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NEW CONTRIBUTIONS TO THE COSMOLOGICAL CONSTANT

by

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ABSTRACT

We show that the quantized gravitino sector of  $N=1$ ,  $D=11$  supergravity leads to new contributions to the  $D=4$  dimensional cosmological constant, in addition to the one coming from the field strength of the three index totally anti-symmetric tensor field. This new contributions arise from the vacuum expectation values of the different auxiliary fields introduced in order to eliminate the quartic gravitino interaction terms.

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Many works can be found in the literature (Deser and Zumino (1977), Cremmer et al. (1983), Rubakov and Shaposhnikov (1983), Hawking (1984), Antoniadis and Tsamis (1984)) which try to find out a mechanism to explain the vanishing of the cosmological constants. In principle one would like to have a symmetry which implied  $\Lambda = 0$ , however one has not found yet such a symmetry or convincing arguments in favour of the choice  $\Lambda = 0$ . Besides the zero point energies of quantum fluctuations, one finds that broken symmetries, supersymmetry, topological fluctuation of the metric (Hawking (1978)), a potential  $V(\phi)$  of a constant scalar field  $\phi$ , contribute to the vacuum energy leading to an effective cosmological constant. One can also find contributions coming from higher dimensional theories of (super) gravity (Rubakov and Shaposhnikov (1983)), Duff and Orzalesi (1982)).

Here we want to point out new contributions to the cosmological constant in  $N=8$   $D=4$  supergravity coming from dimensional reduction of  $N=1$   $D=11$  supergravity theory, where the gravitino sector has been quantized (Jasinschi and Smith (1984)).

We have already shown (Jasinschi and Smith (1983)) that starting from pure  $D=4$  supergravity one can generate dynamically a cosmological constant. Let us now try to follow the same procedure starting from  $N=1$   $D=11$  supergravity.

The Lagrangian is given by

$$L = -\frac{1}{2} eR(e, \omega) - \frac{e}{2} \bar{\psi}_M \Gamma^{MNP} D_N \left( \frac{\omega + \hat{\omega}}{2} \right) \psi_P - \\ - \frac{e}{48} F_{MNPQ}^2 - \frac{3}{4} De \left[ \bar{\psi}_M \Gamma^{MNPQRS} \psi_S + 12 \bar{\psi}^N \Gamma^{PQ} \psi^R \right] \times \\ \times \left[ F_{NPQR} + \hat{F}_{NPQR} \right] + C \varepsilon^{M_1 \dots M_{11}} F_{M_1 \dots M_4} F_{M_5 \dots M_8} A_{M_9 \dots M_{11}} \quad (1)$$

where

$$C = -\sqrt{2} i/3456, \quad D = \sqrt{12}/144 \quad (1.1)$$

$$F_{MNPQ} = \partial_{[M} A_{NPQ]} \quad (1.2)$$

$$\hat{F}_{MNPQ} = F_{MNPQ} - 3\bar{\psi}_{[M} \Gamma_{NP} \psi_{Q]} \quad (1.3)$$

$$D_M = \partial_M + \frac{1}{4} \omega_M^{AB} \Gamma_{AB} \quad (1.4)$$

$$\hat{\omega}_M^{AB} = \omega_M^{AB} - \frac{1}{8} \bar{\psi}_N \Gamma_M^{ABNP} \psi_P \quad (1.5)$$

$$\omega_M^{AB} = \omega_M^{AB}(e) + \kappa_M^{AB} \quad (1.6)$$

$$\kappa_M^{AB} = \frac{1}{8} \bar{\psi}_N \Gamma_M^{ABNP} \psi_P + \frac{1}{4} \left[ \bar{\psi}_M \Gamma^A \psi^B - \bar{\psi}_M \Gamma^B \psi^A + \bar{\psi}^A \Gamma_M \psi^B \right] \quad (1.7)$$

and

$$R(e, \omega) = e_A^M e_B^N R_{MN}^{AB} \quad (1.8)$$

with

$$R_{MN}^{AB} = \partial_M \omega_N^{AB} + \omega_M^{AC} \omega_{NC}^B - (M \leftrightarrow N) \quad (1.9)$$

For further definitions and notations we refer to (P. van Nieuwenhuizen (1981)).

If one uses the gravitino gauge

$$\Gamma^M \psi_M = 0 \quad (2)$$

and Fierz re-arrangement, one can write (1) as

$$\begin{aligned}
 L = & -\frac{1}{2} e R(e) - \frac{e}{2} \left\{ e_A^M e_B^N \left[ \delta_M \left[ \frac{1}{8} (\bar{\psi}_P \Gamma_N^{ABPQ} \psi_Q) + \right. \right. \right. \\
 & + \frac{1}{4} (\bar{\psi}_N \Gamma^A \psi^B - \bar{\psi}_N \Gamma^B \psi^A + \bar{\psi}^A \Gamma_N \psi^B) \left. \right] + \omega_{MC}^A(e) \left[ \frac{1}{8} \bar{\psi}_P \Gamma_N^{BCPQ} \psi_Q + \right. \\
 & + \frac{1}{4} (\bar{\psi}_N \Gamma^B \psi^C - \bar{\psi}_N \Gamma^C \psi^B + \bar{\psi}^B \Gamma_N \psi^C) + \omega_{NC}^B(e) \left[ \frac{1}{8} \bar{\psi}_P \Gamma_M^{ACPQ} \psi_Q + \right. \\
 & + \frac{1}{4} (\bar{\psi}_M \Gamma^A \psi^C - \bar{\psi}_M \Gamma^C \psi^A + \bar{\psi}^A \Gamma_M \psi^C) \left. \right] - (M \leftrightarrow N) \left. \right\} - \frac{29e}{1024} (\bar{\psi}_M \psi^M)^2 + \\
 & - \frac{21}{32} e (\bar{\psi}_{[N} \Gamma_P \psi_{Q]})^2 - \frac{19e}{3072} (\bar{\psi}_M \Gamma_{NPQ} \psi^M)^2 + \frac{5e}{64} (\bar{\psi}_M \Gamma_{NPQ} \psi^M) \bar{\psi}_{[N} \Gamma_P \psi_{Q]} + \\
 & + \frac{181e}{6144} (\bar{\psi}_M \Gamma^{NPQR} \psi^M)^2 + \frac{(23167 - 24\sqrt{2})e}{3072} (\bar{\psi}_M \Gamma^{NPQR} \psi^M) (\bar{\psi}_{[N} \Gamma_{PQ} \psi_{R]}) + \\
 & + \frac{9\sqrt{2}}{32} e (\bar{\psi}_{[N} \Gamma_{PQ} \psi_{R]})^2 - \frac{1}{2} e \bar{\psi}_M \Gamma^{MNP} \partial_N \psi_P - \frac{1}{8} e (\bar{\psi}_M \Gamma^{MNP} \Gamma_{AB} \psi_P) \omega_N^{AB}(e) - \\
 & - \frac{e}{48} F_{MNPQ}^2 - \frac{\sqrt{2}e}{32} \bar{\psi}_{[N} \Gamma_{PQ} \psi_{R]} F_{NPQR} + \frac{\sqrt{2}e}{384} (\bar{\psi}_M \Gamma^{NPQR} \psi^M) F_{NPQR} - \\
 & - \frac{\sqrt{2}i}{3456} \varepsilon^{M_1 \dots M_{11}} F_{M_1 \dots M_4} F_{M_5 \dots M_8} A_{M_9 \dots M_{11}} \quad (3)
 \end{aligned}$$

In order to integrate over  $\psi_M$  one eliminates the quartic gravitino terms by introducing the auxiliary fields  $\sigma(x)$ ,  $\alpha_{MNP}(x)$ ,  $\beta_{MNP}(x)$ ,  $\gamma_{MNPQ}(x)$  and  $\Delta_{MNPQ}(x)$  (Jasinski and Smith (1984)). The fields  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\Delta$  are totally antisymmetric in their indices. The effective action, after quantization is:

$$\begin{aligned}
 S_{\text{eff}} = & \frac{1}{2} \text{Tr} \ln \Delta_{MN} + \int dx^{11} e \left\{ \sigma^2 + \alpha_{MNP}^2 - \beta_{MNP}^2 - \gamma_{MNPQ}^2 - \right. \\
 & - \Delta_{MNPQ}^2 - \frac{1}{48} F_{MNPQ}^2 - \frac{1}{2} R(e) - \\
 & \left. - \frac{\sqrt{2}ki}{3456} \varepsilon^{M_1 \dots M_{11}} F_{M_1 \dots M_4} F_{M_5 \dots M_8} A_{M_9 \dots M_{11}} \right\} \quad (4)
 \end{aligned}$$

where

$$\begin{aligned}
 \Delta_{MN} = & i \left\{ \frac{\partial_Q}{e} \left[ e_A^Q e_B^P e \right] \left[ \frac{1}{8} \Gamma_{PMN}^{AB} + \frac{1}{4} \left[ \delta_{PM} \Gamma^A \delta_N^B - \right. \right. \right. \\
 & - \delta_{PM} \Gamma^B \delta_N^A + \delta_M^A \Gamma_P \delta_N^B \left. \right] + \omega_{QC}^A(e) \left[ \frac{1}{8} \Gamma_{PMN}^{BC} + \right. \\
 & + \frac{1}{4} (\delta_{MP} \Gamma^B \delta_N^C - \delta_{MP} \Gamma^C \delta_N^B + \delta_M^B \Gamma_P \delta_N^C) \left. \right] + \\
 & + \omega_{PC}^B(e) \left[ \frac{1}{8} \Gamma_{QMN}^{AC} + \frac{1}{4} (\delta_{QM} \Gamma^A \delta_N^C - \delta_{QM} \Gamma^C \delta_N^A + \right. \\
 & + \delta_N^A \Gamma_Q \delta_N^C) \left. \right] - (M \leftrightarrow N) \left. \right\} - \frac{1}{4} \Gamma_{MPN} \Gamma_{AB} \omega^{PAB}(e) - \\
 & - \Gamma_{MPN} \partial^P + \frac{\sqrt{29}}{8} \delta_{MN} \sigma + \alpha^{PQR} \left[ \frac{\sqrt{21}}{2} \delta_{M[P} \Gamma_Q \delta_{R]N} + \right. \\
 & + \frac{9\sqrt{21}}{8} \delta_{MN} \Gamma_{PQR} \left. \right] + \frac{\sqrt{5225453}}{693733024} \beta^{PQR} \delta_{MN} \Gamma_{PQR} + \\
 & - \gamma^{PQRS} \left[ \frac{\sqrt{181}}{384} \delta_{MN} \Gamma_{PQRS} + \frac{23167 - 24\sqrt{2}}{181} \frac{\sqrt{181}}{384} \delta_{M[P} \Gamma_{QR} \delta_{N]S} \right] + \\
 & + \frac{\sqrt{536611041 - 799248\sqrt{2}}}{69504} \Delta^{PQRS} \delta_{M[P} \Gamma_{QR} \delta_{N]S} - \\
 & - \frac{\sqrt{2}}{16} F^{PQRS} \delta_{PM} \Gamma_{QR} \delta_{SN} - \frac{\sqrt{2}}{192} \delta_{MN} \Gamma_{PQRS} F^{PQRS} + \\
 & + \bar{C}^\alpha \left\{ \Gamma^J \left[ \frac{1}{8} (\delta_{JM} \delta_N^B \Gamma^A - \delta_{JM} \Gamma^B \delta_N^A + \delta_{AM} \delta_{BN}^B \Gamma_M) \right] \Gamma_{AB} - \right. \\
 & \left. - \frac{\sqrt{2}}{48} (\Gamma_{PQRSJ} - \delta_{PJ} \Gamma_{QRS}) \delta_M^P \delta_N^S \right\} C^\beta \quad (5)
 \end{aligned}$$

and

$C^\alpha$  are the supergravity ghost-fields.

Performing the dimensional reduction of (4) down to

the four-dimensional space-time, one can collect the following contributions to the cosmological constant in the reduced theory:

a) in order to avoid infrared divergences appearing at the quantum level it is possible to turn the gravitino massive by allowing any of the fields  $\sigma(x)$ ,  $\gamma_{\mu\nu\rho\sigma}(x)$ ,  $\gamma_{ijkl}(y)$ ,  $\Delta_{\mu\nu\rho\sigma}(x)$  and  $\Delta_{ijkl}(y)$  to acquire a non-vanishing vacuum expectation value (v.e.v.) and performing the shiftings:

$$\sigma'(x) = \sigma(x) - \sigma_0 \quad (6)$$

$$\gamma'_{\mu\nu\rho\sigma}(x) = \gamma_{\mu\nu\rho\sigma}(x) - \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{e}} C_1 \quad (7)$$

$$\gamma'_{ijkl}(y) = \gamma_{ijkl}(y) - \gamma_{ijkl}^0 \quad (8)$$

$$\Delta'_{\mu\nu\rho\sigma}(x) = \Delta_{\mu\nu\rho\sigma}(x) - \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{e}} C_2 \quad (9)$$

$$\Delta'_{ijkl}(y) = \Delta_{ijkl}(y) - \Delta_{ijkl}^0 \quad (10)$$

where Greek indices run over space-time with coordinates  $x^\mu$  and Latin indices over the extra dimensions with coordinates  $y^m$  and

$$\langle \sigma(x) \rangle = \sigma_0 \quad (11)$$

$$\langle \gamma_{\mu\nu\rho\sigma}(x) \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{e}} C_1 \quad ; \quad \langle \gamma_{ijkl}(y) \rangle = \gamma_{ijkl}^0 \quad (12)$$

$$\langle \Delta_{\mu\nu\rho\sigma}(x) \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{e}} C_2 \quad ; \quad \langle \Delta_{ijkl}(y) \rangle = \Delta_{ijkl}^0 \quad (13)$$

Performing the dimensional reduction of  $\langle \gamma_{[MNPQ]} \rangle$  we see that

Lorentz invariance in four dimensions allows only the terms  $\langle \gamma_{[\mu\nu\rho\sigma]} \rangle$  and  $\langle \gamma_{[ijkl]} \rangle$  to be different from zero;

b) by the reason pointed out in (a), only the terms  $\langle \alpha_{[ijk]} \rangle$  and  $\langle \beta_{[ijk]} \rangle$  survive from the reduction of  $\langle \alpha_{[MNP]} \rangle$  and  $\langle \beta_{[MNP]} \rangle$  respectively. One should notice that at the classical level we have, for example

$$\alpha_{[MNP]} \sim \bar{\Psi}_{[M} \Gamma_N \Psi_{P]} \quad (14)$$

$$\gamma_{[MNPQ]} \sim \bar{\Psi}_{[M} \Gamma_{NP} \Psi_{Q]} \quad (15)$$

so that the v.e.v. of  $\alpha_{[MNP]}$ ,  $\gamma_{[MNPQ]}$  is proportional to  $\bar{\Psi}_{[M} \Gamma_N \Psi_{P]}$  and  $\bar{\Psi}_{[M} \Gamma_{NP} \Psi_{Q]}$  respectively. Duff and Orzalesi (1983) also used non-vanishing v.e.v. for the gravitino bilinears, but in order to establish a geometrical mechanism (parallelizability of the seven sphere (Wu (1984))) by means of which the cosmological constant in four dimensional space-time is zero;

c) the tensor  $F_{MNPQ}$  is allowed to contribute with  $F_{\mu\nu\rho\sigma}$  and  $F_{ijkl}$  in the reduced theory. So

$$\langle F_{\mu\nu\rho\sigma}(x) \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{e} C_3 \quad (16)$$

$$\langle F_{ijkl}(y) \rangle = F_{ijkl}^0 \quad (17)$$

After collecting all these contributions, the cosmological constant reads

$$\Lambda_{\text{eff}} = \sigma^2 + (\alpha_{[ijk]}^0)^2 - (\beta_{[ijk]}^0)^2 - (\gamma_{[ijk]}^0)^2 -$$

$$\begin{aligned}
& - (\Delta_{[ijk\ell]}^c)^2 - \frac{1}{e} (\varepsilon_{\mu\nu\lambda\rho} C_1)^2 - \frac{1}{2} (\varepsilon_{\mu\nu\lambda\rho} C_2)^2 - \\
& - \frac{1}{48} (F_{[ijk\ell]})^2 - \frac{1}{48e} (\varepsilon_{\mu\nu\lambda\rho} C_3)^2. \quad (18)
\end{aligned}$$

Expression (18) leads us to conclude that not only the rank three totally anti-symmetric tensor field in  $N=1, D=11$  supergravity contributes, after dimensional reduction, to the cosmological constant, represented by the last two terms. Other contributions appear when we quantize the gravitino sector in the  $N=1, D=11$  supergravity coming from the various v.e.v. of the auxiliary fields. The mechanism proposed by Duff and Orzalesi for the vanishing of the cosmological constant does not necessarily imply any quantum effect, so that we wonder if after quantizing the gravitino sector their argument continues to be true, because the vacuum structure can be changed.

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