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NEW VALUES FOR NEUTRINO MASS IN THE CONTEXT OF A HORIZONTAL MODEL

by

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ABSTRACT

We study in detail an extended version of the Glashow-Salam-Weinberg model for the electroweak interactions. It has an extra global horizontal symmetry, which allows the appearance of neutrino mass terms. The constraints imposed by the standard cosmological model are used to determine the allowed range of variation of the free parameters of this model. We find that unstable massive neutrinos are permited in a set of values for their masses between 100 eV and ~ 1 GeV.

1. INTRODUCTION

For the last five years, a great number of models in the context of Elementary Particle Physics have been proposed to give masses to the neutrinos. Three facts have motivated physicists to work along this line. The first one is the experiment of the ITEP group[1] that, for the first time, has determined a lower bound to the electron neutrino mass: $14 \text{ eV} \leq m_{\nu_e} \leq 46 \text{ eV}$. The second one is the development of Grand Unified Theories; some of these allowing the existence of non zero neutrino masses[2].

However, it seems that the principal motivation is at the level of Cosmology and Astrophysics; massive neutrinos could, in principle, provide solutions to some cosmological and astrophysical problems like the problem of dark matter in the Universe, the formation of large scale structures in a Robertson-Walker-Friedmann Universe, the halos of galaxies, the solar neutrino puzzle and many others.

Among these models one of the most popular is the Gelmini-Roncadelli model (G.R.)[3], which introduces an extra Higgs triplet into the standard model for the electroweak interactions: the Glashow-Salam-Weinberg model. In the G.R. model, the Majorana neutrino mass term appears as a consequence of the spontaneous breaking of an extra global symmetry. Moreover, in the context of this model, a neutrino ν_H (a heavy mass neutrino) can decay into a ν_L (a light mass neutrino) plus a Goldstone boson, the Majoron χ, that also appears as a consequence of the spontaneous breaking of these symmetry. This decay, is relevant to the solutions of some cosmological problems[12]. Nevertheless, when its free parameters are submitted to the astrophysical and cosmological constraints (the former is necessary since the coupling between Majorons and charged leptons is not null), the resulting neutrinos are not cosmologically relevant, because their contribution to the energy density of the Universe is negligible, as shown by Georgi, Glashow and Nussinov[4].

As an attempt to solve this problem, Valle[5] has proposed an extention of G.R. model that introduces, at least, two extra Higgs triplets into the electroweak lagrangian - the "horizontal model". This allows the elimination of the coupling between Majorons and charged leptons.

We do an explicit verification of this proposal, i.e., once eliminated these coupling, we impose only cosmological constraints on the free parameters of this new model and verify if the problem of the cosmological relevance of the neutrinos is solved in this manner.

In section two we present a summary of the main hypothesis and consequences of the horizontal model and we show in section three, the cosmological restrictions that we impose on the free parameters of the model, and the results obtained.

The conclusions are present in section four. In the Appendix we show a technical result that is associated with the relation between the couplings constants of the model.

2. THE HORIZONTAL MODEL

The main hypothesis of the horizontal model, which will be the subject of our analysis, are the following:

1. The electroweak lagrangian is invariant under an extra global symmetry group, $U(1)_G$ between the families: the "horizontal symmetry". The leptons carry global charge q_α with respect to that group and this charge is in general different for different lepton species;

2. There are not extentions in the leptonic sector;

3. New Higgs triplets, H_a, are introduced (two in the case we are considering; $a = 1,2$), carrying global charge Q_a with respect to $U(1)_G$, distinct for each triplet. The $U(1)_G$ symmetry is spontaneously broken by a non-vanishing vacuum expectation value (V.E.V.) of the triplet;

4. The neutrino mass terms originate from an Yukawa coupling between the neutrinos and the Higgs triplets, after the spontaneous breaking of the new symmetry;

5. The mass eigenstates neutrinos are not the usual lepton number eigenstates.

As consequences, we have the appearance of a Goldstone boson - the Majoron χ and, among others, the coupling between charged leptons and Majorons:

$$g_{ff\chi} \cong \pm \frac{2}{<\phi>} \frac{\sum_{a=1}^{2} Q_a <H_a>^2}{[Q_1^2 <H_1>^2 + Q_2^2 <H_2>^2]^{1/2}} \frac{m_\ell}{<\phi>} , \qquad (2.1)$$

To calculate ρ_{χ_0} , we have used the approximations:

$$\frac{t_D}{\tau_{1/2}} << 1 \quad \text{and} \quad \frac{t_u}{\tau_{1/2}} >> 1 \quad . \qquad (3.14)$$

Finally, after imposing that $\rho_{\chi_0} < \rho_c$, for $\rho_c = 8 \times 10^{-47}\ h_0^2\ \mathrm{GeV}^4$, we arrive at:

$$\frac{m'^{1/8}\ m^{3/4}}{|F|^{5/2}} < 1.6\ h_0^2 \times 10^{25}\ [1\ \mathrm{eV}]^{7/8} \qquad (3.15)$$

where

$h_0 \equiv \frac{H_0}{100} \frac{\sec M_{pc}}{km}$, H_0 is the present value of the Hubble parameter.

Finally, the last restriction to be imposed is:

C. THE PRESENT CONTRIBUTION TO THE ENERGY DENSITY OF THE UNIVERSE DUE TO THE NEUTRINOS MUST BE, AT LEAST, 1% OF ρ_c : $\rho_{\nu_0} \geq 1\%\ \rho_c$.

At the present time, all ν_H^S must have decayed. On the other hand, the ν_L^S can annihilate each other into Majorons. We intend to determine under what conditions the resultant neutrinos can contribute to the energy density of the Universe.

Following reference 4, we assume that at the time

t_i , the light neutrinos become nonrelativistic. For $t < t_i$, they are in equilibrium with Majorons through the reaction (3.2). For $t_i < t < t_u$, their number N in a comoving cube, obeys the equation,

$$\frac{dN}{dt} = -\ n\, N\, \beta\, \sigma \qquad (3.16)$$

where n is the numerical density of neutrinos; β their relative velocity in the center of mass system and σ is the cross section for the annihilation:

$$\sigma \cong \frac{|F|^4}{256\pi} \left(\frac{m'}{m}\right)^2 \frac{\beta}{S} \qquad (3.17)$$

$S = 4m^2$.

It can be shown[4] that N has at present, an asymptotic value N_∞, given in our case by:

$$N_\infty \cong \frac{7}{3} \times \frac{1024\pi\ m^5\ R_i^3}{|F|^4\ m'^2} \qquad (3.18)$$

R_i is the value of the cosmic scale factor $R(t)$, at $t = t_i$.

It follows that

$$\rho_{\nu_0} = n_0 m = \frac{N_\infty}{R^3(t_u)}\, m \cong \frac{2389\pi\ m^5}{t_u\ |F|^4\ m^2} \left(\frac{T_\nu}{m}\right)^{3/2} \qquad (3.19)$$

T_ν is the present neutrinos temperature ($T_\nu \cong 2 \times 10^{-5}$ eV) .

With the imposition that $\rho_{\nu_0} \geq 1\% \, \rho_c$, we obtain:

$$\frac{m^{7/2}}{m'^2} \frac{1}{|F|^4} \geq 5.3 \, h_0^2 \times 10^{23} \, [1\,\mathrm{eV}]^{3/2} \quad , \qquad (3.20)$$

Figure 2 shows the results obtained in this section.

4. CONCLUSIONS

Imposing cosmological constraints on the free parameters of the horizontal model, has allowed us to determine a set of values for m' and F. In particular, the range for m' (100 eV to ~1 GeV) is exactly the same which has been obtained for nonstable neutrinos after imposing that the contribution to the present value of the density parameter of the Universe, due to neutrinos, Ω_ν, obeys: $\Omega_\nu h_0^2 < 1$ ($\Omega_\nu = \rho_{\nu_0}/\rho_T$), as observed[10]. The principal conclusion is that the horizontal model is in accordance with the predictions of the standard cosmological model, but only for a fairly restricted range of the parameters.

APPENDIX – RELATION BETWEEN g_L, g_H and F

We consider only the case of two mass eigenstates neutrinos: ν_H and ν_L. In reference 5, one can find the explicit form of the coupling F:

$$F_{\alpha\beta} \equiv F = \frac{2m_\alpha \, R_{\alpha\beta}}{\left[\sum_{a=1}^{2} Q_a \langle H_a \rangle X_a\right]} \qquad \alpha \neq \beta \qquad (A.1)$$

where:

$$R_{\alpha\beta} \equiv \sum_\gamma U^*_{\gamma\alpha} \, q_\gamma \, U_{\gamma\beta} \quad ;$$

$$X_a = \left[\sum_{a=1}^{2} Q_a^2 \langle H_a \rangle^2\right]^{1/2} \Big/ 2\, Q_a \langle H_a \rangle \quad ; \qquad (A.2)$$

U is the mixing matrix for the leptonic sector.

If we identify $g_L \equiv F_{\alpha\alpha}$, for $\alpha = 1$ and $g_H \equiv F_{\alpha\alpha}$, for $\alpha = 2$, we see that:

$$\frac{g_L}{F} = \frac{R_{11}}{R_{12}} \qquad \text{and} \qquad \frac{g_H}{F} = \frac{R_{22}}{R_{21}} \quad . \qquad (A.3)$$

We can now parametrize the values of the charges in the following way[11]:

$$\begin{aligned} Q_1 &= -2q \\ Q_2 &= -2(2q-1) \\ q_1 &= 1 \\ q_2 &= 2q-1 \end{aligned} \qquad (A.4)$$

where q is an arbitrary parameter.

The Yukawa coupling between the H_a and the lepton number eigenstates neutrinos N_α, has the form:

$$\sum_a N_\alpha \sigma_2 g^{(a)} H_a N_\beta + h.c. \qquad (A.5)$$

Then, using (A.4) we can determine the nondiagonal mass matrix for the neutrinos:

$$m = \begin{bmatrix} 0 & m_* \\ m_* & M \end{bmatrix} \qquad (A.6)$$

where we are assuming that $m_* \sim \langle H_1 \rangle$ and $M \sim \langle H_2 \rangle$; $\langle H_1 \rangle \ll \langle H_2 \rangle$.

The U matrix which diagonalizes (A.6) can be put in the form (assuming CP invariance):

$$U = \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix} . \qquad (A.7)$$

On the other hand, (A.6) can be diagonalized without the use of (A.7). The result is:

$$m_{diag} = \begin{bmatrix} m_*^2/M & 0 \\ 0 & M \end{bmatrix} \equiv \begin{bmatrix} m & 0 \\ 0 & m' \end{bmatrix} . \qquad (A.8)$$

The last step imposes a condition on θ:

$$tg\,\theta \cong \left(\frac{m}{m'}\right)^{1/2} . \qquad (A.9)$$

Now, we can write $R_{12} = R_{21}$ $(\alpha \neq \beta)$, R_{11} and R_{22} explicitly:

$$\begin{aligned} R_{12} &= 2(1-q)\operatorname{sen}\theta\cos\theta \\ R_{11} &= \cos^2\theta - \operatorname{sen}^2\theta + 2q\operatorname{sen}^2\theta \\ R_{22} &= \operatorname{sen}^2\theta - \cos^2\theta + 2q\cos^2\theta \end{aligned} \qquad (A.10)$$

To determine q, we use the relation (2.2) and (A.4), obtaining:

$$q \cong \frac{1}{2 + \left(\frac{m}{m'}\right)} . \qquad (A.11)$$

Introducing (A.9) and (A.11) into (A.10), we can determine (A.3). The result is:

$$\left|\frac{g_H}{F}\right| \cong \frac{1}{2}\left(\frac{m'}{m}\right)^{1/2} \quad \text{and} \quad \left|\frac{g_L}{F}\right| \cong \left(\frac{m'}{m}\right)^{1/2} . \qquad (A.12)$$

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FIGURE CAPTIONS

FIG. 1 - Possible Majoron-neutrino couplings in the horizontal model.

FIG. 2 - Region of allowed values for $|F|$ and m' determined after imposing the three restrictions presented in this section and $\frac{m}{m'} << 1$. [1], [2] and [3] correspond, respectively to $\tau_{1/2} < t_u$, $\rho_{\chi_0} < \rho_c$ and $\rho_{\nu_0} \geq 1\%\ \rho_c$, for $m = 10$ eV and $h_0 = 1$. We see that the intersection of these four regions shows that the allowed values for m' and $|F|$ are: $100 \text{ eV} \leq m' \leq 1 \text{ GeV}$ and $10^{-9} \leq |F| \leq 10^{-6}$.

FIG. 1

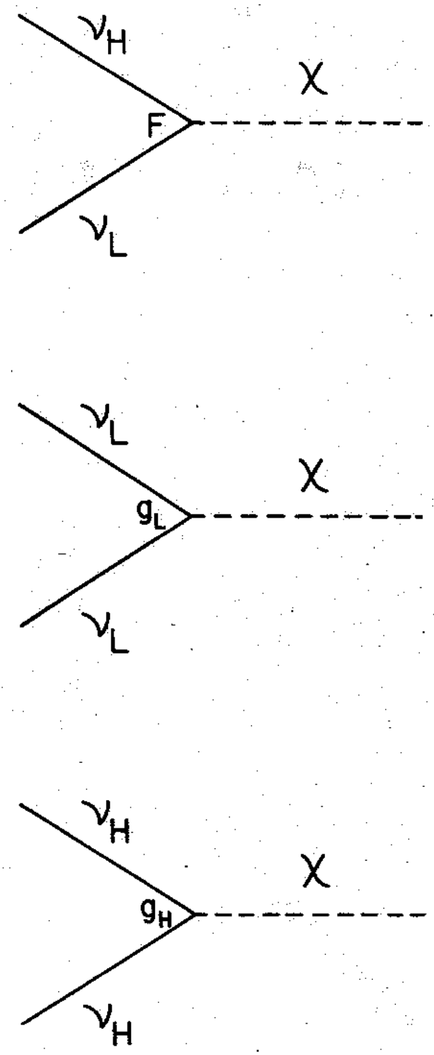
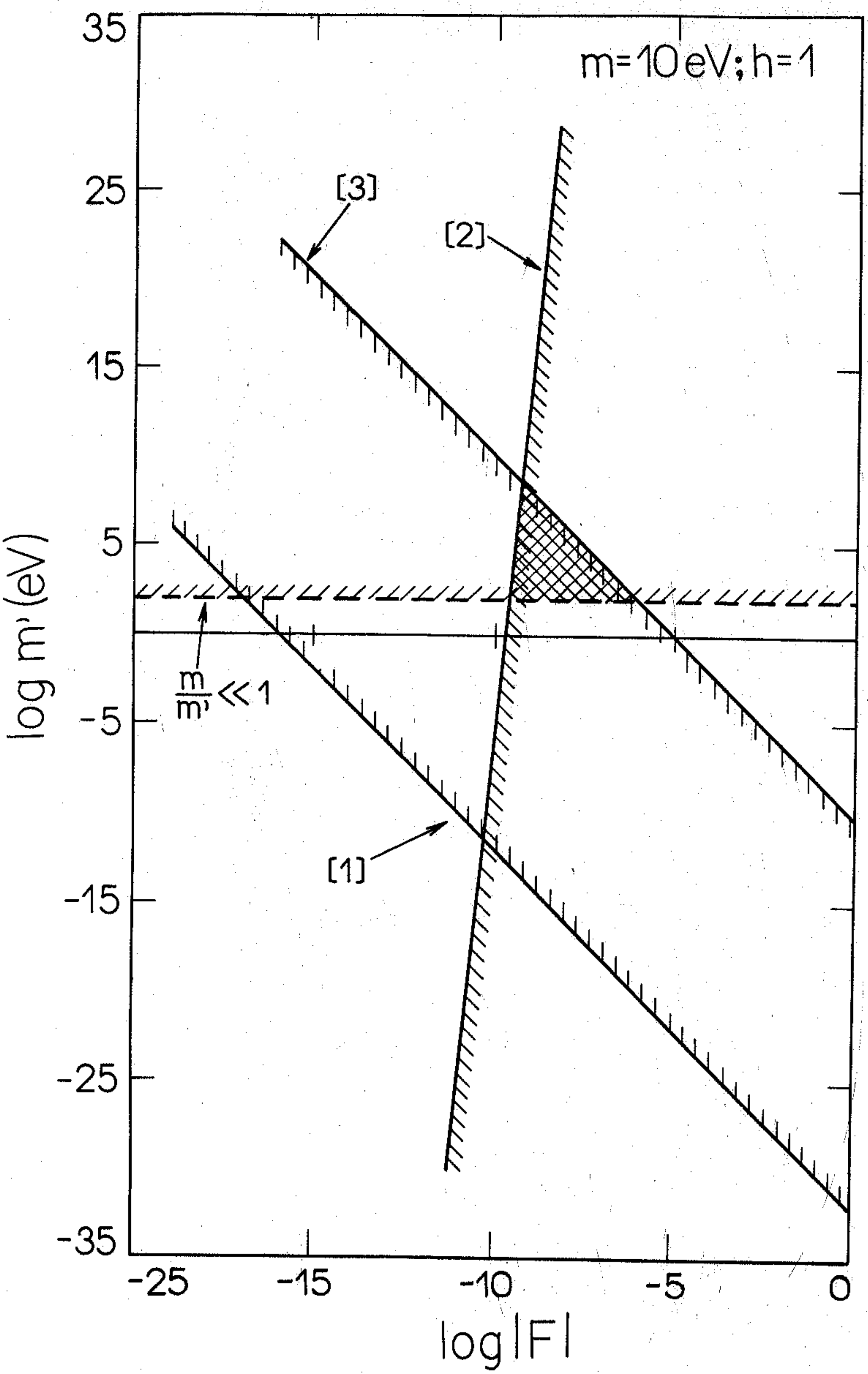


FIG. 2



m=10 eV; h=1
[3]
[2]
[1]
$\frac{m}{m'} \ll 1$
35
25
15
5
-5
-15
-25
-35
log m' (eV)
-25
-15
-10
-5
0
log |F|