

where the second term has all the T-dependence. Pure dimensional analysis yields:

$$\mathcal{G}^{(n)}(\{\vec{k}_i,\omega_i\}) = T^{4-2n}\, g_n\left(\frac{\omega_j}{T}, \frac{\vec{k}_j}{T}, \frac{m}{T}\right) \tag{4.10}$$

where g_n is dimensionless. Putting together (4.8), (4.9) and (4.10):

$$\Omega(T,B(k)) = \Omega_0(B(k)) +$$

$$+ T^2\left\{\sum_{n=1}^{\infty} \frac{1}{n!} T^{2-2n} \prod_{j=1}^{n} \int \frac{d^\nu k_j}{(2\pi)^\nu} \tilde{B}(-\vec{k}_j)\, g_n\left(\frac{\omega_j}{T},\frac{\vec{k}_j}{T},\frac{m}{T}\right) V^{-1}\, \delta^\nu\left(\sum_j \vec{k}_j\right)\right\} \tag{4.11.a}$$

$$\Omega(T,\bar{B}) = \Omega_0(\bar{B}) + T^2\left\{\sum_{n=1}^{\infty} \frac{1}{n!} T^{2-2n}\, g_n(0,\vec{0},\tfrac{m}{T})\,(\bar{B}-B_V)^n\right\} \tag{4.11.b}$$

where the expansions inside the curly brackets are supposed to be well-behaved in the infrared as discussed by Weinberg [11] and the terms Ω_0 are just the zero-temperature values of Ω_T .

In the high-T limit ($T \gg \omega_j, |\vec{k}_j|, m$) both expressions will only depend on the zero-momentum character

of g_n. The leading term of the expansion corresponds to n=1 in (4.11). If we restrict our attention to that term and, furthermore, assume T to be high enough that quantum fluctuations are negligible with respect to thermal ones (so as to take zero-loop for Ω_0), we end up with:

$$\Omega(T,B(k)) = \Delta s_{c\ell}(B(k)) + T^2 g_1(0,\vec{0},0)\tilde{B}(0) \tag{4.12.a}$$

$$\Omega(T,\bar{B}) = \Delta s_{c\ell}(\bar{B}) + T^2 g_1(0,\vec{0},0)(\bar{B}-B_V) \tag{4.12.b}$$

where $\Delta s_{c\ell}$ is the classical action density (per unit area or volume) with respect to ϕ_V and $\tilde{B}(0) = \int_{-\infty}^{\infty} dx\left[B(x)-B_V\right]$

The percolation temperature [12] T_P is the one that makes (4.12.a) vanish for the kink background, ie, $B(x) = \frac{\lambda}{2}\phi_k^2(x)$. Thus, in the high-T limit is is given by:

$$T_P^2 = \frac{-\Delta s_{c\ell}}{\tilde{B}(0) g_1(0,\vec{0},0)} = -\frac{\int_{-\infty}^{\infty} d\xi \left\{\frac{1}{2}\left(\frac{d\phi_k}{d\xi}\right)^2 + \frac{\lambda}{4!}\left[\phi_k^2(\xi) - \phi_V^2\right]^2\right\}}{\left\{\int_{-\infty}^{\infty} d\xi\, \frac{\lambda}{2}\left[\phi_k^2(\xi) - \phi_V^2\right]\right\} g_1(0,\vec{0},0)} \tag{4.13}$$

This expression is quite general and $g_1(0,\vec{0},0)$ is obtained from the renormalized one-point function $\mathcal{G}^{(1)}$. Its one-loop value is 1/24 [2]. For the kink, $\tilde{B}(0)$ is related to the topological charge, Q_T :

$$\int_{-\infty}^{\infty} dx_L \frac{\lambda}{2}(\phi_k^2-\phi_v^2) = -\sqrt{3\lambda}\int_{-\infty}^{\infty}\frac{d\phi_k}{d\xi}\,d\xi = -2\sqrt{3\lambda}\;Q_T \qquad (4.14)$$

$$T_p^2(\text{kink}) = \frac{1}{2\sqrt{3\lambda}}\;\frac{\Delta s_{c\ell}}{Q_T\; g_1(0,\vec{0},0)} \qquad (4.15)$$

It remains to analyze the instability temperature, T_L, of the effective potential. Let us first consider the case without fermions. If we perform a semiclassical expansion around a uniform background, $\bar{\phi}$, the partition function for J=0 may be formally written as[1]:

$$Z(\beta) = e^{-\beta V\Delta s_{c\ell}} \oint[D\eta]\exp\Big\{ -\int_0^\beta d\tau \int d^\nu x \Big[\tfrac{1}{2}(\partial\mu\eta)^2 +$$

$$+\tfrac{1}{2}(2m^2+\tfrac{\lambda}{2}\,[\bar{\phi}^2-\phi_v^2])\eta^2 + \frac{\lambda^{3/2}}{3!}\eta^3 + \frac{\lambda^2}{4!}\eta^4\Big\} \qquad (4.16)$$

where we have set $\phi(x) = \frac{\bar{\phi}}{\sqrt{\lambda}} + \eta(x)$. It should be clear that the quadratic part of the exponent involves the Fourier transform of:

$$I(\omega_n^2,\vec{p}^2) \equiv \omega_n^2 + \vec{p}^2 + 2m^2 + \tfrac{\lambda}{2}(\bar{\phi}^2-\phi_v^2) \qquad (4.17)$$

The "effective mass" I(0,0) will become negative when:

$$2m^2 + \tfrac{\lambda}{2}(\bar{\phi}^2-\phi_v^2) < 0 \Longrightarrow \bar{\phi}^2 < \frac{2m^2}{\lambda} \qquad (4.18)$$

As a result, for values of $\bar{\phi}^2 < \frac{2m^2}{\lambda}$ the loop expansion around the uniform background $\bar{\phi}$ will yield cumplex values to the effective potential. This "instability" of the uniform background occurs at the same value of $\bar{\phi}$ at every order in the expansion, since the inverse of the operator of the quadratic part is used as the unperturbed propagator in the expansion. The terms in η^3 and η^4 are treated as perturbations and we are led to graphs having denominators like (4.17). An explicit computation of the effective potential in Quantum Mechanics at finite temperature, up to two-loops, supports our statement[13].

In order to obtain the temperature T_L all we have to do is to impose the condition that the minima of the high-T expression for the effective potential (4.12.b) coincide with $\phi_L^2 \equiv (2m^2/\lambda)$. Thus, for $T < T_L$ these minima will lie outside the complex region whereas at T_L they just touch its boundary. Then, using $\Delta s_{c\ell} = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_v^2)^2$ we obtain:

$$\frac{\lambda}{4!}(\phi_L^2-\phi_v^2)^2 + T_L^2\, g_1(0,\vec{0},0) = 0 \qquad (4.19)$$

and, finally:

$$T_L^2 = \frac{2m^2}{3\lambda g_1(0,\vec{0},0)} = \frac{\Gamma^{(2)}(0,\vec{0})}{3\lambda g_1(0,\vec{0},0)} \qquad (4.20)$$

The coincidence of T_L^2 and T_p^2 is due to the equality:

$$\frac{-\Delta S_{c\ell}}{\tilde{B}(0)} = \frac{2m^2}{3\lambda} \qquad (4.21)$$

as can be verified by explicit computation. Thus, it depends on the particular relationship between classical action and field at zero-momentum (large-distance behavior). The crucial property of the kink field used in our approach is the interpolation between two regions in different vacua. We then expect that another solution with analogous behavior will have a similar $\tilde{B}(0)$. In that case, the kink being of least action will lead to the smallest T_p^2, justifying its use as a signal for the transition.

If we include other fields, such as fermions or gauge fields, the coincidence will still hold as long as they are weakly coupled to the scalars. As an example, we shall return to the case of fermions interacting through the Lagrangian density (2.2). The partition function becomes:

$$Z(\beta) = Z_B(\beta)\, Z_F(\beta) \qquad (4.22)$$

where $Z_B(\beta)$ is given by (4.16) and $Z_F(\beta)$ is obtained by

integrating over the Fermi fields:

$$Z_F(\beta) \equiv \det\left[\gamma_\mu \partial_\mu + ig\phi(x)\right] \qquad (4.23)$$

In the semiclassical approach we write:

$$\phi(x) = \phi_k(x) + \sqrt{\lambda}\, \eta(x) \qquad (4.24)$$

Therefore:

$$Z_F(\beta) = \det\left[\not{\partial} + ig\phi_k(x) + ig\lambda^{1/2}\eta(x)\right] \qquad (4.25)$$

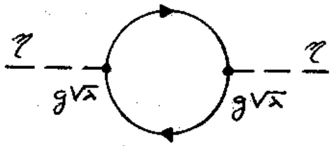
The loop-expansion will then yield

$$F_F(\beta) = -\frac{1}{\beta}\ln Z_F(\beta) = -\frac{1}{\beta}\,\mathrm{tr}\ln\left[\not{\partial} + ig\phi_k(x)\right]\left\{1\!\!1 + \mathrm{tr}\sum_{n=1}^{\infty}(ig\sqrt{\lambda})^n\left[G\eta\right]^n\right\} \qquad (4.26)$$

where $G \equiv \left[\not{\partial} + ig\phi_k\right]^{-1}$. The trace properties of γ-matrices makes the series run over <u>even</u> n. The first correction is then quadratic in η:

$$F_F(\beta) = -\frac{1}{\beta}\,\mathrm{tr}\ln\left[\not{\partial}+ig\phi_k(x)\right]\left\{1\!\!1 - \lambda g^2\int_0^\beta d\tau_1\int d^\nu x_1\int_0^\beta d\tau_2\, d^\nu x_2\, \mathrm{Sp}\left[G(x_1-x_2)\eta(x_2)G(x_2-x_1)\eta(x_1)\right] + O(\eta^3)\right\} \qquad (4.27)$$

If we neglect the $O(\lambda g^2)$ term, we obtain the result of section III which we derived under the assumption that ϕ_L^2 did not change from its value for a pure scalar theory. However, in the loop expansion of (4.25) the presence of the quadratic terms in η, coming from the fermionic determinant, adds to the quadratic kernel a term proportional to $(\frac{g^2}{\lambda}) * \lambda g^2 \sim$ $\sim g^4$. This is small compared to the $\frac{g^2}{\lambda}$ term we had already obtained. If, however, we do not neglect this correction, the value of ϕ_L^2 will change by an amount proportional to the fermion induced term which graphically is

$\left(\frac{g^2}{\lambda}\right)$  (4.28)

Once renormalized this will give the additional contribution that spoils the coincidence. Nevertheless, in the small coupling limit we may neglect this and the coincidence will persist.

Analogous considerations for gauge fields coupled to scalars will also hold. Again, restricting ourselves to the interaction of these with only the classical background, T_L^2 and T_p^2 will be identical in the high-T limit. Gauge field induced corrections will also exist but, once again, are negligible for small gauge-scalar coupling.

V - CONCLUSIONS

We have analyzed in detail the conditions under

which the percolation temperature , T_p, coincides with the limiting temperature T_L. This temperature is defined as the highest temperature for which a semiclassical expansion of the effective potential around a uniform background yields minima which lie outside the complex region.

We have concluded that the coincidence will hold in leading order in the high-T limit and persist even if we include fermions and gauge fields, as long as they couple weakly to the scalars.

As we had argued previously, domain walls are a very useful device to estimate the critical temperature, whenever it coincides with the percolation temperature (as should be the case for $d=\nu+1=4$). Here we conclude that in the high-T limit we can obtain this percolation temperature by just finding the limiting temperature for the validity of the uniform background expansion, which we interpret as a sign of "instability" of such a background - that is, above T_L (or T_p) we expect the state of lowest free energy is described by a condensate of topological deffects (this phenomena is well known in spin systems[14]).

We end up with a coherent picture which allows one to extract T_p and, thus, T_c from either a direct calculation, as done in reference [2], or from the effective potential. In either case, we never have to deal with extrapolations into complex regions which are undesirable.

A point which deserves to be further explored is the analysis of how the coincidence is affected if we go beyond leading order in the high-T expansion. This, however, will be considered elsewhere.

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