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PUBLICAÇÕES

IFUSP/P-486

A GENTILIONIC APPROACH TO QUARK COLOURS

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Setembro/1984

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ABSTRACT

An extended form of Noether's theorem enable us to identify the colour quantum number with the eigenvalue of the invariant of the algebra of $S^{(3)}$. In the gentilionic approach, the composition of the $S^{(3)}$ colour with the symmetric quark model seems to constitute an exact symmetry of Nature. It is also argued some general properties and the universality of Gentile statistics.

1. INTRODUCTION

Within the framework of quantum mechanics and according to the Principle of Indistinguishability, other kind of particles could exist in nature(1). They have been named gentileons to make a clear distinction from the usual bosons and fermions. Thus, the possibility of existence of three kinds of particles embodies the fundamental quantizations found in quantum mechanics. These are related to the representations of the symmetric group in terms of Young diagrams. Bosons and fermions correspond, respectively, to the horizontal and vertical shapes, whereas the intermediate shapes would be associated with the gentileons. The associated wavefunctions are one-dimensional for bosons and fermions and multi-dimensional for gentileons. We have also seen that the gentileons could be understood as "confined entities" with saturation properties. It must be remarked that these features are not "physical" ones but have their origin in the geometrical and topological aspects of the symmetric group. The physical confinement, for instance, that is related to the physical dimensions of the system, would arise from the dynamical features of the particles. We could expect this confinement to be related with the geometrical one but, in this paper, no effort is made to directly connect them.

In this work, in Section 2, we interpret the invariants of the symmetric group algebra in terms of Noether's theorem. Some important physical results such as the appearance of new conserved quantum numbers arise from this analysis.

In Section 3, assuming our gentilionic quark model, we show that the baryonic mass spectra and charges get clarified if a quantum number associated with an algebraic invariant obtained in Section 2 is identified with the colour quantum number. A brief discussion on meson constitution is also given.

Finally, in Section 4 we present some general conclusions on gentilionic statistics. The universality of its validity is suggested and some consequences are drawn.

2. NOETHER'S THEOREM AND THE ALGEBRA OF $S^{(3)}$

All symmetry properties of a system can be described as groups of automorphism which carry over the system into itself. By Noether's theorem, if a system is invariant under a certain group of transformations then, from this symmetry property there follows the conservation of a physical quantity of the system. However, in quantum mechanics we are not restricted to continuous symmetry groups or even to coordinate transformation groups. Indeed, the underlying fundamental group contained in the Principle of Indistinguishability is the symmetric group $S^{(N)}$ which does not appear to be directly related to space time symmetries. We then state Noether's theorem as[2]: "associated with every symmetry principle there exists a unitary operator U in Hilbert space relating state vectors and observables at two different physical points".

To make this statement more precise we will develop an algebraic approach related to $S^{(3)}$. Let us call $AS^{(3)}$ the

algebra[3] of the symmetric group $S^{(3)}$. It is spanned by six vectors $\{\eta_i\}$, $i = 1, 2, \ldots, 6$. In this algebra we define a class operator $K_{(\rho)}$ as being the sum of the $\eta_{(\rho)}$ permutations with the cycle structure (ρ):

$$K_{(\rho)} = \sum_{P_a \in (\rho)} P_a \tag{1}$$

where each P_a corresponds to a vector η_a. As is well known, the class operators form a maximal linearly independent set of elements of $AS^{(3)}$ such that

$$\left[\eta , K_{(\rho)}\right] = 0 \quad \text{with} \quad \eta \in AS^{(3)} \quad . \tag{2}$$

In our case (N=3), the class operators are $K_{(1^3)} = I$, $K_{(2,1)} = \sum (ij)$ and $K_{(3)} = \sum (ijk)$, where each group operator is indicated in the usual notation of substitutional analysis. Now, since the group $S^{(3)}$ admits two generators $\underline{a}$ and $\underline{b}$, we can consider $AS^{(3)}$ as being an associative polynomial algebra generated by $\underline{a}$ and $\underline{b}$: $\{\eta_1, \eta_2, \ldots, \eta_6\} \equiv \{I, ba, ab, a, aba, b\}$. Calling $L(a,b)$ this last algebra, it is easy to see that the commutation relation corresponding to it is

$$ab + ba = -I \quad . \tag{3}$$

It is not difficult to show that $L(a,b)$ contains only the primitive idempotent defined by Eq. (3) itself and that the closure relation (3) also embraces all relations (1).

These definitions of $AS^{(3)}$ and $L(a,b)$ in terms of a and b also reflect the fact that any permutation can be obtained from transpositions. Thus, in our algebraic approach a prominent role is ascribed to the class $(\rho) = (2,1)$. To this class $(2,1)$ do correspond the algebraic invariants $K^{[1^3]}_{(2,1)}$, $K^{[2,1]}_{(2,1)}$ and $K^{[3]}_{(2,1)}$ with eigenvalues given respectively by -3, 0 and 3. The upper square brackets refer to the three representations: $[1^3]$ = fermionic, $[2,1]$ = gentilionic and $[3]$ = bosonic. It is also important to note that the 4-dimensional symmetry adapted state vectors which describe gentilionic states correspond to a 2×2 representation of $AS^{(3)}$ with the representative matrices taken as Kronecker sums since $(S^{(3)})^2 \sim S^{(3)}$.

Usually, for continuous groups, we define the Casimir invariants which commute with all of the generators and are therefore invariants under all group transformations. These simultaneously diagonalized invariants are the conserved quantum operators associated with the symmetry group. In our discrete problem we use the same idea. The class operator $K^{[2,1]}_{(2,1)}$ which corresponds to the genuine gentilionic representation of $AS^{(3)}$ is identified with a quantum operator which gives a new conserved quantum number related to $S^{(3)}$.

Our algebra $L(a,b)$ is a special case of an anticommutative algebra G_n studied long ago by Schönberg[4]. He has shown that G_n is associated with a kind of affine geometry of the Hilbert space. An affine algebra of dimension n contains

a unitary algebra of the same dimensionality as a sub-algebra. From this point of view, it is to be expected that the richer affine structure will enable us to infer some properties more general than those contained in a metric unitary structure as will be seen in section 4.

3. QUARKS AND COLOURS

In the symmetric quark model for baryons[5] where only $(SU(6) \times O(3))_{symmetric} = \phi$ representations exist, the restriction of the wavefunctions ϕ being totally symmetric implies the "ad hoc" introduction of a new threefold degree of freedom for each quark flavour called "colour". We have tried to explain the baryonic mass spectra with an "uncoloured" gentilionic model. That is, we have assumed that the baryons would be described by a 4-dimensional state function defined only on the $SU(6) \times O(3)$ space and we followed the standard calculation procedure adopted in the $SU(6) \times O(3)$ model[5]. Since in our calculation, the $SU(6) \times O(3)$ wavefunction was also found to be given by ϕ, our attempt was unsuccessful due to the symmetry difficulties encountered in the construction of the state vectors. This has lead us to formulate the following model.

We consider the action of $S^{(3)}$ on a colour space (R, B, Y) and we write the baryon 4-dimensional state vector ψ as:

$$\psi = \phi \times Y(\text{colour}) \tag{4}$$

where ϕ is one-dimensional and the 4-dimensional $Y(\text{colour})$ correspond to the intermediate representations of $S^{(3)}$. Since the gentilionic and fermionic models have the same "uncoloured" quark function ϕ, they will give identical results when it is assumed that the physical processes are colour independent.

The algebraic invariant $K^{[2,1]}_{(2,1)}$ of Section 2 associated with the gentileons is, using Noether's theorem, interpreted as a unitary operator taken as the "colour operator". In that manner, the conserved colour quantum number which arises as an intrinsic consequence of the study of $AS^{(3)}$ will be the eigenvalue of the operator $K^{[2,1]}_{(2,1)}$. Now, it is easy to show that this eigenvalue is zero[3]. That this is not a fortuitous result can be seen as follows.

The Gell-Mann-Nishijima relation

$$Q = T_3 + Y/2 = F_3 + F_8/\sqrt{3} \tag{5}$$

defines the charge operator as a function of the SU(3) generators[5]. This relation can be generalized as

$$Q = (T_3 + Y/2) + t/3 \tag{6}$$

with <u>t</u> taken as an arbitrary c-number which in our approach is assumed to be the eigenvalue of $K^{[2,1]}_{(2,1)}$. This assumption is consistent with the definition of a triplet of charges obeying

$$z = (t+2)/3 \tag{7}$$

subject to the constraint that

$$z_R + z_B + z_Y = 2 \tag{8}$$

which follows from the requirement that Δ^{++} $(u_R u_B u_Y)$ have charge $+2$. Thus, the constraint in Eq. (8) corresponds to

$$t_R + t_B + t_Y = 0 \tag{9}$$

assuring the familiar baryon charges satisfying the standard Gell-Mann-Nishijima relation. Expression (9), which is imposed by the experimental data, arises as a natural result in the gentilionic approach: it is exactly the eigenvalue of $K^{[2,1]}_{(2,1)}$ obtained from $AS^{(3)}$.

We must note that Eq. (9) has been deduced for three quarks with different colour states. Since Y (colour) also admits that two particles can occupy the same colour state, an additional condition appears in our scheme:

$$2\,t_n + t_m = 0 \tag{10}$$

where $m \neq n$ and $m, n = R, B$ and Y. Thus, according to Equations (9) and (10) we can conclude that $t_R = t_B = t_Y = 0$, suggesting that the combination of a $S^{(3)}$ quantum colour number with the $SU(6) \times O(3)$ symmetric model could result in an exact symmetry of Nature.

Our approach differs considerably from the one-dimensional quantum description. With the recognition of a

new conserved quantity in physics, the problem arises of finding a corresponding invariance principle. For continuous transformations we define a Hermitian operator as the generator of the symmetry. Here, for discrete transformations, extensive use of the invariants of the operator algebra has been made. We recall that, although the function ϕ is the same in the one-dimensional and in the 4-dimensional approaches, the presence of Y (colour) is very significant since its 4-dimensionality and symmetry properties can lead to an explanation of:
(a) baryonic number conservation, (b) geometrical quark confinement and (c) 3-quark saturation in baryons[1].

Finally, we must note that a very peculiar aspect of the multi-dimensionality of the gentilionic wavefunctions has apparently escaped attention in our previous analysis[1]. This may be seen as follows. Since the wavefunctions associated with gentileons must be multi-dimensional, no <u>two identical gentileons</u> can form a system, for this would imply the representation of the system by one-dimensional wavefunctions. Thus, despite the fact that one-dimensional theories and gentilionic theory give the same results when applied to mesons, the gentilionic approach seems to be more satisfactory in the sense that it can explain why mesons are bound states of two different particles: a quark and a anti-quark.

4. CONCLUSIONS AND COMMENTS

The Pauli exclusion principle plays a fundamental role in the possible ways to define observables in quantum mechanics. It is the most important law which must be obeyed with regard to stability of matter. The statistics defined by it, that of Fermi-Dirac, when applied to the entire field of quantum physics has given unquestionable results. On the other hand, for particles or systems not obeying the Pauli principle we have only one acceptable alternative: that of Bose-Einstein. These two statistics fulfil the requirements of quantum mechanics and of more elaborate theories as e.g. quantum field theory. Nevertheless, recently[1] we have argued about the universality of a third kind of statistics, that of Gentile. It also has been deduced from the general principles of quantum mechanics and is not in contradiction with the Principle of Indistinguishability. But its meaning is not as transparent as that ascribed to the other statistics. One fundamental difficulty is concerned with observability. Whereas Bose-Einstein and Fermi-Dirac statistics have received all experimental confirmation, leading to well established results, no claim has been made of the need of extending these theories to explain experimental results. But quantum mechanics has its own selection rules. Some processes not directly related to experimental observation are allowed or forbidden, according to some quantum rule. We expect that Gentile statistics probably will be of fundamental importance in the formulation of selection rules. This has been shown in detail in this work. The symmetric group $S^{(3)}$ not being directly related to space time symmetries could lead to operators not necessarily related to quantum observables. Our gentilionic interpretation of the multi-dimensional state vectors is consistent with the non observability of some properties which could be attributed to unitary non observables operators.

There is another aspect of observability which is worthwhile to mention. The second quantization method developed by us[1] is a particular case of a general mathematical technique applicable to any formalism involving linear equations of change[6,7]. The multidimensionality of the intermediate state vectors expressed in terms of the second quantization language has, for $N=3$, as a result a tri-linear commutation relation for the creation and annihilation operators. On the other hand either that of Bose-Einstein or that of Fermi-Dirac operators, all expressions connected to observables as the energy and the charge, are written as bilinear combinations of the operators[8]. Thus, since we expect that neither a creation operator nor an annihilation operator can appear alone in any term of the Hamiltonian, a tri-linear commutation relation must be related with unobservable properties. Nevertheless, some selections rules related to the commutation relations could result.

The very strange properties of quarks can be satisfactorily explained if they are assumed to obey Gentile statistics. Colour quantum numbers arise as a consequence of

a broader interpretation of Noether's theorem applied to a 3-quark system invariant under $S^{(3)}$. We think that the theoretical implications of assuming Gentile statistics for collective systems is not restricted to explain SU(3) or SU(6) symmetries and multiplets. The action of $S^{(3)}$ on the colour space considered here is an example of a general procedure which is well known in internal space theory[9]. The appearance of selection rules and conservation laws in our scheme suggests gauge fields and internal spaces as being the most promising beneficiaries of the study of general statistics. This study raises very difficult but fascinating questions that promise a great challenge for future research.

ACKNOWLEDGEMENTS

The authors thank Prof. J. Tiomno for helpful discussions and encouragements.

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