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LONG RANGE ORDER IN THE GROUND STATE OF TWO-DIMENSIONAL ANTIFERROMAGNETS

by

E. Jordão Neves and J. Fernando Perez
Instituto de Física, Universidade de São Paulo

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TWO-DIMENSIONAL ANTIFERROMAGNETS

E. Jordão Neves [(1,2)] and J. Fernando Perez [(3)]

Instituto de Física, Universidade de São Paulo

Caixa Postal 20516, 01498 São Paulo, Brazil

ABSTRACT

We show the existence of long range order in the ground state of the two-dimensional isotropic Heisenberg antiferromagnet for $S \geq \frac{3}{2}$. The method yields also long range order for the ground state of a larger class of anisotropic quantum antiferromagnetic spin systems with or without transverse magnetic fields.

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(2) Permanent Address: Instituto de Matemática e Estatística, USP, Caixa Postal 20570, 01498 São Paulo, Brazil.

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In this letter we show the existence of long range order (ℓ.r.o.) in the ground state of the two-dimensional isotropic quantum antiferromagnetic spin model for spin $S \geq \frac{3}{2}$.

The ground state of this model as opposed to the ferromagnetic one (where all the spins are aligned) is highly non-trivial, even in one-dimension [13]. In two-dimensions numerical results of Betts and Oitmaa [2] indicate ℓ.r.o. already for $S = \frac{1}{2}$. At non zero temperatures the Mermin-Wagner phenomenon [11,12] precludes the existence of ℓ.r.o.. Dyson, Lieb and Simon [3] showed ℓ.r.o. at sufficiently small temperatures for dimension $\nu \geq 3$.

The $\nu = 2$ anisotropic case was considered by Fröhlich and Lieb [1]. From their analysis it follows that the system exhibits ℓ.r.o. at low enough temperatures provided the coupling in the xy spin direction is small enough (depending on S); for the ground state the anisotropy can go to zero only in the classical limit $S \to \infty$.

Our methods involve the combination of infrared bounds (IRB) and sum rules for the Gibbs state of the system at inverse temperature β in a volume Λ. This combination has been used [3] to show ℓ.r.o. at finite inverse temperature β for a class of spin systems having the so called RP property. If the interactions are short ranged the method works well only for dimension $\nu \geq 3$, since for $\nu \leq 2$ the relevant IRB when inserted in the sum rule gives rise to a divergent integral in the limit $\Lambda \to \infty$ (for fixed finite β). Since however we are interested in the ground state of the system we must first take the $\beta \to \infty$ limit and only then the $\Lambda \to \infty$ limit. In $\nu = 2$ dimensions this procedure annihilates the otherwise divergent contribution.

This is connected to the fact that the path space formulation of the $\beta\to\infty$ model would be given by a 3-dimensional classical spin system [6].

The model in a finite volume $\Lambda \subset \mathbb{Z}^2$ is described by the Hamiltonian

$$H = \sum_{\substack{\alpha\in\Lambda \\ m=1,2}} \left(\sum_{i=1}^{3} S_\alpha^i S_{\alpha+\delta m}^i \right)$$

where $\{\delta_m, m=1,2\}$ are the unit vectors of the lattice, S_α^1, S_α^2, S_α^3 are the spin operators at the lattice site $\alpha \in \Lambda$, with the usual commutation rules

$$\left[S_\alpha^i, S_\beta^j \right] = i\,\delta_{\alpha\beta}\,\epsilon_{ijk}$$

and

$$\vec{S}_\alpha \cdot \vec{S}_\alpha = S(S+1)$$

The expectation value of an observable A at inverse temperature β is given by

$$\langle A \rangle = Tr\left(e^{-\beta H} A\right) / Tr\left(e^{-\beta H}\right)$$

Let now

$$g_p^i = \langle S_p^i S_{-p}^i \rangle$$

with

$$S_p^i = \frac{1}{\sqrt{\Lambda}} \sum_{\alpha\in\Lambda} e^{-ip\alpha} S_\alpha^i \quad , \; p \in \Lambda^* \text{ (dual lattice)}$$

The existence of ℓ.r.o. in the ground state of the system is a consequence of

$$m^2 \equiv \lim_{\Lambda\to\infty} \lim_{\beta\to\infty} \frac{1}{\Lambda} \sum_i g_{p=\pi}^i > 0 \tag{1}$$

The theorem below gives sufficient conditions for (1) to hold. It is an extension to the two-dimensional $T=0$ problem of Theorem 5.1 of reference [3].

<u>Theorem</u>: Suppose the model (1) satisfies the following conditions:

i) "Gaussian Domination":

$$(S_p^i, S_{-p}^i) \le \frac{B_p^i}{\beta} \quad ; \; p \ne \pi \quad \text{for all } \beta$$

where $$(A,B) = Z^{-1} \int_0^1 Tr\left(e^{-x\beta H} A\, e^{-(1-x)\beta H} B\right) dx$$

is the Duhamel two-point function.

ii) Existence of an upper bound for the expectation value of the double commutator:

$$\langle [S_p^i, [H, S_{-p}^i]] \rangle \le C_p^i$$

iii) $$\int d^2p \left(B_p^i C_p^i\right)^{\frac{1}{2}} < \infty$$

iv) Existence of a lower bound for the usual two-point function:

$$\sum_i \langle S_\alpha^i S_\alpha^i \rangle \ge D$$

Then, there exists ℓ.r.o. if:

$$2D > \frac{1}{(2\pi)^2}\int_{[-\pi,\pi]^2} d^2p\left[\left(\sum_i B_p^i\right)\left(\sum_i C_p^i\right)\right]^{\frac{1}{2}} \tag{2}$$

<u>Proof</u>: As in [3], from (i), (ii) and the Bruch-Falk inequality [7] we get a domination for the usual two point function:

$$\sum_i g_p^i \leq G_p \tag{3}$$

with $G_p = \frac{1}{2}\left[\left(\sum_i B_p^i\right)\left(\sum_i C_p^i\right)\right]^{\frac{1}{2}} \coth\frac{\beta}{2}\left[\frac{\sum C_p^i}{\sum B_p^i}\right]^{\frac{1}{2}}$

Together with the sum rule

$$\frac{1}{\Lambda}\sum_{p\in\Lambda^*} g_p^i = \langle S_\alpha^i S_\alpha^i\rangle$$

(3) implies

$$\frac{1}{\Lambda}\sum_i g_{p=\pi}^i \geq D - \frac{1}{\Lambda}\sum_{p\neq\pi} G_p \tag{4}$$

Using the inequality [3]:

$$G_p \leq \frac{1}{2}\left[\left(\sum_i B_p^i\right)\left(\sum_i C_p^i\right)\right]^{\frac{1}{2}} + \frac{1}{\beta}\left(\sum_i B_p^i\right) \tag{5}$$

we obtain for the ground state

$$m^2 = \lim_{\Lambda\to\infty}\lim_{\beta\to\infty}\frac{1}{\Lambda}\sum_i g_{p=\pi}^i \geq$$

$$D - \frac{1}{(2\pi)^2}\int d^2p\left[\left(\sum_i B_p^i\right)\left(\sum_i C_p^i\right)\right]^{\frac{1}{2}}$$

Remark: For $\beta<\infty$, inequality (3) is useless (for $\nu=2$) as

$$\lim_{\Lambda\to\infty}\frac{1}{\Lambda}\sum_i g_{p=\pi}^i \geq D - \lim_{\Lambda\to\infty}\frac{1}{\Lambda}\sum_{p\neq\pi} G_p = -\infty$$

As shown in [3], property (i) is fulfilled with B_p^i given by

$$B_p^i = \frac{1}{2E_+(p)}\ , \quad \text{with} \quad E_+(p) = \sum_{i=1}^{2}(1-\cos p_i) \tag{6}$$

The relevant double commutator bound is given by [3]

$$\sum_{i=1}^{3} C_p^i \equiv \langle -A\rangle \leq -a_0 \tag{7}$$

where

$$A = 4\sum_{m=1}^{2}(1-\cos p_m)\langle S_\alpha^1 S_{\alpha+\delta m}^1 + S_\alpha^2 S_{\alpha+\delta m}^2 + S_\alpha^3 S_{\alpha+\delta m}^3\rangle$$

and a_0 is the lowest eigenvalue of A.

For a_0 we have the bound of Anderson [5]:

$$-a_0 \leq 8S\left(S+\frac{1}{4}\right)\left[\sum_{m=1}^{2}(1-\cos p_m)\right] \tag{8}$$

So that we have ℓ.r.o. if

$$\frac{S(S+1)}{\sqrt{2}\,[S(S+\frac{1}{4})]^{\frac{1}{2}}} > \frac{1}{(2\pi)^2}\int d^2p \left[\frac{\sum_m (1-\cos p_m)}{\sum_m (1+\cos p_m)}\right] \qquad (9)$$

Since the r.h.s. is finite, it follows that the system exhibits ℓ.r.o. for sufficiently large S. Numerical integration yields

$$\frac{1}{(2\pi)^2}\int d^2p \left[\frac{\sum (1-\cos p_m)}{\sum (1+\cos p_m)}\right]^{\frac{1}{2}} = 1{,}3$$

and therefore (9) holds for $S \geq \frac{3}{2}$.

With the same techniques we can prove [4,10] ℓ.r.o. for the ground state of a large class of anisotropic models with or without external magnetic fields provided Reflection Positivity is verified. For these models however the bound (7) is more delicate as it involves estimating the lowest eigenvalue of the Hamiltonian of a spin system with a p-dependent anisotropy. For the isotropic xy model our bounds so far are not good enough to imply ℓ.r.o. even for large S !

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