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by

M.S. Hussein
Instituto de Física, Universidade de São Paulo

and

K.W. McVoy
Physics Department, University of Wisconsin,
Madison, Wisconsin 53706

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M. S. Hussein
Instituto de Física, Universidade de São Paulo, São Paulo, Brazil

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K. W. McVoy
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

Abstract

Grazing-angle singles spectra for projectile fragments from nuclear collisions exhibit a broad peak centered near the beam velocity, suggesting that these observed fragments play only a "spectator" role in the reaction. Using only this spectator assumption (but not DWBA), we find that a "prior form" formulation of the reaction leads, via closure, to a $\langle\phi|W|\phi\rangle$-type estimate of the inclusive spectator spectrum, thus relating it to the reaction cross section for the "participant" with the target. We show explicitly that this expression includes an improved multi-channel version of the Udagawa-Tamura formula for the "breakup-fusion" or incomplete fusion cross section, and identifies it as the fluctuation part of the participant-target reaction cross section.

A Glauber-type estimate of the distorted wave functions which enter clearly shows how the width of the peak in the spectator spectrum arises from the "Fermi motion" within the projectile, as in the simple Serber model, but is modified by the "overlap geometry" of the collision.

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A class of nuclear reactions which has attracted considerable attention in recent years is that of projectile fragmentation, identified by the detection of at least one fragment of the projectile ("the spectator") near its grazing angle, where its spectrum is found to peak near the beam velocity. The simplest measurable aspect of the reaction, both experimentally and theoretically, is the inclusive or "singles" cross section, i.e., the determination of the energy and/or angular distribution of the spectator alone. Our purpose here is to provide an especially simple and direct formulation of the theory of this most elementary reaction, in order to correlate several previous treatments and to provide considerable insight into elaborate numerical calculations which have been performed within the DWBA model.

In the notation which has become conventional, the reaction is described as

$$a + A \rightarrow b + x + A^* \rightarrow b + X\ , \qquad (1.1)$$

indicating that only $\underline{b}$ is detected (thus including both bound and unbound states of $X = x + A$). Although in general $\underline{b}$ could of course have interacted strongly with the target A, it is presumed, in those cases where its spectrum is found to peak near the beam velocity, to have played a passive or spectator role in the reaction. Employing this assumption, all models constructed to date have considered the reaction to be basically a collision of the "participant" $\underline{x}$ with A, in which $\underline{b}$ is indeed treated as a spectator, and permitted to scatter on A, if at all, only elastically. By far the simplest description possible is that of the Serber model[1], in which both the spectator and the incident projectile are described by plane waves. This leads ~~to~~ the appealingly simple on-shell formula

$$\frac{d^2\sigma}{d\Omega_b dE_b} \sim \sigma_{xA}^T |\tilde{\phi}_a(q_b)|^2 \ \rho(E_b) \ , \qquad (1.2)$$

where σ_{xA}^T is the total cross section for xA scattering, and $|\tilde{\phi}_a(q_b)|^2$ is the momentum distribution of the x-b relative motion inside the projectile, evaluated at the momentum transfer to b; $\rho(E_b)$ is the final-state density for b. $|\tilde{\phi}_a(q_b)|^2$ peaks at $q_b = 0$ (b remains at beam velocity), and in this simplified model it is entirely responsible for the energy and angular distribution of the spectator. As we shall see, this is the physical origin of these distributions even in a more realistic calculation, but they are substantially modified by the absorption which the plane-wave model of course omits. The appearance of the total xA cross section means that all possible xA reactions are included, from elastic scattering to complete fusion (including what has been called "breakup-fusion").

The Serber model has met with modest success in explaining the shapes of forward-angle spectra, but considerably more realistic calculations are now available. Aarts[2] generalized the plane wave approach to include Coulomb interactions (but not nuclear absorption) and compared it with his extensive coincidence as well as singles data, in addition to more accurate DWBA calculations. Friedman[3] argues that the absorptive bA interaction requires the collision to be a fairly peripheral one, making it sensitive not to the entire interior of the projectile wave function $\phi_a(\vec{r}_b-\vec{r}_x)$, but only to its surface region, and he finds an impressive correlation between the width of the quasi-free

bump in $d\sigma/dE_b$ ~~and the binding energy for the a → b + x~~ breakup mode. For relativistic data, Hüfner and Nemes[4] come to much the same conclusion within the limits of the Glauber approximation, employing closure to sum ~~over the~~ unobserved states of the xA system.

The alternative theoretical approach which has been studied, for lower energy data, is the DWBA (or DWIA), again a spectator approach and again using closure. Bauer and collaborators[5-7] have performed an extensive series of DWBA calculations for the breakup of light projectiles like deuterons and alphas. They employed the post form of DWBA, as well as a zero-range approximation for $\phi_a(\vec{r}_b-\vec{r}_x)$ and a surface approximation for $\chi(\vec{r}_x-\vec{r}_A)$. They were quite successful in fitting both singles and coincidence data, and in particular found that elastic breakup, in which A is left in its ground state, is generally a small component of the b spectrum. Finally, Udagawa et al.[8-10], considering what they designate as a "breakup fusion" reaction, have been able to write the cross section for observing the spectator particle in the appealing form

$$\frac{d^2\sigma}{d\Omega_b dE_b} = 2\langle\phi_x^{(+)}|W_{xA}|\phi_x^{(+)}\rangle \ \rho(E_b)/\hbar v_a \qquad (1.3)$$

in which they identify $\phi_x^{(+)}$ as the wave function for x-A relative motion, after the a → x + b breakup. They find this result to produce a fairly acceptable fit to E_b spectra for lighter heavy ions, at the upper ends of the spectra, but in general to underestimate these spectra by as much as a factor of 10 at their lower ends.

We conjecture that part of the reason for this underestimate is due to the constraint imposed by these authors that the projectile breakup occur before the xA interaction. In general these events could occur in either order, or simultaneously, and since there is no experimental way to distinguish the various orders, all should be summed over to obtain the singles b-spectrum. As we demonstrate below, if this is done, one obtains for the net b-spectrum (employing only the spectator approximation, but no DWBA assumption) the similar but even simpler result for the "xA reaction cross section" (i.e., excluding elastic breakup),

$$\frac{d^2\sigma_R}{d\hat{\Omega}_b dE_b} = 2\langle \rho_x^{(+)} | \hat{W}_{xA} | \rho_x^{(+)} \rangle \ \rho(E_b)/\hbar v_a \ , \qquad (1.4)$$

with

$$\rho_x(\vec{r}_x) \equiv (\chi_b^{(-)}(\vec{r}_b) | \phi_a(\vec{r}_b - \vec{r}_x) \chi_a^{(+)}(\vec{r}_b, \vec{r}_x) \rangle \ , \qquad (1.5)$$

employing Udagawa's notation that (| > implies integration only over the coordinates of particle b; $\chi_a^{(+)}$ and $\chi_b^{(-)}$ are optical wave functions for a and b scattering elastically from A, and ϕ_a is the internal wave function for the projectile. $\rho_x^{(+)}(\vec{r}_x)$ thus clearly plays the role of an optical wave function for the elastic scattering of x (riding inside the projectile) on A, and $\hat{W}_{xA}$ is the imaginary part of the corresponding optical potential. $\langle \rho | \hat{W} | \rho \rangle$ thus represents the reaction cross section for x on A. It contains $\phi_a(\vec{r}_b - \vec{r}_x)$, which is basically the source of the width in the E_b-spectrum, but it also contains the absorption of both x and b by A, in their optical wave functions. This can be made very explicit by employing a WKB approximation to these wave functions, to produce a very clear qualitative picture of the reaction, as we demonstrate below.

II. BACKGROUND ON REACTION CROSS SECTIONS

Motivated by the form of Udagawa's result, Eq. (3), we recall the familiar derivation of a similar expression for a reaction cross section from a simple optical-potential Schrödinger equation, written in the form

$$-\frac{\hbar^2}{2m} \nabla^2 \phi^{(+)} + (V - iW)\phi^{(+)} = E\phi^{(+)} \ , \qquad (2.1)$$

where we adhere to the customary convention of taking W(r) positive to describe absorption. The usual Wronskian manipulation with ϕ and ϕ^* yields

$$-\hbar \int \vec{j} \cdot d\vec{A} = 2\langle \phi^{(+)} | W | \phi^{(+)} \rangle \ , \qquad (2.2)$$

where the integral is over any surface surrounding the potential, in a region where the potential has vanished, and describes the net inward flux due to the absorption. Dividing it by the incident current $|\phi^{(+)}|^2 v_o = v_o$ (which defines the normalization of $\phi^{(+)}$) gives the familiar expression for the total reaction cross section,

$$\sigma_R = 2\langle \phi^{(+)} | W | \phi^{(+)} \rangle / \hbar v_o \ . \qquad (2.3)$$

I.e., the reaction cross section out of a specific entrance channel is given by the expectation value of the imaginary part of the optical potential in that channel, calculated with the corresponding optical wave function in that entrance channel.

~~It is almost possible to~~ apply this expression, unchanged, to the three-body or fragmentation problem of Eq. (1.1), and in any practical calculation that is doubtless what one would do, using Eq. (1.5) to provide the obvious definition of the "negative energy entrance channel" wave function, $\beta_x(\vec{r}_x)$. The fact that the spectator carries away a range of possible energies, however, requires certain care, which can best be seen by employing the Feshbach projection operator formalism.

We consider the multi-channel problem in which the channels are defined by the states $|n\rangle$ of the target, and choose the projection operators

$$P = |0\rangle\langle 0| \ , \quad Q = 1 - P \ , \tag{2.4}$$

so that $P\phi$ is the elastic-channel projection of ϕ, i.e., the optical model wave function for this channel.

As usual, the coupled equations are

$$(E - H_{QQ})Q\phi = V_{QP}P\phi \ , \tag{2.5a}$$

with solution

$$Q\phi = \frac{1}{E - H_{QQ}} V_{QP} \, P\phi \tag{2.5b}$$

(no incident wave in the Q space), and

$$\begin{aligned} (E - H_{PP})P\phi &= V_{PQ} \, Q\phi \\ &= V_{PQ} \frac{1}{E - H_{QQ} + i\varepsilon} V_{QP} \, P\phi \ , \end{aligned} \tag{2.6}$$

giving the customary expression for the imaginary part of the optical potential in channel P,

$$\begin{aligned} -W_P(E) &= |0\rangle \{ \mathrm{Im}\langle 0| V_{PQ} \frac{1}{E - H_{QQ} + i\varepsilon} V_{QP} |0\rangle \} \langle 0| \ , \\ &= \pi |0\rangle\langle 0| V_{PQ} \ \delta(E - H_{QQ}) V_{QP} |0\rangle\langle 0| \ , \end{aligned} \tag{2.7}$$

which will be non-zero at any energy where Q-channels are open.

We now consider any <u>reaction</u> from P to Q. Considering the Hamiltonian in the form $H^P_{opt} + V_{QP}$ for this purpose, where H^P_{opt} is the above optical potential in channel P, its eigenfunction $P\phi^{(+)}$ is the "unperturbed wave" in this context (containing none of the reaction channels produced by V_{QP}), so we write it as $\chi_P^{(+)}(\vec{r})|0\rangle$; this includes the full elastic optical distortion in the definition of the incident wave. Then if ϕ_f is any <u>exact</u> final state in the Q space (we are going to sum over <u>f</u> by closure, and so need not restrict our considerations to DWBA final states), the net <u>reaction</u> cross section out of channel P is

$$\begin{aligned} \sigma_R &= \frac{2\pi}{\hbar v_o} \sum_f |\langle \phi_f^{(-)} | V_{QP} \chi_P^{(+)} |0\rangle|^2 \ \delta(E_f - E_o) \\ &= \frac{2\pi}{\hbar v_o} \sum_f \langle \chi_P^{(+)} | \langle 0 | V_{PQ} | \phi_f^{(-)} \rangle \langle \phi_f^{(-)} | \ \delta(E - H_{QQ}) V_{QP} |0\rangle | \chi_P^{(+)} \rangle \\ &= \frac{2\pi}{\hbar v_o} \langle \chi_P^{(+)} \langle 0 | V_{PQ} \ \delta(E - H_{QQ}) V_{QP} |0\rangle | \chi_P^{(+)} \rangle \\ &= 2\langle \chi_P^{(+)} | W_P | \chi_P^{(+)} \rangle / \hbar v_o \ , \end{aligned} \tag{2.8}$$

[using Eq. (2.7)], as we found via the simpler argument. Note that the closure sum was done within the Q-space.

III. THE SPECTATOR MODEL FOR INCLUSIVE FRAGMENTATION

Now consider the spectator model for "fragmentation", where by inclusive "fragmentation" we mean that a fragment of the projectile is observed, but we sum over all possible final states of the rest of the projectile interacting with the target. This includes "breakup", in which both fragments escape, as well as total or incomplete fusion of x with the target.

III.1. Direct Reactions Only

We divide the possible xA reactions into two extremes, direct and compound (i.e., fluctuating). In the present section we consider only the direct reactions, meaning that the xA cross section (or, in the present context, the E_b spectator spectrum) exhibits no energy fluctuations; Udagawa and Tamura interpret this to mean no incomplete fusion. Mathematically it means that H_{xA}^{QQ} is real, like the above H_{QQ}, so that Eq. (2.7) holds for the xA system.

Like Udagawa *et al.*, we find it most natural to write the (exact, not DWBA) matrix elements in the "prior" form, which considers the interaction causing the fragmentation to be the entrance-channel potential,

$$V_{QP} = V_{xA}^{QP} + V_{bA}^{QP} \ . \qquad (3.1)$$

If a given trajectory causes A to interact more strongly with x than with b, the resultant "tidal force" can fragment the projectile. The reaction in this case is caused more by V_{xA} than by V_{bA}. In the limit that the V_{bA} interaction (and final-state V_{bx}) is neglected altogether,

we have the (b) spectator model. Of course, other trajectories can equally make x the spectator (and so leave it travelling forward at beam velocity); the basic assumption of the spectator model is that these two cases can be cleanly separated experimentally.

Choosing b as the spectator means that we neglect $QV_{bA}P$ as far as the reaction is concerned. Its elastic scattering component is retained, however, by incorporating it into the optical model wave function $\chi_b(\vec{r}_b)$ used to describe the outgoing spectator, which contains the important effect of nuclear absorption of b if the projectile impact parameter is too small. χ_a should actually include the full effect of (x+A) (i.e., not just of A) on b, but if $M_A \gg M_x$, the distinction is not important.

Thus the spectator-model matrix element in the prior (but non-DWBA) form is

$$T_{fi} = \langle \chi_b^{(-)} \ \phi_f^{(-)} | V_{xA} | \ \phi_a \chi_a^{(+)} \Phi_o \rangle \ , \qquad (3.2)$$

where $\chi_a^{(+)}$ is the optical model wave function of the projectile, ϕ_a its internal state and $\Phi_o = |0\rangle$ the ground-state wave function for the target. Within the spectator model, this matrix element is exact, for $\phi_f^{(-)}$ is the exact (and unknown) wave function for any state of the xA* system, including the internal state of A*.

*We should, of course, include internal wave functions for x and for particle-stable states of b, but they will be summed over and so can be understood to be in ϕ_f. We also note[11] that the prior form requires $T_{fi} = \langle \Psi_f | V_i | \phi_i \rangle$, with $(H-V_i)\phi_i = E_i \phi_i$.

By slightly generalizing the Q space of Sec. II to include all states of xA, not just of A, the same manipulation gives, for the net reaction cross section out of the entrance channel $\underline{a}$,

$$\frac{d^2\sigma_R^{dir}}{dE_b d\Omega_b} = \frac{2\pi}{\hbar v_a}\sum_f |T_{fi}|^2\ \delta(E_b+E_f-B_a-E_i)\,\rho(E_b)$$

$$= \frac{2\pi}{\hbar v_a}\,\rho(E_b)\langle\chi_a^{(+)}\chi_b^{(-)\dagger}\phi_a\langle 0|V_{xA}\ \delta(E_i+B_a-E_b-H_{xA}^{QQ})$$

$$\times V_{xA}|0\rangle\phi_a\chi_b^{(-)\dagger}\chi_a^{(+)}\rangle \qquad (3.3)$$

$$\equiv \frac{2}{\hbar v_a}\,\rho(E_b)\langle\chi_a^{(+)}\chi_b^{(-)\dagger}\phi_a|\hat{W}_{xA}(E_i+B_a-E_b)|\phi_a\chi_b^{(-)\dagger}\chi_a^{(+)}\rangle\ ,$$

where $B_a > 0$ is the binding energy of $\underline{x}$ to $\underline{b}$, and $\rho(E_b) = \mu_b k_b/(2\pi)^3\hbar^2$ is the asymptotic phase space density for $\underline{b}$. $\hat{W}_{xA}(E_i+B_a-E_b)$ is, by definition, the imaginary part of the xA optical potential appropriate to $\underline{x}$ entering in the wave function $\chi_a^{(+)}\phi_a$, rather than as a simple distorted wave. This $\hat{W}_{xA}$ is of course experimentally inaccessible, but in a practical calculation one would assume, with Udagawa, that it could be approximated by the normal empirical xA optical potential, evaluated at $E_x = E_i + B_a - E_b$.

If we now define "negative energy entrance channel" wave functions for $\underline{x}$ as in Sec. I,

$$\rho_x^{(+)}(\vec{r}_x) \equiv (\chi_b^{(-)}(\vec{r}_b)\,|\,\phi_a(\vec{r}_b-\vec{r}_x)\,\chi_a^{(+)}(\vec{r}_b,\vec{r}_x)\rangle\ , \qquad (3.4)$$

where the round bracket indicates integration over b-coordinates only, we have the advertised result,

$$\frac{d^2\sigma_R^{dir}}{d\Omega_b dE_b} = \frac{2}{\hbar v_a}\,\rho(E_b)\ \langle\rho_x^{(+)}|\hat{W}_{xA}^{dir}(E_i+B_a-E_b)|\rho_x^{(+)}\rangle\ . \qquad (3.5)$$

This has much the appearance of Udagawa's result, but is in fact very different. In particular it lacks the Green's function factors (which he includes in β), the physical reason being that the $\hat{W}_{xA}^{dir}$ of Eq. (3.5) arises from flux loss into other $\underline{\text{open}}$ xA channels (hence the "direct" superscript), whereas Udagawa's W_{xA} arises from $\underline{\text{closed}}$ xA channels, as we show explicitly in the next section.

We recall that Eq. (3.5) describes only $\underline{\text{reactions}}$ of the xA system but omits its elastic scattering. I.e., it is the cross section for inelastic fragmentation, omitting elastic fragmentation. An experimentally measured singles spectrum of course includes both, so an estimate (perhaps in DWBA) of the elastic contribution should be included in any confrontation with experimental data; recent theoretical and experimental results[5-7] suggest, however, that elastic fragmentation is a factor of 5 or more smaller than the inelastic.

In summary, Eq. (3.5) is our central result. It is essentially "true by definition", obtainable (within the spectator approximation) directly from the elementary Eq. (1.3) by recognizing that Eq. (1.3) is valid for $\underline{\text{any}}$ entrance channel. Equation (3.5) employs an entrance channel in which $\underline{x}$ enters bound to $\underline{b}$. It therefore has a kinetic energy $E_x = E_i - E_b + B_a$, and indeed Eq. (3.5) requires that we employ its optical potential evaluated at this energy, thus relating the E_b-spectrum directly to the motion of $\underline{b}$ within the projectile before the reaction. The inclusive reaction cross section of Eq. (3.5) includes the processes computed by Bauer $\underline{\text{et al.}}$,[5-7] but should be more accurate, in that it makes neither ~~the DWBA approximation nor any~~ further approximations like surface or zero-range estimates.

III.2. Fluctuation in the (xA) Cross Section: Incomplete Fusion

The method of derivation employed in obtaining Eq. (3.5) was based on the assumption that the xA cross section contained no strong energy dependence (e.g., resonances or "fluctuations"). One might expect such fluctuations to occur if the xA interaction leads to the formation of a compound nucleus, and indeed Udagawa et al.[8-10] assume that it is exactly these fluctuations (appearing here in the E_b spectrum) which signal the fusion of x with A, i.e., what would be called "incomplete fusion" or "breakup-fusion" in the present context. This identification of an xA fluctuation cross section with breakup-fusion seems quite plausible, and in any case, if such fluctuations are present, the inclusive cross section for detecting b becomes, upon energy-averaging,

$$\frac{d^2\sigma}{d\Omega dE_b} = \frac{d^2\sigma^{dir}}{d\Omega dE_b} + \frac{d^2\sigma^{f\ell}}{d\Omega dE_b} \,, \tag{3.6}$$

as we show by generalizing our previous Eq. (3.5) to include fluctuations.

To do so, the previous spectator model must be generalized to include the (xA) resonances, coming from closed channels which were previously neglected. Thus, we change our previous notation, which used $Q = 1-P$ to identify open channels different from the entrance channel. Instead let us write

$$1 = p_o + p' + Q \equiv P + Q \tag{3.7}$$

where p_o is the entrance channel, p' the other open channels and Q the closed channels. If we eliminate the closed channels, the effective hamiltonian in P (i.e., the effective coupled-channels hamiltonian matrix) is

$$H_P(E) = H_{PP} + H_{PQ}\frac{1}{E - H_{QQ}} H_{QP} \,. \tag{3.8}$$

If we then eliminate p' also, we obtain the effective one-channel hamiltonian in p_o,

$$H_{p_o}(E) = H_{p_o p_o} + H_{p_o p'}\frac{1}{E - p' H_P(E) p' + i\varepsilon} H_{p' p_o} \,. \tag{3.9}$$

It will show the Q-space fluctuations, but if we average it (by $E \rightarrow E + iI$), the denominator will contain exactly the coupled-channels optical hamiltonian, $H_{opt}(E)$, which will be complex because of the fluctuations.

We use the fact that the imaginary part of the remaining Green's function can be written[12)]

$$\begin{aligned} \mathrm{Im}\frac{1}{E - H_{opt} + i\varepsilon} &= -\pi \sum_c |\phi_c^{(-)}\rangle\langle\phi_c^{(-)}| + G_{opt}^{(+)\dagger}\,\mathrm{Im}(H_{opt})G_{opt}^{(+)} \\ &= -\pi \sum_c |\phi_c^{(-)}\rangle\langle\phi_c^{(-)}| - \pi G_{opt}^{(+)\dagger} H_{p'Q}\,\overline{\frac{|\phi_q\rangle\langle\tilde{\phi}_q|}{d_q}}\, H_{Qp'}\, G_{opt}^{(+)} \,, \end{aligned} \tag{3.10}$$

in which

$$(E - H_{opt})|\phi^{(+)}\rangle = 0 \tag{3.11}$$

defines the coupled-channel optical wave functions,

$$(E - H_{QQ})|\phi_q\rangle = 0 \tag{3.12}$$

defines the closed-channel resonant states, and the average in the second term of (3.10) is over these q-states; G_{opt} is the coupled-channel optical (matrix) Green's functions in p'. From this we recognize

$$-\pi H_{p'Q}\,\frac{\overline{|\phi_q\rangle\langle\tilde{\phi}_q|}}{D_q}\,H_{Qp'} \equiv W^{f\ell}_{p'} \tag{3.13}$$

as the absorptive part of the open-channel optical hamiltonian matrix in p', which is generally assumed[12] to be diagonal,

$$W^{f\ell}_{cc'} = \delta_{cc'}\, W^{f\ell}_{c}\,, \tag{3.14}$$

so the implied double channel sum in (3.10) reduces to a single one. Hence we have directly the total imaginary part of the p_o-channel optical potential,

$$-\mathrm{Im}\, V_{p_oP}\,\frac{1}{E - H_{opt} + i\varepsilon}\,V_{Pp_o} = \hat{W}^{dir}_{p_o} + \hat{W}^{f\ell}_{p_o}\,, \tag{3.15}$$

the two terms coming from the two terms of Eq. (3.10); in fact, $\hat{W}^{f\ell}_{p_o}$ contains (from $H_{p_op_o}$) a similar term from p_o directly to Q. If we define a new G_{opt} to include all open channels, we can write (see also Ref. 12)

$$\hat{W}^{f\ell}_{p_o} = V_{p_oP}\left[G_P^{(+)opt}\right]^{\dagger} W_P\, G_P^{(+)opt}\, V_{Pp_o}\,, \tag{3.16}$$

in which the implied sums over the open channels in P show that $W^{f\ell}_{p_o}$ comes from flux loss out of all open channels into Q, not just that out of the entrance channel. We remark in passing that Eq. (3.5), with (3.16) for $\hat{W}^{f\ell}_{p_o}$, bears a close resemblance to the formula obtained by Kasano and Ichimura[13] from post-form DWBA. Though very similar in structure, their expressions and ours differ in containing the post and prior interaction potentials, respectively, as is to be expected from their different starting points.

With the aid of Eq. (3.16), a rather simple structure may be immediately obtained for $\frac{d^2\sigma^{fl}}{d\Omega dE_b}$ of Eq. (3.6). We first identify $G_p^{(+)\,opt}\,V_{pp_o}|\hat{\rho}_x^{(+)}\rangle$ with the exact wave function in the p space and call it $|\hat{\rho}_p^{(+)}\rangle$. Using our assumption that W_p is diagonal in the channels, and identifying $\frac{2}{\hbar v_c}\langle\hat{\rho}_c^{(+)}|W_c|\hat{\rho}_c^{(+)}\rangle$ with the fusion cross section σ_F^c in channel c (contained in p), we finally obtain

$$\frac{d^2\sigma^{fl}}{d\Omega dE_b} = \rho(E_b)\sum_{c\in p}\frac{v_c}{v_a}\,\sigma_F^c(E_i + B_a - E_b)\,, \tag{3.17}$$

an incoherent sum of fusion cross sections[14] in all p (xA) channels. We should mention at this point that the angular dependence of the inclusive fluctuacting b cross-section (incomplete fusion) is entirely contained in these angle-dependent xA fusion cross sections. This is made transparent with the aid of Glauber theory in the next section.

In summary, Eq. (3.5) for the inclusive spectator spectrum still holds, provided only that W^{dir} is replaced by $\hat{W}^{dir} + \hat{W}^{f\ell}$, which is in fact the imaginary part of the empirical optical potential in the (xA) entrance channel. If one wishes to decompose this inclusive spectrum into "direct" and "incomplete fusion" (break-up-fusion) parts, the latter does indeed give a term of the Udagawa-Tamura form. However (as pointed out in Ref. 12), the $\hat{W}^{f\ell}$ which appears in the formula, as Eq. (3.16) makes clear, is really a sum over the $\hat{W}_c$'s from all open channels. These are quantities which are more difficult to obtain than the single $\hat{W}_{p_o}$, since they require fitting a complex channel optical matrix to the energy-averaged (xA) data. Our conclusion is that the UT formula, if corrected as indicated above, is a possible means of calculating incomplete fusion (i.e., fluctuation) cross sections, but it requires optical parameters from all open channels.

IV. FURTHER INSIGHT VIA GLAUBER DISTORTION

In spite of the appealing simplicity of Eq. (3.5 or its generalization), it does not display very explicitly the dependence of the E_b-spectrum on the zero-point internal motion of the projectile, nor the influence of absorption of x and b by the target. All these can be demonstrated very nicely by employing a WKB or Glauber approximation to the optical wave functions for b and a. At sufficiently high energies, this should be quite accurate, but our purpose here is primarily to provide helpful insight into complex numerical computations.

The Glauber approximation to an elastic-scattering distorted wave is

$$\chi_{\vec{k}}^{(+)}(\vec{r}) = \chi_{\vec{k}}^{(+)}(z,\vec{b}) = e^{i\vec{k}\cdot\vec{r}}\, e^{+i\int_{-\infty}^{z} \Delta k(z',b)dz'} ,$$

$$\chi_{\vec{k}}^{(-)}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}\, e^{-i\int_{z}^{\infty} \Delta k(z',b)dz'} . \tag{4.1}$$

The incident momentum $\vec{k}$ is taken to point along the positive z-axis, and $\vec{b}$ is the component of $\vec{r}$ perpendicular to z, assumed to vary little along a small-angle trajectory. The exponent of the second factor in $\chi_{\vec{k}}^{(+)}$ is the amount of (generally complex) Born approximation phase shift accumulated along the trajectory up to the point $(z,\vec{b})$, with the integrand defined as

$$\Delta k(z',b) \equiv - \frac{k}{2E} U(z,b) , \tag{4.2}$$

where $-U(r) = V(r) + iW(r)$ ~~is the optical potential. The customary~~ optical phase shift, accumulated along the entire trajectory, is given by

$$2\delta(b) = \int_{-\infty}^{\infty} \Delta k(z',b)dz' = 2\int_{0}^{\infty} \Delta k(z',b)dz' \ , \qquad (4.3)$$

and the partial-wave optical S-matrix element is

$$S(b) = e^{2i\delta(b)} \ . \qquad (4.4)$$

The optical potential U_{aA} is

$$U_{aA} = U_{bA} + U_{xA} \ , \qquad (4.5)$$

and since the Glauber phase is linear in the potential, the phase shift for a composite particle is simply the sum of those for its components, each at its respective impact parameter,

$$\delta_a(b_a) = \delta_b(b_b) + \delta_x(b_x) \ , \qquad (4.6)$$

i.e., the Glauber distorted wave for the projectile is just

$$\chi_a^{(+)} = e^{i(\vec{k}_x\cdot\vec{r}_x+\vec{k}_b\cdot\vec{r}_b)}\, e^{i\int_{-\infty}^{z_x} \Delta k_x(z',b_x)dz'}\, e^{i\int_{-\infty}^{z_b} \Delta k_b(z',b_b)dz'} \ , \qquad (4.7)$$

with

$$\vec{k}_x = (m_x/m_a)\vec{k}_a,\ \vec{k}_b = (m_b/m_a)\vec{k}_a \qquad (4.8)$$

as the average momenta of x and b in a. Substituting this and the analogous expression for $\chi_b^{(-)}$ into Eq. (3.4), we see that, within the spectator assumption, the two phase integrals for particle b combine to produce the net optical phase shift $2\delta_{bA}(b_b)$, whose exponential is $S_{bA}(b_b)$, giving for the "entrance channel wave function"

$$\begin{aligned}\rho_x(\vec{r}_x\vec{q}) &= \int d^3r_b\, (\chi_b^{(-)*}|\phi_a\rangle\, \chi_a^{(+)}\rangle \\ &= e^{i\vec{k}_x\cdot\vec{r}_x}\, e^{i\int_{-\infty}^{z_x} \Delta k_x(z',b_x)dz'} \\ &\times \int d^3r_b\, e^{i\vec{q}\cdot\vec{r}_b}\, S_{bA}(b_b)\, \phi_a(\vec{r}_b-\vec{r}_x) \ ,\end{aligned} \qquad (4.9)$$

with

$$\vec{q} = \vec{q}_b = \vec{k}_b - \vec{k}_b' \qquad (4.10)$$

the average momentum transferred from b to A by elastic scattering. The dependence of the scattering amplitude on the energy and angle of the spectator thus appears explicitly in $\vec{q}$, in the form of a Fourier transform of the product $S_{bA}\ \phi_a$. If S_{bA} were not present (as in the plane-wave or Serber model), this would be just the Fourier transform $\tilde{\phi}_a(q)$ of the internal wave function $\phi_a(\vec{r})$ of the projectile, giving the spectrum in the form $|\tilde{\phi}_a(q)|^2$ of the Serber model.

The factor $S_{bA}(b_b)$ modifies this, however, for it is small at small b_b where b is strongly absorbed. The spectator is thus required to miss the target in order to avoid being absorbed. This will both reduce the magnitude of the cross section from that of the Serber model and also broaden the transverse momentum spectrum, by narrowing the range of the b_b integration. The z_b integration, on the other hand, is unaffected by S_{bA}, so, for a <u>given</u> $\vec{r}_x$, the longitudinal (i.e., grazing-angle) momentum spectrum, $d\sigma/dq_\parallel$, will be the same as in the Serber model. However, $\vec{r}_x$ values which cause the spectator to overlap the target are eliminated by absorption. The result is generally to permit only those z_b integrations which pass through the <u>surface</u> region of $\phi_a(r)$. This shows explicitly that $d\sigma/dq_\parallel$ is primarily sensitive to the surface parameters of ϕ_a, rather than to its entire volume, in complete agreement with Friedman[3] and Hüfner and Nemes.[4]

If we define the integral in Eq. (4.9) to be

$$\int d^3r_b\, e^{i\vec{q}\cdot\vec{r}_b}\, S_{bA}(b_b)\ \phi_a(\vec{r}_b-\vec{r}_x) \equiv e^{iq_\parallel z_x}\ \hat{\phi}_{a,b}(\vec{q},\vec{b}_x)\ , \qquad (4.11)$$

then

$$\langle\beta_x|\hat{W}_{xA}|\beta_x\rangle = \int d^2b_x\ |\hat{\phi}_{a,b}(\vec{q},\vec{b}_x)|^2$$

$$\times \int_{-\infty}^{\infty} dz_x\ \hat{W}_{xA}\ e^{-2\frac{kx}{2E_x}\int_{-\infty}^{z_x} W_{xA}dz'} \qquad (4.12)$$

$$= \frac{E_x}{k_x}\int d^2b_x\ |\hat{\phi}_{a,b}(\vec{q},\vec{b}_x)|^2[1-|S_{xA}(b_x)|^2]\ ,$$

using the fact that Glauber distortion neglects the difference between $\hat{W}_{xA}$ and W_{xA}, and neglecting its energy dependence. This is clearly just an expansion in terms of partial waves of the participant <u>x</u>, so if we recall that the partial-wave xA reaction cross section is

$$\sigma_{xA}^R(b_x) = \pi\lambda_x^2(2\ell_x+1)[1-|S_{xA}(b_x)|]^2\ , \qquad (4.13)$$

and use $\ell_x = b_x k_x$ and $v_x = v_a$, then Eqs. (3.5) and (4.12) yield

$$\frac{d^2\sigma_R}{d\Omega_b dE_b} = \rho(E_b)\sum_{\ell_x}\ \sigma_{xA}^R(\ell_x)P_{a,b}(q,\ell x)\ , \qquad (4.14)$$

where

$$P_{a,b}(q,\ell_x) = \frac{1}{2\pi}\int_0^{2\pi} d\phi\ |\hat{\phi}_{a,b}(\vec{q},\vec{\ell}_x/k_x)|^2\ , \qquad (4.15)$$

ϕ being the angle between $\vec{q}$ and $\vec{\ell}_x = k_x\vec{b}_x$.

Equation (4.14) is our final result, in the Glauber-distortion spectator model. The factor σ_{xA}^R explicitly indicates that the reaction occurred because of a collision of A with <u>x</u>, leading to any possible final state different from the entrance channel. And it shows clearly that the momentum distribution arises from the zero-point relative motion ("Fermi motion") of <u>x</u> and <u>b</u> within the projectile, which is broadened in the transverse direction by absorption of the spectator. In particular, if this absorption were absent ($S_{bA} = 1$), we would have $P_{a,b}(q,\ell_x) = |\tilde{\phi}_a(q)|^2$, and

$$\frac{d^2\sigma_R}{d\Omega_b dE_b} = \rho(E_b)\ |\tilde{\phi}_a(q)|^2\ \sigma_{xA}^R\ , \qquad (4.16)$$

which is the original Serber model.

Finally, if one is not interested in the momentum spectra of <u>b</u>, but only in the momentum-integrated cross sections giving relative abundances of various b's, one sees from Eqs. (4.9) and (4.12) that the total yield of fragment b is

$$Y_b \equiv \int \frac{d^2\sigma_R}{d\Omega_b dE_b} d^3q = \frac{2}{v_a}(2\pi)^3 \frac{E_x}{\hbar k_x} \int d^3r_b d^3r_x \tag{4.17}$$
$$\times |S_{bA}(b_b)|^2 \; |\phi_a(\vec{r}_b-\vec{r}_x)|^2 [1 - |S_{xA}(b_x)|^2] \; .$$

This displays the "overlap geometry" of the reaction very clearly. It can be thought of as basically an integral over $|\phi_a(\vec{r}_b-\vec{r}_x)|^2$, but with two absorption factors. As Fig. 2 indicates, one of them excludes small values for b_b, and the other excludes large values of b_x, exactly in accord with one's intuition. In fact the latter constraint, that the participant have some overlap with the target, is just the "fireball geometry" constraint; it is the basis of a recent model by Harvey and Homeyer,[15] which is remarkably successful in predicting relative cross sections for different spectators.

The integrand of Eq. (4.17) becomes particularly simple in the limit that the projectile is much smaller than the target. Replacing $\phi_a(\vec{r}_b-\vec{r}_x)$ by $\delta(\vec{r}_b-\vec{r}_x)$ the $\vec{r}_b$-integral sets $b_b = b_x$, so the remaining integrand becomes $|S_{bA}(b_x)|^2[1 - |S_{xA}(b_x)|^2]$, which clearly peaks at the surface of A. E.g., if $S_{bA} = S_{xA} = [1 + \exp\beta]^{-1}$, $\beta = (b-R)/a$, the integrand is just $\cosh^2(\beta/2)$, very much like the parametrization Bauer et al.[7] have found to be an accurate description of their DWBA amplitudes.

V. CONCLUSION

In a two-body scattering problem described by a complex potential, the reaction cross section out of any entrance channel c is given by the familiar expression

$$\sigma_{R,c} = \frac{2}{\hbar v_c} \langle \psi_c^{(+)} | W_c | \psi_c^{(+)} \rangle \; , \tag{5.1}$$

where $-W_c$ is the imaginary part of the optical potential in channel c and $\psi_c^{(+)}$ its corresponding optical wave function. In a three-body model of the projectile fragmentation reaction

$$a + A \rightarrow b + x + A^* \; , \tag{5.2}$$

projectile-fragment spectra measured in certain regions of the final-state phase space (e.g., b near its grazing angle) strongly suggest that one of the two fragments, here taken as b, does not participate directly in the breakup. In this spectator model, the breakup is produced by A interacting in any possible way with x, but doing so only elastically with b. In this limit it is plausible that the inclusive "fragmentation-reaction" cross section, for producing a b, but summed over all xA final states which leave A in an excited state (including fusion), might be written in the analogous form

$$\frac{d^2\sigma_R}{d\Omega_b dE_b} = \frac{2}{\hbar v_a} \rho(E_b) \langle \beta_x | \hat{W}_{xA} | \beta_x \rangle \; , \tag{5.3}$$

where $\hat{W}_{xA}$ and β_x are the absorptive xA potential and corresponding wave function for this peculiar xA entrance channel. We find that this is indeed the case, and hence suggest that this may provide a useful theoretical approach to the problem. The above cross section, however, excludes elastic fragmentation, in which the target remains in its ground state, so before comparing it to singles data for spectator particles, their elastic fragmentation cross section must be estimated (DWBA should suffice, since it is known to be small) and added to Eq. (5.3).

Considerable further insight can be gained by using a Glauber approximation to the distorted waves entering the problem, since it puts all particles on the energy shell. Within this further approximation the above cross section takes the very simple partial-wave form

$$\frac{d^2\sigma_R}{d\Omega_b dE_b} = \rho(E_\rho) \sum_{\ell x} \sigma_{xA}^R(\ell_x) P_{a,b}(q,\ell_x) \; . \tag{5.4}$$

$P_{a,b}(q,\ell_x)$ is here the probability that the spectator has momentum $\vec{k}_b' = \vec{k}_b + \vec{q}$ within the projectile at the instant $\underline{x}$ is removed, provided this occurs when $\underline{b}$ misses the target and the xA relative angular momentum is ℓx; and $\sigma_{xA}^R(\ell_x)$ is the xA reaction cross section at the same ℓ_x. If the absorption of the spectator by the target is neglected, $P_{a,b}(q,\ell_x)$ reduces to $|\tilde{\phi}_a(q)|^2$, the square of the momentum-space internal wave function for the projectile, giving the Serber-model limit,

$$\frac{d^2\sigma_R}{d\Omega_b dE_b} = |\phi_a(q)|^2 \; \sigma_{xA}^R \; \rho(E_b) \; . \tag{5.5}$$

Thus the Glauber-distortion approximation clearly illustrates the following general features of inclusive fragmentation:

1. The reaction occurs because $\underline{x}$ interacts with A; in a $\underline{b}$-inclusive measurement, <u>all</u> final xA states must be included, not just the "breakup-fusion" ones.

2. The spectator must not be absorbed by the target. This requires that the longitudinal momentum spectrum of $\underline{b}$ come from the surface region of the projectile, and broadens its transverse momentum spectrum.

3. Although the absorptive effects of the bA and xA optical potentials are large, their refractive effects are minimal. The real part of the xA optical phase shift vanishes from $|S_{xA}(\ell)|^2$, and that of the A potential appears in δ_x, but only where $|S_{bA}|$ is small, suggesting a minimal effect. Both phases would presumably be much more important in less inclusive reactions.

REFERENCES

1. R. Serber, Phys. Rev. 72 (1947) 1008.
2. E.H.L. Aarts, Thesis, Groningen (1983); F.H.L. Aarts et al., Nucl. Phys. A380 (1982) 465.
3. W.A. Friedman, Phys. Rev. C27 (1983) 569.
4. J. Hüfner and M. C. Nemes, Phys. Rev. C23 (1981) 2538.
5. See G. Baur and D. Trautmann, Phys. Rep. 25C (1976) 293.
6. G. Baur, R. Shyam, F. Rösel and D. Trautmann, Phys. Rev. C29 (1983) 946.
7. G. Baur, F. Rösel, D. Trautmann and R. Shyam, GSI preprint 83-32 (1983).
8. T. Udagawa and T. Tamura, Phys. Rev. Lett. 45 (1980) 1311.
9. T. Udagawa and R. Tamura, Phys. Rev. C24 (1981) 1348.
10. T. Udagawa, Lecture Notes of 1983 RCNP Kikuchi Summer School (ed. H. Ogata, M. Fujiwara and A. Shimiza), Kyoto, 1983.
11. See, e.g., L. S. Rodberg and R. M. Thaler, Introduction to the Quantum Theory of Scattering (Academic Press, N.Y., 1967), p. 197, for a careful discussion of rearrangement collisions.
12. A.K. Kerman and K.W. McVoy, Ann. Phys. (N.Y.) 122 (1979) 197.
13. A. Kasano and M. Ichimura, Phys. Lett. 115B (1982) 81.
14. M.S. Hussein, Phys. Rev. C30 (1984) 1962.
15. B. Harvey and H. Homeyer, LBL preprint.