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DEFORMATION EFFECTS IN THE HEAVY ION QUARTER-POINT ANGLE

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ABSTRACT

The effects of static and dynamic deformation on the heavy-ion elastic scattering quarter-point angle are discussed and analyzed in the sudden approximation. Simple expressions are derived within the Fresnel model and applications to several heavy-ion systems are presented.

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I. INTRODUCTION

One of the most conspicuous features of heavy-ion elastic scattering at above-barrier energies is the quarter-point angle. This angle is defined, through the quarter-point recipe, as that at which the ratio of the elastic differential cross section to the Rutherford cross section attains the value 0.25. At an optical model level, it is also defined as the angle that is related, through a Coulomb deflection relation, to an angular momentum at which the transmission coefficient is 0.5 (or the elastic reflection coefficient is $\sqrt{0.5}$). This angular momentum is invariably referred to as the grazing or strong absorption angular momentum, and it represents, using an optical analogy, the Fresnel diffraction boundary, in angular momentum space (see Born and Wolf, ref. 1).

The utility of the quarter-point recipe resides in the immediate, albeit approximate, determination of the total reaction cross section through the knowledge of $\theta_{1/4}$ as

$$\sigma_R = \frac{\pi}{k^2}\left(\ell_{1/4}+1\right)^2 \qquad (1)$$

with

$$\ell_{1/4} = \eta \cot\left(\tfrac{1}{2}\theta_{1/4}\right) \qquad (2)$$

where k is the asymptotic wave number of relative motion and η is the Sommerfeld parameter. The quarter-point recipe, first investigated by Blair[2], has been used extensively in the analysis of heavy-ion elastic scattering.

Clearly the quarter-point recipe, as described by Eq. (1) and (2), does not take into account any nuclear structure effect aside from the over all optical behavior exemplified by the optical model description.

In this paper, we investigated the effect of nuclear deformation on the quarter-point angle. We work out approximate analytic forms for the corrections, arising from static deformation, to $\theta_{1/4}$. We consider our treatment based on the sudden approximation, as approximation to a full coupled channels calculation, having, as a clear advantage, the closed forms, which facilitates the understanding of the physics involved.

In Section II, we present our sudden treatment of the deformation effects on $\theta_{1/4}$. Our starting point for the cross section ratio to Rutherford is the strong absorption Fresnel model[3].

In Section III, we discuss the two major corrections to $\theta_{1/4}$ arising from, what we call, dynamic deformation effects due to Coulomb excitation, and static deformation effects due to Nuclear excitation, both treated within the sudden approximation.

In Section IV, we present our numerical results for several heavy-ion systems. And, finally, in Section V, we present several concluding remarks.

II. THE SHARP CUT-OFF MODEL OF $\sigma(\theta)/\sigma_{Ruth}(\theta)$, AND STATIC DEFORMATION EFFECTS

Within the sharp cut-off model of heavy-ion elastic scattering, the ratio to Rutherford is given by[3]

$$\frac{\sigma}{\sigma_{Ruth}}(\theta) = F\left\{ \eta^{1/2} \frac{\sin\left(\frac{1}{2}(\theta-\theta_{1/4})\right)}{\sin\left(\frac{1}{2}\theta_{1/4}\right)} \right\} \tag{3}$$

where the function F is given by

$$\begin{aligned} F(x) &= \frac{1}{4}\left|\mathrm{erfc}\left(e^{i\pi/4}x\right)\right|^2 \\ &= \frac{1}{2}\left\{\left[\frac{1}{2} - C\left(\sqrt{\tfrac{2}{\pi}}\,x\right)\right]^2 + \left[\frac{1}{2} - S\left(\sqrt{\tfrac{2}{\pi}}\,x\right)\right]^2\right\} \end{aligned} \tag{4}$$

The Fresnel integrals $C(z)$ and $S(z)$ are given by

$$\begin{aligned} C(z) &= \int_0^z \cos\left(\frac{\pi}{2}t^2\right) dt \quad ; \quad C(z) = -C(-z) \\ S(z) &= \int_0^z \sin\left(\frac{\pi}{2}t^2\right) dt \quad ; \quad S(z) = -S(-z) \end{aligned} \tag{5}$$

Though completely not adequate at $\theta > \theta_{1/4}$, owing to the sharp cut-off approximation, Eq. (3), nevertheless supplies a reasonable description of the heavy-ion elastic scattering cross section ratio to Rutherford at $\theta \leq \theta_{1/4}$. Furthermore, it exhibits explicitly the quarter-point property of the scattering, namely at $\theta = \theta_{1/4}$, $\frac{\sigma}{\sigma_{Ruth}} = \frac{1}{4}$, since $F(0) = \frac{1}{4}$.

For further use, we give below the asymptotic behavior of $F(x)$. For $\theta \ll \theta_{1/4}$, we have, using the asymptotic forms of $C(z)$ and $S(z)$,

$$F(x) \simeq 1 + \frac{1}{4\pi|x|^2} + \frac{1}{\sqrt{\pi}\,|x|}\sin\left(x^2 - \frac{\pi}{4}\right) \; ; \; x \ll 0 \tag{6a}$$

$$F(x) \simeq \frac{1}{4\pi |x|^2} \quad ; \quad x \gg 0 \tag{6b}$$

Equation (6a) shows that the Fresnel oscillations seen in σ/σ_{Ruth} at small angles are described by $\sin\left\{\eta \frac{\sin^2(\frac{1}{2}(\theta-\theta_{1/4}))}{\sin^2(\frac{1}{2}\theta_{1/4})} - \frac{\pi}{4}\right\}$. The local period of these oscillations is given by

$$P_\theta \simeq \frac{\pi}{\frac{\partial}{\partial\theta} x^2} = \frac{2\pi}{\eta}\,\frac{\sin^2(\frac{1}{2}\theta_{1/4})}{\sin(\theta-\theta_{1/4})}$$

$$= \frac{2\pi\eta}{\eta^2+\lambda_{1/4}^2}\,\frac{1}{\sin(\theta-\theta_{1/4})} \tag{7}$$

clearly, P_θ increases as θ approaches $\theta_{1/4}$ and as η decreases.

Equation (6b) indicates that the cross section ratio σ/σ_{Ruth} decreases with angle, in the shadow region, as $\sin^{-2}(\frac{1}{2}(\theta-\theta_{1/4}))$. This is, of course, an artificial feature associated with the sharp cut-off approximation and is far from the exponential damping, which results if a smooth absorption is considered. However, since our aim in this work is to investigate the quarter angle region, which is adequately described by Eq. (3), we feel quite comfortable in using the sharp cut-off model.

Before introducing the deformation aspect to the problem we give below positions of maxima and minima of $F(x)$,

$$\theta_{max}^{(m)} = \theta_{1/4} - 2\sin^{-1}\sqrt{\left(\frac{3\pi}{4}+\frac{4m\pi}{4}\right)\frac{\sin^2(\frac{1}{2}\theta_{1/4})}{\eta}} \qquad m = \text{even}$$

$$\theta_{min}^{(m)} = \theta_{1/4} - 2\sin^{-1}\sqrt{\left(\frac{3\pi}{4}+\frac{4m\pi}{4}\right)\frac{\sin^2(\frac{1}{2}\theta_{1/4})}{\eta}}\,, \quad m = \text{odd} \tag{8}$$

The first major maximum measured from $\theta_{1/4}$ occurs at

$$\theta_{max}^{(0)} = \theta_{1/4} - 2\sin^{-1}\sqrt{\sqrt{\frac{3\pi}{4\eta}}\,\sin(\tfrac{1}{2}\theta_{1/4})}$$

$$= \theta_{1/4} - \cos^{-1}\left[1 - \frac{3\pi}{2\eta}\sin^2(\tfrac{1}{2}\theta_{1/4})\right] \tag{9}$$

The value of $\frac{\sigma}{\sigma_{Ruth}}$ at this angle is

$$\frac{\sigma}{\sigma_{Ruth}}(\theta_{max}^{(0)}) \simeq 1 + \frac{1}{3\pi^2} + \frac{2}{\pi\sqrt{3}} = 1.40\,. \tag{10}$$

Therefore, in the angle interval $\theta_{max}^{(0)} < \theta < \theta_{1/4}$, the cross section ratio changes by ~460%, a rather major drop. It is in this region that we expect static deformation to play a major rôle.

III. STATIC AND DYNAMIC DEFORMATION EFFECTS IN $\theta_{1/4}$

In several heavy-ion systems, the usual Fresnel form of the ratio $\frac{\sigma}{\sigma_{Ruth}}$ at small angles, come out quite modified. As an example, we show in Fig. 1, the data on ^{84}Kr scattered by ^{208}Pb and ^{232}Th at 500 MeV measured by Colombani et al[4]. In Fig. 2, we exhibit the elastic data of the system $^{18}O + ^{184}W$ at 90 MeV measured by Thorn et al[5]. Clearly, several specific nuclear structure effects not accounted for by Eq. (1), are involved in this case.

In particular, the strong Coulomb excitation of low-lying excited states both target and projectile may have a major role in bringing in the clear deviation in the data from pure Fresnel diffraction. Aside from Coulomb excitation, nuclear excitation also inflicts several changes in the diffraction scattering.

In this chapter, we consider these two effects separately due to the clear difference in their nature, the Coulomb effects, represented by an adequate polarization potential, is of a long range nature both in r and ℓ, whereas the nuclear excitation is short ranged and can be accounted for approximately, as we show below, by performing an adequate average over the orientation angle.

As has been demonstrated in refs. 5-7), the dynamic deformation effects arising from the Coulomb excitation of low-lying collective states can be nicely accounted for quite adequately by an ℓ and r dependent polarization potential, which has the form

$$V^{\ell}_{pol}(r) = \frac{-i\hbar^2}{2\mu}\left[\frac{a_\ell}{r^3} + \frac{b_\ell}{r^4} + \frac{c_\ell}{r^5}\right] \tag{11}$$

where, the ℓ and E dependent coefficients a_ℓ, b_ℓ and c_ℓ are given by

$$\begin{aligned} a_\ell &= \frac{2}{5k}\eta^2 q^2_{0\to2}\left[\frac{3\hat{\ell}^2+1}{\hat{\ell}^2(\hat{\ell}^2+1)^2} - \frac{1}{\hat{\ell}^3}\tan^{-1}(\hat{\ell})\right] \\ b_\ell &= \frac{8a}{5k}\eta^2 q^2_{0\to2}\frac{\hat{\ell}^2}{(\hat{\ell}^2+1)^2} \\ c_\ell &= \frac{4a^2}{5k}\eta^2 q^2_{0\to2}\frac{\hat{\ell}^4}{(\hat{\ell}^2+1)^2} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \hat{\ell} &= (\ell+1/2)/\eta \\ a &= \frac{Z_1 Z_2 e^2}{2E} = \eta/k \\ q_{0\to2} &= \sqrt{\frac{\pi}{5}}\,\frac{\eta}{a^2}\,\frac{\langle 2\|\mathcal{M}(E2)\|0\rangle}{Z_2\, e} \end{aligned} \tag{13}$$

In the above equations, it is assumed that only the target nucleus (A_2, Z_2) is deformed and only the 2^+ excited state is considered. Generalization to include multistep excitation processes $(4^+, 6^+, \ldots)$ and projectile excitation has been done in ref. 8). For a detailed discussion of polarization potentials see Hussein et.al[9].

At sub-barrier energies, where nuclear excitation and absorption is very small, the cross section ratio to Rutherford is easily evaluated and comes out to be

$$\frac{\sigma}{\sigma_{Ruth}}(\theta) = \exp\left\{-\frac{1}{\hbar}\int_{-\infty}^{\infty}dt\, V^{\ell}_{pol}(r(t))\right\} \tag{14}$$

The above formula has been applied to several cases of heavy ion elastic scattering involving deformed targets and its agreement with the data has been quite satisfactory.

Because of the long range nature of $V^{\ell}_{pol}(r)$, we expect that the damping in $\frac{\sigma}{\sigma_{Ruth}}$ arising from it and given by Eq. (14) to be still valid even at above-barrier energies, where Eq. (14) should be replaced by

$$\frac{\sigma}{\sigma_{Ruth}}(\theta) = \frac{\sigma^0}{\sigma_{Ruth}}(\theta)\,\exp\left\{-\frac{1}{\hbar}\int_{-\infty}^{\infty}dt\, V^{\ell}_{pol}(r(t))\right\} \tag{15}$$

To support our claim above, we apply Eq. (15) to the data on $^{18}O + {}^{184}W$, Fig. 3. We generate $\frac{\sigma^0_{el}}{\sigma_{Ruth}}$ from spherical optical model potential suggested by Love et al[6]. Fig. 3 summarize our finding. Clearly Eq. (15) is quite reasonable.

It is important at this point to point out that $\frac{\sigma^0_{el}}{\sigma_{Ruth}}$ is still, in general, not obtainable, in principle, from a straightforward optical model calculation, since at above barrier energies, the short-ranged nuclear excitations come into play. We turn in to the calculation of $\frac{\sigma^0_{el}}{\sigma_{Ruth}}(\theta)$.

Having isolated the long-range Coulomb polarization effect on $\frac{\sigma_{el}}{\sigma_{Ruth}}$, we now treat $\frac{\sigma^0_{el}}{\sigma_{Ruth}}$, Eq. (15), as containing only short-range nuclear coupling effect.

We adopt the sudden approximation, which amounts to neglecting the excitation energies of the excited states. In this limit the coupled channels problem simplifies significantly. If all states of the rotor are included, one reduces the problem to that of an equivalent sphere calculation. Accordingly, one needs but to evaluate the elastic amplitude for a given value of the angle x that specifies the orientation of the symmetry axis with respect to the line that join the centers of the two colliding heavy-ions, at asymptotic distances.

Therefore the "elastic" scattering amplitude, that goes into the calculation of $\frac{\sigma^0_{el}}{\sigma_{Ruth}}$, is given by

$$f(\theta,x) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)\,P_\ell(\cos\theta)\left\{S_\ell^N(x)\,e^{2i\sigma_\ell}-1\right\} \tag{16}$$

The elastic scattering amplitude $f(\theta)$ is obtained from $f(\theta,x)$ by averaging the latter over all orientations

$$f(\theta) = \langle f(\theta,x)\rangle_{\Omega_x} \equiv \frac{1}{4\pi}\int d\Omega_x\, f(\theta,x) \tag{17}$$

It is easy to recognize that the total nuclear inelastic cross section is given by

$$\frac{d\sigma_{inel}}{d\Omega} = \langle |f(\theta,x)|^2\rangle_{\Omega_x} - |\langle f(\theta,x)\rangle_{\Omega_x}|^2 \tag{18}$$

This is so since

$$\frac{d\sigma_{inel}}{d\Omega} = \sum_{IM\neq 0} |\langle IM|f(\theta,x)|00\rangle|^2 \tag{19}$$

where the states $|IM\rangle$ are connected with the x-degree of freedom. Adding and subtracting $|\langle 0|f(\theta,x)|0\rangle|^2 \equiv |\langle f(\theta,x)\rangle_{\Omega_x}|^2$, and using closure on the IM sum, we recover Eq. (18).

It is important to recognize that one may distinguish, using the averaging procedure above, between the real elastic scattering cross section described by $\langle f(\theta,x)\rangle_{\Omega_x}$ and, what may be called, the generalized elastic scattering cross section given by

$$\frac{d\sigma_{gen.}}{d\Omega} = \langle |f(\theta,x)|^2\rangle_{\Omega_x} \tag{20}$$

which, according to Eq. (18), is the sum of the elastic cross section and all inelastic cross sections.

Therefore the "generalized" total reaction cross section attached to $\frac{d\sigma_{gen}}{d\Omega}$, given by

$$\sigma_R^{gen} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left\{1 - \langle |S_\ell^N(x)|^2\rangle_{\Omega_x}\right\} \tag{21}$$

is different from the total reaction cross section, extracted from $\frac{d\sigma}{d\Omega} = |\langle f(\theta,x)\rangle_{\Omega_x}|^2$,

$$\sigma_R = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left\{1 - \left|\langle S_\ell^n(x)\rangle_{\Omega_x}\right|^2\right\} \tag{22}$$

by an amount which corresponds to the total angle-integrated inelastic cross section

$$\sigma_R^{gen.} = \sigma_R - \int d\Omega \, \frac{d\sigma_{inel}}{d\Omega} \tag{23}$$

with $\frac{d\sigma_{inel}}{d\Omega}$ given by Eq. (19). Obviously $\sigma_R^{gen} < \sigma_R$.

The above equations, Eqs. (16)-(23), constitute our sudden approximation to the calculation of $\frac{\sigma^0_{e\ell}}{\sigma_{Ruth}}$.

In our application to heavy ion systems we shall use the above equations both within an optical model description of the scattering, as well as in the sharp cut-off approximation exemplified by Eq. (1).

Before presenting our numerical results obtained within the optical model, we give below the changes, arising from static (short-ranged) deformation effects, on the sharp cut-off cross section, Eq. (1).

In the sharp cut-off limit, equation (16) becomes

$$f(\theta,x) = \frac{1}{2ik}\sum_{\ell=0}^{\ell_{1/4}(x)}(2\ell+1)\, P_\ell(\cos\theta)\, e^{2i\sigma_\ell} \tag{24}$$

where $\ell_{1/4}(x)$ is connected with the x-dependent nuclear radius, through

$$\frac{\hbar^2 \ell^2_{1/4}(x)}{2\mu} = R^2(x)\left\{E - V_C(R(x)) - V_N(R(x))\right\} \tag{25}$$

writing for R(x)

$$\begin{aligned} R(x) &= D + X(x) \\ X(x) &= r_0 A_2^{1/3}\,\beta\sqrt{\frac{5}{4\pi}}\,P_2(\cos x) \end{aligned} \tag{26}$$

The barrier radius R(x) is determined from

$$\frac{d}{dr}\left\{V_C(r) + V_N(r,x) + \frac{\hbar^2\ell^2_{1/4}(x)}{2\mu}\right\} = 0 \tag{27}$$

Simple algebra leads to

$$\frac{\hbar^2}{2\mu}\ell^2_{1/4}(x) = \left(1+\frac{X(x)}{D}\right)^2\left[\frac{\hbar^2}{2\mu}\ell^2_{1/4}(0) + D^2 V_C(D)\left[1-\frac{1}{1+\frac{X(x)}{D}}\right]\right] \tag{28}$$

where D is the nuclear radius when $\beta = 0$.

The expression $f(\theta,x)$ can be evaluated approximately using the stationary phase method, to give

$$f(\theta,x) = \frac{1}{2} f_{Ruth}(\theta)\, \mathrm{erfc}\left\{e^{-i\pi/4}\left[\frac{1}{2}\left|\Theta'_{Coul.}(\lambda_{1/4}(x))\right|\right]^{1/2}(\lambda_{1/4}(x)-\lambda_{sp})\right\} \tag{29}$$

where $\lambda = \ell + 1/2$

Therefore, the cross section ratio to Rutherford is given by

$$\frac{\sigma^0}{\sigma_{Ruth}}(\theta) \simeq F\left\{\left[\frac{\lambda_{1/4}(x)}{2\sin(\vartheta_{1/4}(x))}\right]^{1/2} 2\sin\left(\frac{1}{2}(\theta-\vartheta_{1/4}(x))\right)\right\} \tag{30}$$

where $\theta_g(x)$ is defined by

$$\vartheta_{1/4}(x) = 2\tan^{-1}\left(\frac{\eta}{\lambda_{1/4}(x)}\right) \tag{31}$$

If we perform the average over x on Eq. (28), we obtain the result of Rowley[10]. Such a calculation would result in a "generalized" quarter-point angle related to σ_R^{gen}, Eq. (21).

On the other hand if we average $f(\theta,x)$ and then calculate $\frac{\sigma_{el}}{\sigma_{Ruth}}$, Eq. (30), we should obtain the genuine elastic scattering cross section. In the vicinity of the grazing angle $\theta_{1/4}(x)$, we may write

$$\frac{\sigma^{\circ}}{\sigma_{Ruth.}}(\vartheta) \simeq \frac{1}{4}\left|1 - \frac{2}{\sqrt{\pi}}\left\langle e^{i\pi/4}\, u(\vartheta,\vartheta_{1/4}(x))\right\rangle_{\Omega_x}\right|^2 \tag{32}$$

where $u(\theta,\theta_{1/4}(x))$ is given by

$$u(\vartheta,\vartheta_{1/4}(x)) = \left[\frac{\lambda_{1/4}(x)}{2\sin\left(\frac{1}{2}\vartheta_{1/4}(x)\right)}\right]^{1/2} 2\sin\left(\tfrac{1}{2}(\vartheta-\vartheta_{1/4}(x))\right) \tag{33}$$

To second order in β, the new, deformation-modified, quarter-point angle may be obtained simply by setting

$$\begin{aligned}\left\langle u(\vartheta^{el.}_{1/4},\vartheta_{1/4}(x))\right\rangle_{\Omega_x} &= 0\\ &\approx \frac{\eta^{1/2}}{\sin\left(\frac{1}{2}\vartheta^{\circ}_{1/4}\right)}\Big\{\sin\left(\tfrac{1}{2}(\vartheta^{el.}_{1/4}-\vartheta^{\circ}_{1/4})\right) + \frac{1}{2}\left\langle\frac{X^2}{D^2}\right\rangle_{\Omega_x}\\ &\cdot\sin\left(\tfrac{1}{2}\vartheta^{el.}_{1/4}\right)\cos\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)\Big\{1+4\alpha\tan^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)+\\ &+2\cos^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)\left[1+2\alpha\tan^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)\right]^2\Big\}\Big\}\end{aligned} \tag{34}$$

with $\alpha = \frac{E}{V_0(D)}$, and $\theta^{0}_{1/4}$ is the quarter-point angle when $\beta = 0$

Solving for $\theta^{el}_{1/4}$, we obtain

$$\vartheta^{el.}_{1/4} = \vartheta^{\circ}_{1/4} - \Delta_{static} \tag{35}$$

where

$$\begin{aligned}\Delta_{Static} = \frac{1}{2}\left\langle\frac{X^2}{D^2}\right\rangle_{\Omega_x}\sin(\vartheta^{\circ}_{1/4})\Big\{1+2\frac{D}{a}\tan^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)+\\ +2\cos^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)\left[1+\frac{D}{a}\tan^2\left(\tfrac{1}{2}\vartheta^{\circ}_{1/4}\right)\right]^2\Big\}\end{aligned} \tag{36}$$

where

$$\left\langle\frac{X^2}{D^2}\right\rangle_{\Omega_x} = \frac{1}{4\pi}\frac{R_2^2}{D^2}\beta^2 \tag{37}$$

Clearly when $\beta = 0$, we have $\theta^{el}_{1/4} = \theta^{0}_{1/4}$, as expected.

It is important to recognize that $\theta^{el}_{1/4}$ is still not the quarter-point angle since it represent $\theta_{1/4}$ for $\frac{\sigma^{0}_{el}}{\sigma_{Ruth}}$. The quarter-point angle that should be compared to the data is obtained by setting

$$\frac{\sigma}{\sigma_{Ruth.}}(\vartheta_{1/4}) = \frac{\sigma^{\circ}_{el}}{\sigma_{Ruth}}(\vartheta_{1/4})\exp\left\{-\frac{16}{45}q^2_{0\to2}\,g_2(\xi)\,I(\vartheta_{1/4})\right\} = 1/4 \tag{38}$$

which results in

$$\vartheta_{1/4} = \vartheta^{\circ}_{1/4} - \Delta_{static} + \Delta_{dynamic} \tag{39}$$

where

$$\Delta_{dynamic} = \sqrt{\frac{\pi}{8\eta}}\ \sin(\vartheta^{el}_{1/4})\left\{\exp\left[-\frac{16}{45}q^2_{0\to2}\,g_2(\xi)\,I(\vartheta_{1/4})\right] - 1\right\} \tag{40}$$

The exponent in Eq. (18) was obtained by calculating the integral of Eq. (15), with $V^{\ell}_{pol}(r)$ given by Eq. (11). The factor $g_2(\xi = \eta\,\frac{\Delta E}{2E})$ is a semi-classical energy-loss factor ($g_2(0) = 1$). Finally $I(\theta)$ is given by[8)]

$$I(\theta) = \frac{3}{4}\left\{\sin^4\frac{\theta}{2} + 3\tan^2\frac{\theta}{2}\left\{1 - \frac{1}{2}(\pi-\theta)\tan\frac{\theta}{2}\right\}^2\right\} \tag{41}$$

Our expression for $\theta_{1/4}$, Eq. (39) should supply a reasonable estimate of the total reaction cross section in the case of scattering of deformed heavy-ions. It is interesting to observe that the two correction $\Delta_{dynamic}$ and Δ_{static} are of same signs: both corrections tend to push $\theta_{1/4}$ to small values.

In cases where the generalized elastic scattering differential cross section is considered, e.g., when the experimental resolution is such as not able to isolate the elastic peak from the 2^+ or 4^+ peaks in the spectrum, this are antecipate, as Rowley have done several years ago, that the $\theta^{gen}_{1/4}$ is given by

$$\vartheta^{gen.}_{1/4} = \vartheta^{\circ}_{1/4} - \Delta^{gen.}_{static} \tag{42}$$

Where $\Delta^{gen}_{st.}$ is different from $\Delta_{stat.}$ since it involves the solution of the equation $\langle F(\theta_{1/4}(x))\rangle_{\Omega_x} = \frac{1}{4}$, and not $|\langle f(\theta_{1/4}(x))\rangle_{\Omega_x}|^2 = \frac{1}{4}$.

Substituting Eqs. 4 and 5 in 17.6, and using

Eq. (42), we find

$$\Delta^{gen.}_{stat.} = -\frac{S_1(\vartheta^{\circ}_{1/4})}{S_2(\vartheta^{\circ}_{1/4})}\ , \tag{43}$$

$$S_1(\theta) = \sin\theta\left\{\left[2\sqrt{\frac{\eta}{2\pi}}\cos\frac{\theta}{2} - 2\cos^2\frac{\theta}{2}\right]\left[1 + \frac{D}{a}\tan^2\frac{\theta}{2}\right]^2 - 1 - 2\frac{D}{a}\tan^2\frac{\theta}{2}\right\}$$

$$S_2(\theta) = 2\left\langle\frac{D^2}{X^2}\right\rangle_{\Omega_x} + \cos^2\frac{\theta}{2}\left[2\eta + \cos^2\frac{\theta}{2} - 8\sqrt{\frac{\eta}{2\pi}}\cos\frac{\theta}{2}\right]\cdot\left[1 + \frac{D}{a}\tan^2\frac{\theta}{2}\right]^2 - 2\sqrt{\frac{\eta}{2\pi}}\cos\frac{\theta}{2}\left[1 + 2\frac{D}{a}\tan^2\frac{\theta}{2}\right]$$

In Obtaining Eq. (43), we have used the condition $\frac{\sigma_{gen}}{\sigma_{Ruth}}(\theta^{gen}_{1/4}) = \frac{1}{4}$.

In contrast to the static deformation correction to the pure elastic quarter-point angle, $\Delta^{gen}_{stat.}$ comes out to be negative thus resulting in a <u>larger</u> $\theta^{gen}_{1/4}$. As was discussed by Rowley[10)], this features explains several erroneous interpretations of experimental data when the quarter-point recipe is applied directly to deformed systems, such as $^{84}Kr+^{232}Th$. One may easily end up with an extracted strong absorption radius which is much too small. (E.g. at 500 MeV, it was concluded to Colombini at that the S.A. radius parameter in Kr+Pb is <u>larger</u> than that in Kr+Th!).

IV. NUMERICAL RESULTS

We present in this section the numerical results of our sudden approximation calculation of $\Delta^{gen}_{stat.}$, $\Delta_{stat.}$ and $\Delta_{dyn.}$. The results were obtained both through an optical model calculation and a subsequent averaging, as well as using the approximate analytical results of the preceeding section. In our calculation we take only the target nucleus as deformed. The quadrupole deformation parameter, β, is then related to the experimental $\beta(E2)$ values according to[12)]

$$\beta(1+0.16\beta) = \frac{4\pi}{3}\frac{\sqrt{B(E2)}}{R_2^2 \, Z_2 e} \tag{44}$$

The nuclear radius, R, and the Coulomb barrier position, D, are taken to be[11)]

$$R_i = 1.233\, A_i^{1/3} - 0.978\, A_i^{-1/3} \,[fm]$$
$$D = 1.07\,(A_1^{1/3}+A_2^{1/3}) + 2.72\,[fm] \tag{45}$$

To start with, we present in Fig. 4, the result of the equivalent sphere optical model calculation of $\frac{\sigma_{gen}}{\sigma_{Ruth}}$ for the system $^{16}O + {}^{28}Si$ at $E_{C.M.} = 35$ MeV, using for the spherical potential. The E18 complex interaction[13)], with the parameters

$$V = 10.0\ \text{Mev},\quad r_{oV} = 1.35\,fm,\quad a_V = 0.618\,fm.$$
$$W = 23.4\ \text{Mev},\quad r_{oW} = 1.23\,fm,\quad a_W = 0.552\,fm.$$

Although the $^{16}O + {}^{28}Si$ system is not so strongly deformed, we have chosen it for our discussion and varied the

deformation parameter of ^{28}Si arbitrarily. Figure 4 shows our results for the generalized elastic cross-section ratio to Rutherford, indicating clearly the shifting to higher values of $\theta^{gen.}_{1/4}$ in accordance with our discussion above. In figure 5 we show $\theta^{gen.}_{1/4}$ vs. β. Also shown is the result obtained by Rowley using the sharp cut-off model. At small β, both calculations yield similar $\theta^{gen}_{1/4}$. At larger β, however, significant deviation occur. For qualitative purposes, however, the Fresnel model should be quite adequate.

In Table 1, we compare our approximate calculation of $\theta^{gen}_{1/4}$, Eq. (42), with those of Refs. 14), 15) obtained from coupled channels calculation, $\theta_{1/4}$. The background spherical $\theta^{0}_{1/4}$ is extracted from optical model calculation. It is clear that our approximate formula for $\theta^{gen}_{1/4}$ is quite adequate. In Table 2, we present our results for the genuine elastic quarter-point, angle $\theta_{1/4}$ and the extracted spherical $\theta^{0}_{1/4}$, which comes out very close to the one $\theta^{sph.}_{1/4}$ obtained from optical model calculation. In our calculation of $\theta^{0}_{1/4}$ we employed Eq. (39). We note that $\Delta_{st.}$ is intrinsically positive, in clear contrast to Δ^{gen}_{st}. Once the value of $\theta^{0}_{1/4}$ is extracted, a deduced value of the strong absorption radius may then be obtained. We have attempted to extract these radii for Kr+Pb and Kr+Th using our approximate formulae. The results were not satisfactory owing to the large deformation effects in Kr+Th scattering. On the other hand the general trend comes out reasonable.

V. CONCLUSIONS

In this paper we have investigated the deformation effects on the heavy-ion quarter-point angle. In cases where the summed elastic plus inelastic cross section is measured (for very strongly deformed systems), the generalized quarter-point angle is obtained in the form

$$\theta_{1/4}^{gen.} = \theta_{1/4}^{0} - \Delta_{static}^{gen.}$$

where $\theta^0_{1/4}$ corresponds to zero deformation and Δ^{gen}_{stat} is negative.

When the elastic component is clearly identified and measured, we have then

$$\theta_{1/4} = \theta_{1/4}^{0} - \Delta_{static} - \Delta_{dynamic}$$

when the corrections $\Delta_{stat.}$ and $\Delta_{dyn.}$, which are both positive, arise from static (short-range) and dynamic (long-range) deformation effects. The above expression can be used in conjunction with the data to obtain the spherical quarter-point angle, through which an unambiguous strong absorption radius may be extracted. Our analyses of several systems demonstrated the reasonableness of our procedure.

It is obvious that our discussion can be easily extended to the treatment, within the sudden approximation, vibrational nuclei. The averaging then corresponds to taking into account the zero-point motion[17), 18)].

APPENDIX A

In this appendix we present the details of the calculation of the generalized elastic scattering cross section in the short cut-off model (Fresnel model) and for small values of the deformation parameter.

Using the identity

$$\frac{2\mu}{\hbar^2} = \frac{4E}{D^2 V_c^2(D)}\eta^2 = \frac{4\alpha}{D^2 V_c(D)}\eta^2 \tag{A.1}$$

and the approximation $\lambda_{1/4} \approx \ell_{1/4}$ in Eq. (31), we may rewrite Eq. (28) as[10)]

$$\cot^2\left(\tfrac{1}{2}\theta_{1/4}(x)\right) = \cot^2\left(\tfrac{1}{2}\theta_{1/4}^0\right) + 2\frac{X}{D}\left\{\cot^2\left(\tfrac{1}{2}\theta_{1/4}^0\right) + 2\alpha\right\} + \frac{X^2}{D^2}\left\{\cot^2\left(\tfrac{1}{2}\theta_{1/4}^0\right) + 4\alpha\right\} \tag{A.2}$$

Writing now

$$\theta_{1/4}(X) = \theta_{1/4}^0 + \Delta \tag{A.3}$$

we have from (A.2),

$$\Delta = -\frac{1}{2}\sin\left(\theta_{1/4}^0\right)\left\{\frac{2X}{D}\left[1 + 2\alpha\tan^2\left(\tfrac{1}{2}\theta_{1/4}^0\right)\right] + \frac{X^2}{D^2}\left[1 + 4\alpha\tan^2\left(\tfrac{1}{2}\theta_{1/4}^0\right)\right]\right\} \tag{A.4}$$

We remind the reader that the averages of X and X^2 are

$$\langle X \rangle_{\Omega_x} = 0$$

$$\langle X^2 \rangle_{\Omega_x} = \frac{1}{4\pi} R_2^2 \, \beta^2 \tag{A.5}$$

Using Eqs. (A.4), (20) and (3), we find for the generalized elastic scattering cross section in the vicinity of the quarter-point angle,

$$\begin{aligned} \frac{d\sigma_{gen.}}{d\sigma_{Ruth}}(\theta) &= \langle F[u(\theta, \theta_{1/4}(x))] \rangle \\ &\approx F[u(\theta, \theta^0_{1/4})] \\ &+ \left\langle \frac{X^2}{D^2} \right\rangle_{\Omega_x} \Big\{ \sin^2(\theta^0_{1/4}) \left[1 + 2\alpha \tan^2\left(\frac{\theta^0_{1/4}}{2}\right)\right]^2 \cdot \\ &\qquad \cdot F''[u(\theta, \theta^0_{1/4})] \\ &\qquad - \frac{1}{2} \sin(\theta^0_{1/4}) \left[1 + 4\alpha \tan^2\left(\frac{\theta^0_{1/4}}{2}\right)\right] \cdot \\ &\qquad \cdot F'[u(\theta, \theta^0_{1/4})] \Big\} \end{aligned} \tag{A.6}$$

where

$$u(\theta, \theta^0_{1/4}) = \eta^{1/2} \frac{\sin\left[\frac{1}{2}(\theta - \theta^0_{1/4})\right]}{\sin\left(\frac{1}{2}\theta^0_{1/4}\right)} \tag{A.7}$$

In Eq. (A.6), F' and F" are the first and second derivation of F at $\theta^0_{1/4}$.

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TABLE CAPTIONS

TABLE 1 : Calculation of $\theta^{gen}_{1/4}$ for the systems $^{134}Xe + ^{152}Sm$ and $^{40}Ar + ^{238}U$ (see Ref. 14 and text).

TABLE 2 : Calculation of $\theta^{0}_{1/4}$ from $\theta_{1/4}$ for $^{12}C + ^{184}W$ and $^{18}O + ^{184}W$ (see Refs. 5, 6 and text).

FIGURE CAPTIONS

Fig. 1 - Elastic scattering angular distributions of ^{84}Kr on ^{208}Pb and ^{232}Th at $E_{Lab.} = 500$ MeV (from Ref. 4).

Fig. 2 - Elastic scattering angular distributions of ^{18}O on ^{208}Pb and ^{184}W at $E_{Lab.} = 90$ MeV (from Ref. 5).

Fig. 3 - Elastic scattering angular distribution of ^{18}O on ^{184}W at $E_{Lab.} = 90$ MeV. The solid curve corresponds to Eq. (15) (see text) while the dashed curve is obtained by including the dynamic polarization potential in a conventional optical model calculation[6].

Fig. 4 - Generalized elastic scattering angular distribution for $^{16}O + ^{28}Si$ at $E_{C.M.} = 35$ MeV for $\beta = 0$ (dashed curve) and $\beta = 0.3$ (solid curve). The equivalent sphere optical model was employed.

Fig. 5 - $\theta^{gen.}_{\frac{1}{4}}$ vs. β for $^{16}O + ^{28}Si$ at $E_{C.M.} = 35$ MeV. The dashed curve was obtained using the optical model, whereas the solid curve is based on the Fresnel model. The equivalent sphere method was employed.

SYSTEM	E_{LAB}	$\Theta^0_{1/4}$	$\Theta_{1/4}$	$\Theta^{gen}_{1/4}$
$^{134}Xe + ^{152}Sm$	690 MeV	94.0	96.0	95.3
$^{40}Ar + ^{238}U$	340 MeV	51.9	52.6	52.4

TABLE 1

SYSTEM	E_{LAB}	$\Theta_{1/4}$	$\Theta^{sph}_{1/4}$	$\Theta^0_{1/4}$
$^{12}C + ^{184}W$	70 MeV	80.5	84.7	84.3
^{18}O ^{184}W	90 MeV	84.0	90.5	91.0

TABLE 2

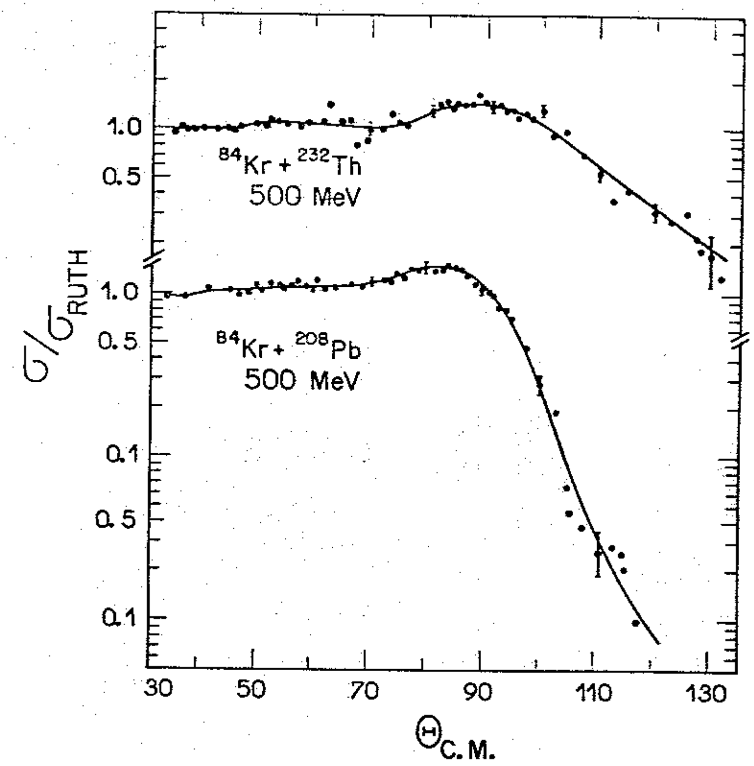
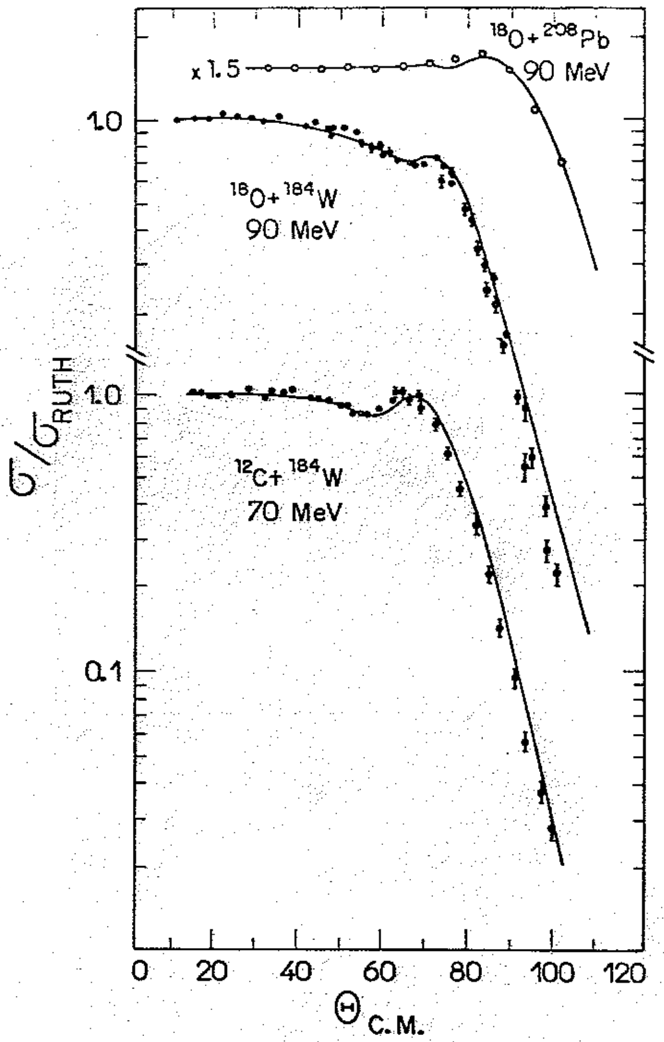
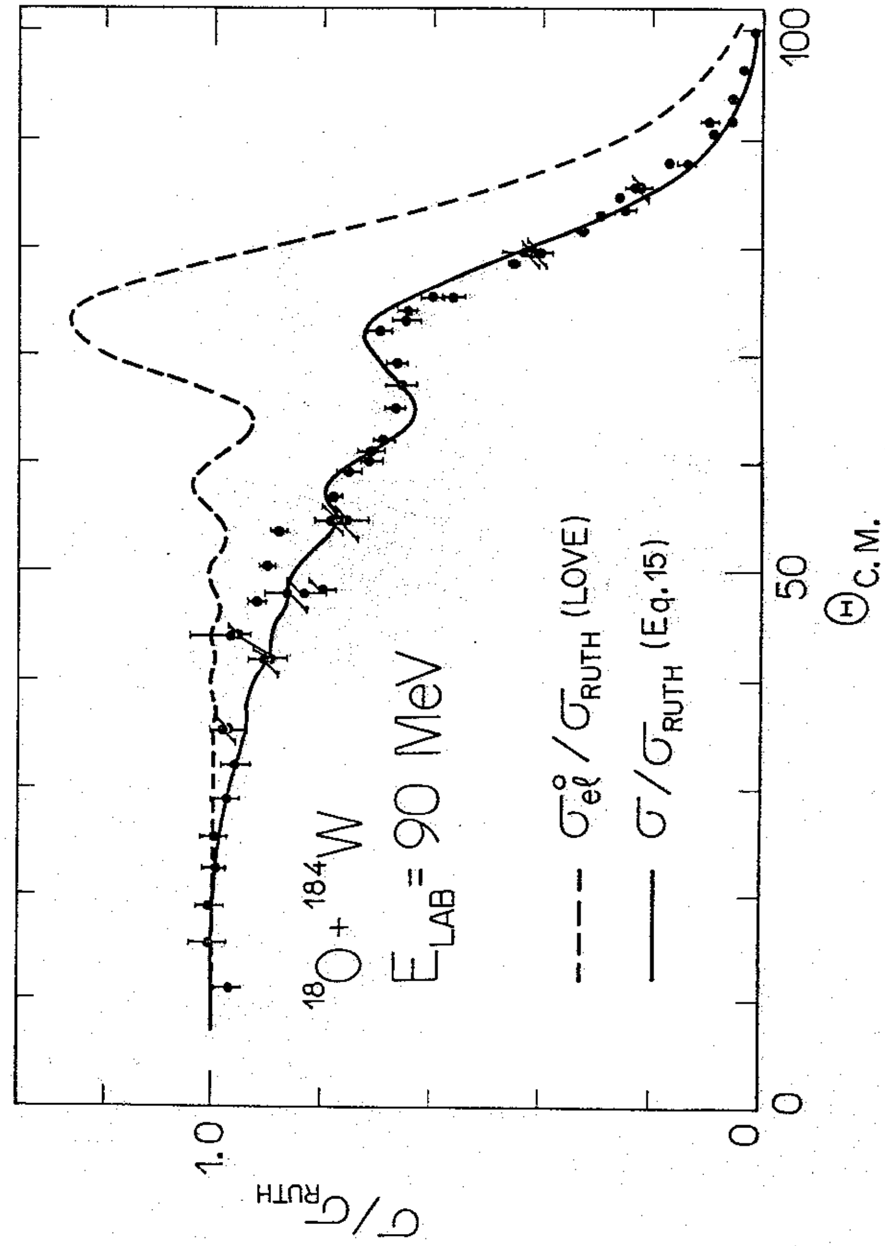


Fig. 1



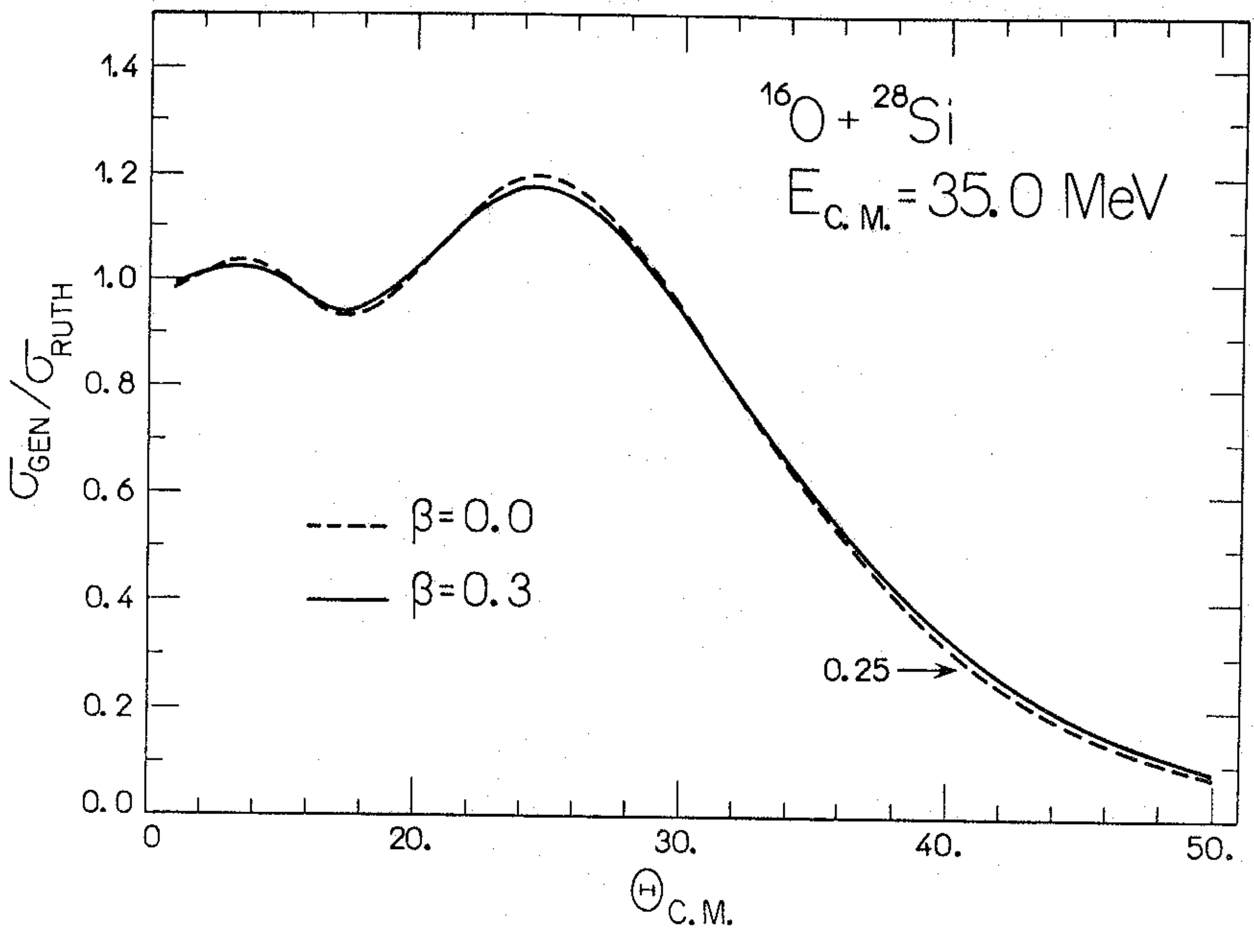
18O+208Pb
90 MeV
x 1.5
1.0
18O+184W
90 MeV
1.0
12C+184W
70 MeV
0.1
σ/σRUTH
0 20 40 60 80 100 120
ΘC.M.

Fig. 2



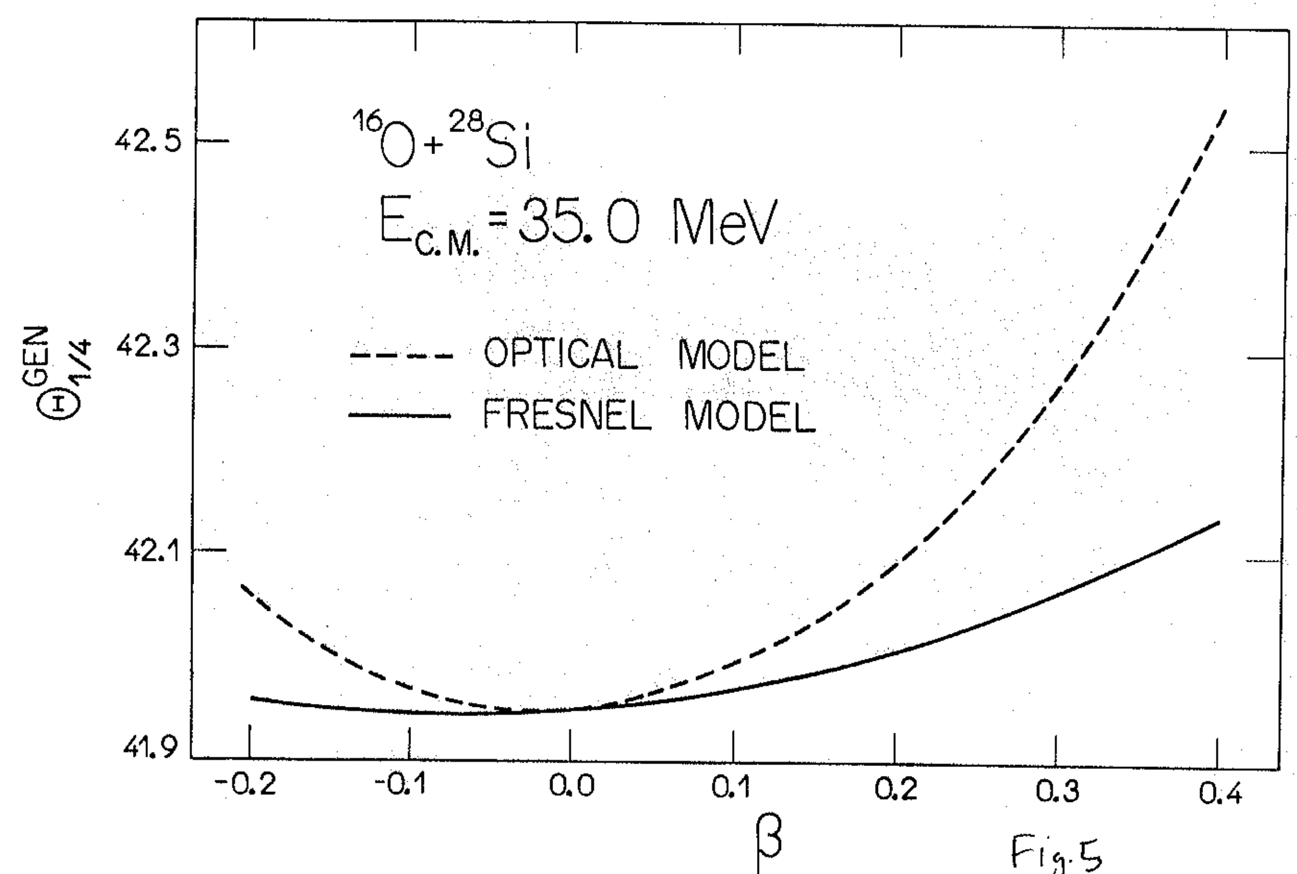
18O+184W
ELAB = 90 MeV
σ°el/σRUTH (LOVE)
σ/σRUTH (Eq. 15)
1.0
0
σ/σRUTH
0
50
100
ΘC.M.

Fig. 3



$^{16}O + ^{28}Si$
$E_{C.M.} = 35.0$ MeV
$\beta = 0.0$
$\beta = 0.3$
0.25
$\sigma_{GEN}/\sigma_{RUTH}$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20.
30.
40.
50.
$\Theta_{C.M.}$

Fig. 4



$^{16}O + ^{28}Si$
$E_{C.M.} = 35.0$ MeV
OPTICAL MODEL
FRESNEL MODEL
$\Theta^{GEN}_{1/4}$
42.5
42.3
42.1
41.9
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
β

Fig. 5