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by

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ON THE EQUIVALENCE OF DILUTE ANTIFERROMAGNETS AND FERROMAGNETS IN RANDOM EXTERNAL FIELDS: CURIE-WEISS MODELS

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ABSTRACT

Using a method proposed by van Hemmen we compute the free energy of the Curie-Weiss version of the site-dilute antiferromagnetic Ising model in the presence of an uniform magnetic field. The solution displays an exact correspondence between this model and the Curie-Weiss version of the Ising model in the presence of a random magnetic field. The phase diagrams are discussed and a tricritical point is shown to exist.

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Considerable attention has been devoted in the literature to the study of the Ising model in the presence of random magnetic fields (RMF) [1,6,7]. The critical behavior and the phase diagram of this model are believed to be related to those of a site dilute antiferromagnetic Ising model in the presence of an applied uniform magnetic field (DAF) [2,3]. This is specially relevant since site-dilute magnetic materials can be prepared and the magnetic-ordering transition as a function both of the external field and of the dilution of the magnetic species has been experimentally investigated [4].

In this letter we use an approach introduced by van Hemmen [5] to compute the free energy and the phase diagram of a Curie-Weiss mean field version of a DAF model. We then compare our results to those obtained by Salinas and Wrezinski [6] for the Curie-Weiss version of a RMF model and establish an exact correspondence between the parameters and phase diagrams of the two-models, as to show their complete equivalence.

Our mean field DAF model is described in a finite-volume $\Lambda \subset \mathbb{Z}^\nu$ by the hamiltonian

$$H_{DAF} = -\frac{J}{N}\sum_{i,j\in\Lambda_e}\epsilon_i\epsilon_j\sigma_i\sigma_j - \frac{J}{N}\sum_{i,j\in\Lambda_o}\epsilon_i\epsilon_j\sigma_i\sigma_j + \frac{J}{N}\sum_{\substack{i\in\Lambda_e\\ j\in\Lambda_o}}\epsilon_i\epsilon_j\sigma_i\sigma_j + H\sum_{i\in\Lambda}\epsilon_i\sigma_i \qquad (1)$$

where $\Lambda_e = \Lambda \cap \mathbb{Z}_e^\nu$, $\Lambda_o = \Lambda \cap \mathbb{Z}_o^\nu$ with $\mathbb{Z}_e^\nu$ ($\mathbb{Z}_o^\nu$) being the sublattice of $\mathbb{Z}^\nu$ for which the sum of coordinates of each site are even (odd) integers. The interaction is antiferromagnetic ($J > 0$) between sites in different sublattices and ferromagnetic between sites in the same sublattice. The random

variables $\epsilon_i \in \{0,1\}$ describe the site dilution and they are taken to independent and identically distributed, with

$$\epsilon_i = \begin{cases} 1, & \text{probability } p \\ 0, & \text{probability } 1-p \end{cases}$$

The spin variables σ_i are, for simplicity, taken to be of Ising type: $\sigma_i = \pm 1$. The external magnetic field H is uniform and deterministic. N denotes the number of points in Λ.

Introducing the sub-lattice magnetizations:

$$m_e = \frac{1}{N}\sum_{i\in\Lambda_e} \epsilon_i \sigma_i \quad , \quad m_o = \frac{1}{N}\sum_{i\in\Lambda_o} \epsilon_i \sigma_i$$

the hamiltonian (1) can be written in the form

$$H_{DAF} = -N\frac{J}{2}(m_e - m_o)^2 + NH(m_e + m_o) \tag{2}$$

As proved by van Hemmen [5] the quenched free energy density $f(\beta)$ of the system at inverse temperature β is given by:

$$-\beta f(\beta) = \max_{\vec{m}\in\mathbb{R}^2} \{Q(\vec{m}) + c^*(\vec{m})\} \tag{3}$$

where $\vec{m}$ is the two-component vector $\vec{m} = (m_e, m_o)$,

$$Q(\vec{m}) = -\frac{\beta}{N} H_{DAF} \tag{4}$$

$$c^*(\vec{m}) = -\sup_{\vec{t}\in\mathbb{R}^2} (\vec{m}\cdot\vec{t} - c(\vec{t})) \tag{5}$$

and

$$c(\vec{t}) = \lim_{N\to\infty} \frac{1}{N} \ln\left[\frac{1}{2^N}\sum_{\{\sigma_i=\pm 1\}} \exp N\vec{t}\cdot\vec{m}\right] \tag{6}$$

We then compute:

$$c(\vec{t}) = \frac{1}{2}\lim_{N\to\infty}\left\{\frac{2}{N}\sum_{i\in\Lambda_e} \ln\cosh(t_e\epsilon_i) + \frac{2}{N}\sum_{i\in\Lambda_o} \ln\cosh(t_o\epsilon_i)\right\}$$

$$= \frac{1}{2}\left\{\overline{\ln\cosh(t_e\epsilon)} + \overline{\ln\cosh(t_o\epsilon)}\right\} \tag{7}$$

where $\bar{A}$ denotes the average of the quantity $A(\epsilon)$ with respect to the probability distribution of the random variable ϵ. The last equality in (7) is a consequence of the strong law of large numbers, which is a key ingredient in the technique of van Hemmen. Therefore

$$c(\vec{t}) = \frac{p}{2}\ln\cosh t_e + \frac{p}{2}\ln\cosh t_o \tag{8}$$

If we now introduce the order parameter $m = m_e - m_o$, the free energy is given by

$$f_{DAF}(\beta, J, p, H) = \frac{1}{2}Jm^2 - \frac{p}{2\beta}\ln[\cosh\beta(Jm-H)]$$

$$- \frac{p}{2\beta}\ln[\cosh\beta(Jm+H)] \tag{9}$$

with m determined by the equation

$$\frac{2m}{p} = \tanh[\beta(Jm-H)] + \tanh[\beta(Jm+H)] \tag{10}$$

or equivalently

$$f_{DAM}(\beta,J,p,H) = \inf_m g_{DAM}(\beta,J,p,H,m) \qquad (11)$$

where g_{DAM}, the free energy density in the magnetization ensemble, is defined by the r.h.s. of (9).

Let us now consider the Curie-Weiss RMF hamiltonian

$$H_{RMF} = -\frac{J}{N}\sum_{i,j\in\Lambda}\sigma_i\sigma_j + \sum_{i\in\Lambda}h_i\sigma_i \qquad (12)$$

where the random variables h_i, $i\in\Lambda$ are independent and identically distributed, being equal to $\pm h$ $(h>0)$ with probability $\frac{1}{2}$. Applying the method of van Hemmen to H_{RMF}, the free energy density can be computed [6]:

$$f_{RMF}(\beta,J,h) = \frac{1}{2}Jm^2 - \frac{1}{2\beta}\ln[\cosh\beta(Jm-h)] - \frac{1}{2\beta}\ln[\cosh\beta(Jm+h)] \qquad (13)$$

with the order parameter m determined by

$$2m = \tanh[\beta(Jm-h)] + \tanh[\beta(Jm+h)] \qquad (14)$$

or equivalently

$$f_{RMF}(\beta,J,h) = \inf_m g_{RMF}(\beta,J,h,m) \qquad (15)$$

with g_{RMF} defined as the r.h.s. of (13).

Comparing expressions (9) and (13) we see that:

$$f_{DAF}(\beta,J,p,H) = p\, f_{RMF}(\beta,pJ,H) \qquad (16)$$

and

$$g_{DAF}(\beta,J,p,H,m) = p\, g_{RMF}(\beta,pJ,H,\tfrac{m}{p}) \qquad (17)$$

which establishes the correspondence between the two models.

As in [6] the phase diagram of the DAF model can be analysed by expanding g_{DAM} in powers of m around $m=0$:

$$g_{DAF}(m) = A + Bm^2 + Cm^4 + Dm^6 + \cdots \qquad (18)$$

where

$$A = -\frac{p}{\beta}\ln\cosh\beta H$$

$$B = \frac{J}{2} - \frac{p}{2\beta}(\beta J)^2\,\mathrm{sech}^2\beta H$$

$$C = \frac{p}{2\beta}(\beta J)^4(\mathrm{sech}^2\beta H)(1-3\tanh^2\beta H) \qquad (19)$$

$$D = -\frac{p}{6\beta}(\beta J)^6(\mathrm{sech}^2\beta H)\left(\tanh^4\beta H - \tanh^2\beta H + \frac{2}{15}\right)$$

There is a critical line determined by the condition $B=0$ i.e.

$$p\beta J = \cosh^2\beta H \qquad (20)$$

On this critical line, there is a tri-critical point determined by the conditions $B=C=0$, $D>0$:

$$p\beta J = \frac{3}{2} \quad , \quad \tanh^2\beta H = \frac{1}{3} \qquad (21)$$

The critical line represents a first order phase transition

for $p\beta J > \frac{3}{2}$ (since $c < 0$) and a second order phase transition for $p\beta J < \frac{3}{2}$ (since $c > 0$).

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