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BRASIL

PUBLICAÇÕES

IFUSP/P-544

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Setembro/1985

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ABSTRACT

Fundamental considerations of indistinguishability in quantum mechanics lead to the possibility of existence of three kinds of particles (bosons, fermions and gentileons) in nature. This is expressed in terms of a Statistical Principle. Assuming that quarks are spin 1/2 gentileons we see that quantum chromodynamics can be used, with additional conditions, to estimate the properties of gentilionic hadrons.

During the last two decades, the theory of elementary particles has been developed in terms of the quark hypothesis of Gell-Mann-Zweig. The SU(3) (or SU(6)) symmetry combined with several other group transformations has served as a guide to classify all known strongly interacting particles. Nevertheless, in spite of its great success in classifying the several multiplets found in nature, Gell-Mann's theory leaves some major questions open for investigation. Among these great problems we are interested in, we can quote the statistical one which arises with the fermionic character of quarks and the two intriguing peculiar problems posed by the quark hypothesis: the quark saturation in hadrons and the quark confinement. Although being powerfully attacked by QCD, some aspects of these problems remain unsolved. With these difficulties in mind we have proposed, within the last few years,[1,2] an alternative approach to the statistical problem of quarks, assuming that they obey Gentile statistics instead of Fermi statistics.

In our preceding works[1,2] we have shown, according to the postulates of quantum mechanics and with the principle of indistinguishability, that three kinds of particles could exist in nature: bosons,fermions and gentileons. These results can be synthesized in terms of the following principle (Statistical Principle): "Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively". Bosonic and fermionic systems are represented by one-dimensional totally symetric and totally anti-symmetric wavefunctions, respectively. Subsystems of bosonic or fermionic states are also bosonic or fermionic in the sense that they have the same symmetry properties of the original systems. Since the commutation relations for the creation and annihilation operators are bi--linear, bosons and fermions obey the spin-statistics theorem:[3-5] bosons have integer spin and fermions half-integer spin.

Because gentilionic systems are represented by intermediate Young shapes, their wavefunctions are multi-dimensional with mixed symmetries. Only three or more gentileons can form a system of indistinguishable particles. This means that two identical gentileons are prohibited to constitute a system of indistinguishable particles. This implies that a gentileon cannot appear as a free particle. Indeed, if this were possible, two free gentileons could interact constituting, consequently, such a system. With regard to symmetry properties, a subsystem of a gentilionic system cannot be defined.[6] As the creation and annihilation operators for gentileons obey multi-linear commutation relations,[1] the spin-statistics theorem is meaningless for gentileons. Finally, due to intrinsic geometric features of the intermediate state-vectors, the gentilionic systems exhibit a saturation property.[1] This means, for instance, that two systems like (ggg) and (gggg) cannot coalesce into a composite system (ggggggg). Only bound states (ggg) - (gggg) could be possible. Up to now, only systems consisting of similar gentileons have been considered. Let us consider also systems formed by two different kinds of gentileons, g and G . Taking

into account the Statistical Principle, we must expect that systems like (gG), (gggG), (gggGGG), and so on, are allowed. On the other hand, systems like (ggG), (ggGG), (gggGG), ... , seem to be prohibited since (gg) and (GG) are not allowed.

Another requirement of the Statistical Principle is that only bosons and fermions can appear as free particles in nature. Confinement and saturation are basic intrinsic properties of gentileons. This last result has led us to the suggestion that bosons and fermions are aggregates of gentileons.[1] It would be more natural that gentileons, instead of bosons and fermions, would be permanently confined inside a very small volume ΔV, like that occupied by an elementary particle. This is particularly clear in view of the fact that Schrödinger equation is a partial differential equation, the solutions of which are unique only when initial, boundary or other equivalent conditions are given. In a physical problem, the Schrödinger equation and the boundary conditions are inextricably connected and the latter can in no way be considered less important than the former. Thus, the following primordial boundary conditions must be obeyed, otherwise the Statistical Principle would be violated : "the flux ϕ of gentileons througt the surface which encloses the volume ΔV must be zero". We do not know, the exact confinement mechanism. It could be produced by a very peculiar confining interaction potential between quarks, by an impermeable bag as is proposed in the bag model, or something else. This means that the gentilionic scheme must be formulated in order to have

$\phi = 0$, otherwise when taken over directly into the quantum theory it would get inconsistencies. In this sense, our approch does not signify any departure from or modifications in the general pinciples of quantum mechanics.

Assuming that quarks are spin 1/2 gentileons[1,2] we see that, according to the Statistical Principle, the systems (q) and (qq) are rigorously prohibited and systems like $(qq\bar{q})$ and $(qq\bar{q}\bar{q})$, for instance, seem to be prohibited too. Thus, the mesons $(q^2\bar{q}^2)$ could exist only as bound states $(q\bar{q})$ - $(q\bar{q})$ of the mesons $(q\bar{q})$.[7] We will not treat here the systems with more than three quarks: we assume the baryons as formed by three identical gentileons (qqq) and the mesons by two different gentileons $(q\bar{q})$. We will sketch now, in a first approximation, some properties of gentilionic hadrons in the spirit of quantum field theory.

We begin by analysing the mesons $(q\bar{q})$. If the masses m_q and $m_{\bar{q}}$ of the quarks obey the condition $m_{\bar{q}} >> m_q$, the hamiltonian of the system can be written assuming that q is submitted to an effective external field generated by $\bar{q}$. Since q has spin 1/2 it would obey a Dirac equation. This would imply, in a field theory, that q can be taken as a fermion, from the algebraic point of view. Since q and $\bar{q}$ must be permanently confined, our formalism must be consistently enounced to be able to predict this unusual property. In other words, a gentileon submitted to an effective external field can be approximated by a "confined fermion".

When $m_q \cong m_{\bar{q}}$, or in the case of baryons, in the absence of a better hint, we will assume that the gentileons interact via "effective external" gluon fields. With this drastic simplifying hypothesis we see that the gentileons are reduced to "confined fermions". For baryons this means that the 4-dimensional gentilionic state-vector is changed into a one--dimensional fermionic wavefunction. Under these conditions, we conclude that Quantum Chromodynamics could be suited to estimate the properties of gentilionic hadrons. However, some additional conditions, which appear naturally for gentileons, must be obeyed in QCD approximation : (a) confinement, (b) saturation, (c) only the existence of colour singlet hadrons is admitted and (d) the electromagnetic colour charges are equal to zero.[2] These conditions show the distinguishing features between the two approaches : in the fermionic QCD they are imposed "ad hoc" whereas in the gentilionic theory they are deduced in a simple way from the Statistical Principle only with the assumption that quarks are gentileons.

Concluding this note, we must remark that the hypothesis of regarding gentileons as the "building blocks" of the elementary particles can have very far reaching consequences. Some very fundamental problems like the proton decay in supersymmetric theories, the cosmological "Big Bang" and the gravitational collapse must be reviewed.

ACKNOWLEDGEMENTS

The authors thank Prof. H. Fleming and Prof. J. Frenkel for helpful discussions on QCD and the Conselho Nacional de Pesquisas for the finatial support.

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