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## FROM QUANTUM TO SEMICLASSICAL KINETIC EQUATIONS: NUCLEAR MATTER ESTIMATES

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## ABSTRACT

Starting from the exact microscopic time evolution of the quantum one body density associated with a many fermion system we derive semiclassical approximations to it. In the limit where small momentum transfer two body collisions are dominant we get a Fokker-Planck equation and work out friction and diffusion tensors explicitly for nuclear matter. If arbitrary momentum transfers are considered a Boltzmann equation is derived and used to calculate the viscosity coefficient of nuclear matter. We also give a derivation of the collision term used by Landau to describe the damping of zero sound waves at low temperatures in Plasmas. Memory effects are essential for this. The damping of zero sound waves in nuclear matter is also calculated and the value so obtained associated with the bulk value of the damping of giant resonances in finite nuclei. The bulk value is estimated to be quite small indicating the importance of the nuclear surface for the damping.



## I. INTRODUCTION

The purpose of this paper is the derivation and interpretation of semiclassical approximations to the collisional dynamics of many fermion systems, starting from a quantum microscopic theory for the time evolution of the one body density. In the quantum limit, the exact dynamics of the one body density contains contributions of two distinct types: the usual mean field contribution and correlation corrections to it which entail nonunitary effects[1]. If correlation effects are treated perturbatively their contribution to the dynamics acquires a very simple form[2] which is nonlinear and non-markovian. Relaxation properties of the one body density are essentially connected to these correlation corrections and have been investigated in a one dimensional uniform system, which allows for a numerical solution including nonlinear and non-markovian effects[3]. The small amplitude limit of the one body dynamics has also been studied and shown to provide for a self consistent treatment of occupation probabilities, as well as for a systematic procedure to introduce corrections to the RPA amplitude[2]. In this paper we study semiclassical approximations to this collisional dynamics, both in the long and small amplitude limits. Memory effects are, of course, also present in the semiclassical limit of the theory and are shown to be essential for the derivation of the collision integral Landau used to describe the damping of zero sound waves in infinite systems. We study along the same lines the collisional damping of zero sound waves in nuclear matter and interpret the

result obtained as representative of the bulk value of the damping of collective modes in finite nuclei. The damping turns out to be too small if compared with experimental values, indicating the importance of the nuclear surface for the description of the damping mechanism.

If memory effects are neglected, we arrive at Boltzmann's collision integral with Fermi statistics. This type of semiclassical equation has been wildely used to describe intermediate energy heavy ion collisions[4],[5]. We investigate the viscosity coefficient of nuclear matter and compare our estimate with empirical values obtained by adjusting spreading widths of giant resonances and the kinetic energy of fission fragments[6] by means of a Navier-Stokes equation. We conclude that such dissipation mechanism, which is based on hydrodynamical hypothesis of local equilibrium gives a poor description of dissipative phenomena in nuclei, emphasizing once more the important role played by the nuclear surface.

The Boltzmann collision integral as derived is valid for arbitrary momentum transfers. In the limit where small momentum transfer two body collisions are dominant, a Fokker-Planck equation can be obtained. Friction and diffusion coefficients for nuclear matter are derived. An interesting application of this friction coefficient is to the inelastic scattering of 800 MeV protons in nuclei at very forward angles. In this case the momentum transfer is small, of the order of half the Fermi momentum[7], and this limit of the theory should be applicable.

In section II we review briefly the microscopic



quantum equations which will be our starting point. In section III the semiclassical limit of the mean field part is derived and a quantitative basis to the usual "$\hbar \to 0$" limit is given. Section IV contains the derivation of the three semiclassical approximations to the collision integral discussed above Section V the three applications to nuclear matter. Our conclusions are given in section VI.

## II. QUANTUM COLLISIONAL DYNAMICS OF THE ONE BODY DENSITY: A BRIEF REVIEW

The equation governing the time evolution of the one body density operator is given by[1]

$$i\dot{\hat{\rho}} = (\hat{\hat{\ell}}_o(t) + \hat{\hat{\ell}}(t))\,\hat{\rho}(t) + \hat{\pi}(t) \qquad \text{(II.1)}$$

where

$$\left(\hat{\hat{\ell}}_o \hat{\rho}(t)\right)_{\lambda\mu} = \mathrm{Tr}\left(c_\mu^+ c_\lambda \left[H, F_o(t)\right]\right) \qquad \text{(II.2)}$$

$$\left(\hat{\hat{\ell}}(t)\hat{\rho}(t)\right)_{\lambda\mu} = -i\,\mathrm{Tr}\left(c_\mu^+ c_\lambda \int_o^t dt' \left[H, G(t,t')\mathcal{Q}(t')\left[H, F_o(t')\right]\right]\right) \qquad \text{(II.3)}$$

and

$$\hat{\pi}_{\lambda\mu}(t) = \mathrm{Tr}\left(c_\mu^+ c_\lambda \left[H, G(t,o) F_I^{\dagger}(o)\right]\right) \qquad \text{(II.4)}$$

The uncorrelated Fock space density $F_o(t)$ is best written in terms of fermions operators $c_\lambda$, $c_\lambda^+$ associated with the time dependent natural orbitals $|\lambda(t)>$, which make $\hat{\rho}(t)$

diagonal,

$$\hat{\rho}(t) = \sum_{\lambda} |\lambda(t)\rangle\, p_{\lambda}(t)\, \langle\lambda(t)| \qquad \text{(II.5)}$$

In this case

$$\hat{F}_0(t) = \prod_{\lambda} \left[ (1-p_{\lambda}) c_{\lambda} c_{\lambda}^{+} + p_{\lambda} c_{\lambda}^{+} c_{\lambda} \right] \qquad \text{(II.6)}$$

In this representation it is very simple to disentangle unitary ($\lambda \neq \mu$ in eq. (II.3)) and nonunitarity ($\lambda = \mu$ in eq. (II.3)) correlation contributions to the dynamics.

The propagator $G(t,t')$ is formally written as

$$G(t,t') = T_{exp}\left[-i \int_{t'}^{t} dt'' \, Q(t'') \cdot L \right] \qquad \text{(II.7)}$$

where $L$ is the liouvillian generator associated with the hamiltonian $H$, and the superoperator $Q(t)$ essentially eliminates uncorrelated parts of the objects on which it acts. It is discussed in detail in ref. (1).

$F_I'$ is the correlation part of the initial full density matrix $F(t)$, i.e.

$$F_I'(0) = F(0) - F_0(0) \qquad \text{(II.8)}$$

and we assume it to be zero throughout this paper. The first term on the r.h.s. of eq. (II.1) represents the usual time dependent (mean field) Hartree-Fock contribution to the dynamics

$$\left(\hat{\ell}_0 \hat{\rho}(t)\right) = \left[ h[\rho], \rho \right] \qquad \text{(II.9)}$$

where $h[\rho]$ is the Hartree-Fock hamiltonian for the given two



body potential. The second term on the r.h.s. of eq. (II.1) carries the correlation contributions to the dynamics. They are responsible for collisional effects. In order to implement the collisional contributions, we consider correlation corrections up to second order in the two body coupling potential, which amounts to replace $G(t,t')$ by $g_{MF}(t,t')$ the unitary time displacement operator associated with the correlation free mean field propagation. In this case, the collisional corrections can be cast into a very simple form

$$\hat{\Gamma}_{\lambda\mu} \equiv \left[ \hat{\hat{\ell}}(t) \rho(t) \right]_{\lambda\mu} \cong$$

$$\cong \frac{i}{2} \sum_{\rho\gamma\delta} \int_{0}^{t} dt' \Big[ \langle\gamma\delta|\tilde{v}|\mu\rho\rangle_t \langle\lambda\rho|\tilde{v}|\gamma\delta\rangle_{t'} \left(p_{\lambda} p_{\rho} q_{\gamma} q_{\delta} - \rho \leftrightarrow q\right)_{t'}$$

$$+ \langle\lambda\rho|\tilde{v}|\gamma\delta\rangle_t \langle\gamma\delta|\tilde{v}|\mu\rho\rangle_{t'} \left(p_{\mu} p_{\rho} q_{\gamma} q_{\delta} - \rho \leftrightarrow q\right)_{t'} \Big] \qquad \text{(II.10)}$$

## III. SEMICLASSICAL LIMIT OF THE COLLISIONLESS DYNAMICS: THE VLASOV EQUATION

Neglecting collisional contributions, the equation governing the dynamics of the one body density is given by

$$i\hbar \dot{\rho}_{\alpha\beta} = \left[ h[\rho], \rho \right]_{\alpha\beta} \qquad \text{(III.1)}$$

The traditional way to study its semiclassical limit is to write the above equation in Wigner representation as follows

$$\frac{\partial \rho_\omega(\vec{p},\vec{q})}{\partial t} = \frac{2}{\hbar}\sin\left[\frac{\hbar}{2}\left(\frac{\partial^{(h)}}{\partial \vec{q}}\frac{\partial^{(\rho)}}{\partial \vec{p}} - \frac{\partial^{(h)}}{\partial \vec{p}}\frac{\partial^{(\rho)}}{\partial \vec{q}}\right)\right] h_\omega(\vec{p},\vec{q})\,\rho_\omega(\vec{p},\vec{q}) \qquad \text{(III.2)}$$

where

$$f_\omega(\vec{p},\vec{q}) = \int e^{i\vec{p}\cdot\vec{r}} \left\langle \vec{q}+\frac{\vec{r}}{2}\middle|\rho\middle|\vec{q}-\frac{\vec{r}}{2}\right\rangle \qquad \text{(III.3)}$$

The usual way to obtain the semiclassical limit of eq. (III.2) is by considering an expansion in powers of $\hbar$, and retaining the zero$^{\underline{th}}$ order contribution. We can give more quantitative meaning to this expansion. If we write

$$h_\omega(\vec{p},\vec{q}) = \int h_\omega(\vec{p},\vec{k}_1)\, e^{i\vec{k}_1\cdot\vec{q}}\, d\vec{k}_1 \qquad \text{(III.4a)}$$

and

$$\rho_\omega(\vec{p},\vec{q}) = \int \rho_\omega(\vec{p},\vec{k}_2)\, e^{i\vec{k}_2\cdot\vec{q}}\, d\vec{k}_2 \qquad \text{(III.4b)}$$

we can cast eq. (III.2) into the form

$$\frac{\partial \rho_\omega}{\partial t} = \frac{1}{i\hbar}\left[e^{i\frac{\hbar}{2}\left(\frac{\partial^{(h)}}{\partial \vec{q}}\frac{\partial^{(\rho)}}{\partial \vec{p}} - \frac{\partial^{(h)}}{\partial \vec{p}}\frac{\partial^{(\rho)}}{\partial \vec{q}}\right)} - c.c.\right] h_\omega \rho_\omega =$$

$$= \frac{1}{i\hbar}\int d\vec{k}_1 \int d\vec{k}_2 \left[e^{-\frac{i\hbar}{2}\left(\vec{k}_1\cdot\frac{\partial^{(\rho)}}{\partial \vec{p}} - \vec{k}_2\cdot\frac{\partial^{(h)}}{\partial \vec{p}}\right)} - c.c.\right] h_\omega \rho_\omega =$$

$$= \frac{1}{i\hbar}\int d\vec{k}_1 \int d\vec{k}_2 \left[h_\omega\left(\vec{p}+\frac{\hbar\vec{k}_2}{2},\vec{k}_1\right)\rho_\omega\left(\vec{p}-\frac{\hbar\vec{k}_1}{2},\vec{k}_2\right)\right.$$

$$\left. - h_\omega\left(\vec{p}-\frac{\hbar\vec{k}_2}{2},\vec{k}_1\right)\rho_\omega\left(\vec{p}+\frac{\hbar\vec{k}_1}{2},\vec{k}_2\right)\right] e^{i(\vec{k}_1+\vec{k}_2)\cdot\vec{q}} \qquad \text{(III.5)}$$



Now we see clearly what is required for the lowest contribution in an "$\hbar$ expansion" to be a good approximation: $h_\omega$ must be smooth in $\vec{p}$ on the scale of the inverse spatial size of $\rho_\omega$ and vice-versa. If this condition is fullfilled we obtain the Vlasov equation as a lowest order contribution

$$\frac{\partial \rho_\omega}{\partial t} \simeq \left(\frac{\partial^{(h)}}{\partial \vec{q}}\frac{\partial^{(\rho)}}{\partial \vec{p}} - \frac{\partial^{(h)}}{\partial \vec{p}}\frac{\partial^{(\rho)}}{\partial \vec{q}}\right) h_\omega(\vec{p},\vec{q})\,\rho_\omega(\vec{p},\vec{q}) \qquad \text{(III.6)}$$

We notice in this case that the key elements which control the quality of the approximation are the spatial properties of the density, since in $h_\omega$ the two body potential is folded with the density.

One of the essential differences between classical and quantum mechanics is that in the latter the states involve an essential nonlocal character when expressed in terms of classical phase space variables on the account of the uncertainty principle, which requires special correlations between position and momentum variables. When one expresses the dynamics in the Wigner representation this nonlocality manifests itself through the appearance of derivatives of arbitrarily high order inside the sine function. The truncation of the sine series by retaining only first derivatives (the zero$^{\underline{th}}$ order term in $\hbar$) amounts in fact to discarding quantum effects by rendering the dynamics blind to an essential ingredient of quantum kinematics. This is of course permissible only to the extent that this limitation can be sustained without completely obliterating the physics. In the case of the nuclear one body density one must

in particular give up the description of such essentially quantum features as sharp single nucleon occupation numbers and shell effects in favor of a suitably smoothed density which, on the other hand allows for a meaningful local interpretation (in phase space). As is well known, this can be achieved either through the use of a Strutinsky-type averaging process of fully quantal densities[8] or by working with phase space representations associated with wave-packets rather than with sharp position and momentum states[9]. We shall adhere to the Wigner representation and assume that the proper Strutinsky averaging (whenever necessary) process has been carried out in what follows.

## IV. SEMICLASSICAL APPROXIMATIONS TO THE COLLISIONAL INTEGRAL

The conditions for validity of semiclassical approximations are more restrictive in the case of collisional contributions (eq. (II.9)) due to the new element which comes into play there, namely the two body potential. Now we have two possibly distinct length scales: the one given by spatial properties of the density, as discussed before, the other given by the range of the two body potential. We shall consider three different cases: a) the momentum transfers involved in the process are small with respect to the smallest inverse length and memory effects negligible, b) the momentum transfers are arbitrary and memory effects are negligible, c) the momentum transfers are arbitrary and memory effects important.



### IV.A. LOW MOMENTUM TRANSFERS: THE FOKKER-PLANCK EQUATION

In the case where mainly low momentum transfers are involved, we can argue as in the previous section, write the collision integral in Wigner representation and expand it to first order in $\hbar$. This leads to the following semiclassical equation for the one body density in Wigner representation

$$\frac{\partial}{\partial t}\rho_\omega(\vec{p},\vec{q},t) \quad \{h_\omega,\rho_\omega\} = \Gamma_\omega^{FP}(p,\vec{q},t) \tag{IV.A.1}$$

where

$$\Gamma_\omega^{FP} = \sum_{ik}\frac{\partial}{\partial p_i}\left(-A_i + \frac{1}{2}\frac{\partial}{\partial p_k}B_{ik}\right)\rho(\vec{p},\vec{q},t)$$

$$+\sum_{ik}\frac{\partial}{\partial p_i}\left(\frac{\partial}{\partial r_k}C_{ik} - D_{ik}\right)\rho(\vec{p},\vec{q},t) \tag{IV.A.2}$$

where

$$A_i = \frac{C_\rho}{(2\pi\hbar)^3}\int d\vec{p}_2\left\{T_{ik}(1-\rho)\frac{\partial\rho_2}{\partial p_{2k}} + \rho_2(1-\rho_2)\frac{\partial}{\partial p_{1k}}T_{ik}\right\} \tag{IV.A.3a}$$

$$B_{ik} = \frac{C_\rho}{(2\pi\hbar)^3}\int d\vec{p}_2\, T_{ik}\,\rho_2(1-\rho_2) \tag{IV.A.3b}$$

$$C_{ik} = \frac{C_r}{(2\pi\hbar)^3}\int d\vec{p}_2\, S_{ik}\,\rho_2(1-\rho_2) \tag{IV.A.3c}$$

$$D_{ik} = \frac{C_r}{(2\pi\hbar)^3}\int d\vec{p}_2\, S_{ik}(1-\rho)\frac{\partial\rho_2}{\partial r_{2k}} \tag{IV.A.3d}$$

$$T_{ik} = \frac{(\vec{p}-\vec{p}_2)^2 \delta_{ik} - (\vec{p}-\vec{p}_2)_i (\vec{p}-\vec{p}_2)_k}{|\vec{p}-\vec{p}_2|^3}$$

$$S_{ik} = \frac{(\vec{p}-\vec{p}_2)^2 \delta_{ik} - 4(\vec{p}-\vec{p}_2)_i (\vec{p}-\vec{p}_2)_k}{|\vec{p}_1-\vec{p}_2|^3}$$

$$c_p = \frac{m}{8\pi}\int_0^\infty dq\, v_\omega^2(q)\, q^3 \qquad C_r = \frac{m}{2\pi^2}\int_0^\infty dq\, v_\omega^2(q)\, q^2$$

The coefficients A and B are the friction and diffusion coefficients respectively. The other two coefficients arise when one considers spatial changes in the system. C represents a modification in the mean field potential caused by the two body interaction and D modifies the relations between variances[10].

## IV.B. THE BOLTZMANN COLLISION INTEGRAL

If we are dealing with processes involving large momentum transfers, the two body potential in eq. (II.9) is to be treated as a Born amplitude. If furthermore one writes the collision integral in momentum representation and makes a local density approximation,

$$\rho(\vec{p},\vec{p}') \simeq \rho(\vec{p}',\vec{q})\,\delta(\vec{p}-\vec{p}') \qquad \text{(IV.B.1)}$$

where $\rho(\vec{p},\vec{q})$ depends only parametrically on $\vec{q}$, one gets for the collision integral



$$\Gamma^B(\vec{p},\vec{q},t) = \int_0^t dt'\, e^{i(\epsilon_p+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4})(t-t')} \cdot \int d\vec{p}_2 \int d\vec{p}_3 \int d\vec{p}_4 \;\; |\langle \vec{p}\,\vec{p}_2|\tilde{v}|\vec{p}_3\,\vec{p}_4\rangle|^2 \cdot \left[\rho(\vec{p},\vec{q})\rho(\vec{p}_2,\vec{q})(1-\rho(\vec{p}_3,\vec{q}))(1-\rho(\vec{p}_4,\vec{q})) - \rho \leftrightarrow 1-\rho\right]_{t'} \text{(IV.B.2)}$$

where $\epsilon_p$ represents the mean field energy of the state $|\vec{p}\rangle$. The usual form of the Boltzmann collision term can be obtained if we assume that the densities vary slowly in time as compared to the oscillating phase, and make the limit $t \to \infty$. In this case we obtain the usual Markovian form of the collision integral used in the theory of Fermi liquids, namely

$$\Gamma^B(\vec{p},\vec{q},t) = \pi \int d\vec{p}_2 \int d\vec{p}_3 \int d\vec{p}_4 \;\; |\langle \vec{p}\,\vec{p}_2|\tilde{v}|\vec{p}_3\,\vec{p}_4\rangle|^2 \cdot \delta(\epsilon_p+\epsilon_{p_2}-\epsilon_{p_3}-\epsilon_{p_4}) \cdot \left[\rho(\vec{p},\vec{q})\rho(\vec{p}_2,\vec{q})(1-\rho(\vec{p}_3,\vec{q}))(1-\rho(\vec{p}_4,\vec{q})) - \rho \leftrightarrow 1-\rho\right]_t \text{(IV.B.3)}$$

## IV.C. MEMORY EFFECTS: THE MODIFIED BOLTZMANN EQUATION

In this section we derive the collision integral used by Landau to study the damping of collective modes in infinite systems at zero temperature. Apart from linearization, his collision term differs from the usual Boltzmann collision term derived in the previous section by the presence of the

phonon energy ($\hbar\omega$) in the energy conserving $\delta$-function[11]. The energy $\hbar\omega$ is crucial to allow for excitations above the Fermi level at zero temperature. We show next that we can get the modified Boltzmann collision integral by including memory effects of the density, before taking the limit $t \to \infty$. Let us begin with eq. (IV.B.2), where no Markovian approximation has been made. If we linarize the equation i.e., write

$$\rho = \rho_0 + \frac{\partial \rho_0}{\partial \epsilon} \left[ \rho_+' e^{-i\omega t} + \rho_+^{*} e^{i\omega t} \right] \qquad \text{(IV.C.1)}$$

and consider terms only up to first order in $\rho_\pm'$ we get (notice that $\rho$ is still a function of $t'$)

$$\Gamma^{MB} = -\frac{1}{T} \int d\vec{p}_2 \int d\vec{p}_3 \int d\vec{p}_4 \; |\langle \vec{p}\,\vec{p}_2 | \tilde{v} | \vec{p}_3 \vec{p}_4 \rangle|^2_x$$

$$\times \rho_0(\vec{p},\vec{q})\,\rho_0(\vec{p}_2,\vec{q})\,(1-\rho_0(\vec{p}_3,\vec{q}))(1-\rho_0(\vec{p}_4,\vec{q}))_x \left[\rho_+'(\vec{p},\vec{q}) + \rho_+'(\vec{p}_2,\vec{q}) - \rho_+'(\vec{p}_3,\vec{q}) - \rho_+'(\vec{p}_4,\vec{q})\right]$$

$$\times \int_0^t dt' \exp i\left[\epsilon_p + \epsilon_{p_2} - \epsilon_{p_3} - \epsilon_{p_4} + \hbar\omega\right](t-t') \qquad \text{(IV.C.2)}$$

where $\rho_0$ has been taken as a Fermi function at temperature $T$.

If we now take the limit $t \to \infty$, we recover the collision integral in the form used by Landau

$$\Gamma^{MB} = -\frac{1}{T} \int d\vec{p}_2 \int d\vec{p}_3 \int d\vec{p}_4 \; |\langle \vec{p}\,\vec{p}_2 | \tilde{v} | \vec{p}_3 \vec{p}_4 \rangle|^2 \times \delta(\epsilon_p + \epsilon_{p_2} - \epsilon_{p_3} - \epsilon_{p_4} + \hbar\omega)$$

$$\cdot \rho_0(\vec{p},\vec{q})\,\rho_0(\vec{p}_2,\vec{q})\,(1-\rho_0(\vec{p}_3,\vec{q}))(1-\rho_0(\vec{p}_4,\vec{q}))\cdot$$

$$\times \left[\rho_+'(\vec{p},\vec{q}) + \rho_+'(\vec{p}_2,\vec{q}) - \rho_+'(\vec{p}_3,\vec{q}) - \rho_+'(\vec{p}_4,\vec{q})\right]. \qquad \text{(IV.C.3)}$$



## V. NUCLEAR MATTER APPLICATIONS

In this section we discuss applications of the three semiclassical collisional dynamics derived in the previous section.

### V.A. FRICTION AND DIFFUSION COEFFICIENTS FOR NUCLEAR MATTER FOR LOW MOMENTUM TRANSFERS

The inelastic scattering of high energy photons (~ 800 MeV) to forward angles is an example of a low momentum transfer process in nuclei. The incoming proton in this situation transfers momenta of the order of half the Fermi momentum of the target nucleons. The friction and diffusion coefficients associated with its path through the target nucleus can therefore be evaluated as follows. If we assume the finite temperature Thomas-Fermi density for target nucleons

$$\rho = \frac{1}{1 + \exp\left(\frac{\hbar(\vec{p}\,\vec{r}) - \epsilon_F}{kT}\right)} \qquad \text{(V.A.1)}$$

we can immediately evaluate eqs. (IV.A.3a) and (IV.A.3b) to get

for the friction and diffusion tensors respectively

$$\vec{A} = -\frac{4}{3}\pi^{3/2}\frac{C_p}{(2\pi\hbar)^3}\left(\frac{p_F}{p}\right)^3\left[1+\frac{3kTm}{p_F^2}\right], \quad p > p_F \qquad (V.A.2)$$

and

$$B_{ij} = -2\pi^{3/2} m kT C_p \left\{\frac{p_F}{p}\left(\hat{p}_i\hat{p}_j - \delta_{ij}\right) - \left(\frac{p_F}{p}\right)^3\left(\hat{p}_k\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\right\} \qquad (V.A.3)$$

where $\hat{p}_i$ indicates the angles of the vector $\vec{p}$, and $p_F$ the local Fermi momentum. It is interesting to note that the damping of the surface collective mode generated by the incoming nucleon cannot be described within a low momentum transfer approximation. The two body damping of this collective mode will involve arbitrary momentum transfers and one would need collision terms as the ones derived in the next two sections.

## V.B. THE NUCLEAR HYDRODYNAMICAL VISCOSITY COEFFICIENT

Many nuclear phenomena, whose interpretation is given in terms of dissipative processes (widths of giant resonances, fission dynamics) can be adjusted within the framework of fluidynamical descriptions with viscous terms like viscosity coefficient in the Navier-Stokes equation. This viscosity coefficient, in the particular case of the damping of nuclear vibrations has been adjusted to the data and is given by

$$\eta \sim 1.0 \times 10^{-23} \text{ MeV s fm}^{-3} \qquad (V.B.1)$$



To our knowledge there has been no theoretical effort to connect this number to a microscopic description of the nuclear fluid, although this connection can easily be made with the help of results for Fermi liquids available in the literature[11],[13]. As it is well known, for low temperatures, $\eta$ grows like $T^{-2}$ in Fermi systems[14]. An estimate, taking into account only the Landau parameter $F_0$ gives

$$\eta \sim 0.7 \times 10^{-22}\left(\frac{\bar{F}_0+1}{\bar{F}_0}\right)^2\left(\frac{\epsilon_F}{T}\right)^2 \qquad (V.B.2)$$

where $\epsilon_F$ is the Fermi energy and $T$ the temperature (in units of energy). It is a well known fact that in the case of the nucleus the set of Landau parameters is not unique. We take the point of view that the Landau parameter to be used should reproduce the ground-state energy and collective excitations of large nuclei. We take the value for $F_0$ in the interior of the nucleus given in ref. (15), which should represent the bulk nuclear matter value. In this case $F_0 = .2$. For reasonable values of the temperature the viscosity coefficient turns out to be too large by orders of magnitude. The value for $F_0$ which would give the correct order of magnitude is grossly overattractive, $-.94$. Knowing that the underlying theory is perfectly adequate for liquid $^3He$[13], our estimate for the nucleus stresses the inadequacy of the Navier-Stokes viscosity as a mechanism for the damping of nucleon vibrations.

## V.C. DAMPING OF ZERO SOUND IN NUCLEAR MATTER

At very low temperatures, the collision processes between (quasi-)fermions is strongly inhibited due to the exclusion principle and to the conservation laws. This is the fact ultimately responsible for the $T^{-2}$ divergence in the viscosity coefficient. In this regime, collective excitations ("zero sound") can be propagated through the modifications they induce in the mean field. For very slow modes $\hbar\omega << kT$ of this nature, the collisional damping decreases with temperature like $T^2$ [14]. However, when $\hbar\omega >> kT$ the energy associated with the mode allows for the possibility of dissipative mechanisms involving the promotion of (quasi-)particles above the Fermi level. This is the mechanism of the damping at zero temperature. Such dissipation is contained in the modified Boltzmann collision integral eq. (IV.C.3). The effect of the modification introduced by the phonon energy in the energy-conserving δ-function for the damping can be estimated through the correction derived by Landau[11] and others[16], which consists in multiplying the value obtained with the usual Boltzmann collision integral (proportional to $T^2$) by the correction factor

$$1 + \left(\frac{\hbar\omega}{2\pi kT}\right)^2 \qquad \text{(V.C.1)}$$

An application to finite nuclear systems is difficult due to the boundary conditions to be imposed on the dispersion relation and also due to the spatial dependence of the equilibrium density. In the simpler case of infinite nuclear matter, the



width of a mode with frequency $\omega$ and wave number $k$, which satisfy usual dispersion equation in the Landau limit

$$\frac{s}{2}\ln\frac{s+1}{s-1} = \frac{1}{F_0} + 1 \qquad \text{(V.C.2)}$$

with

$$s = \frac{\omega}{k v_F} \qquad \text{(V.C.3)}$$

where $v_F$ is the Fermi velocity, can be easily estimated as[13]

$$\Gamma \sim 0.02\,\frac{s(s^2-1)(F_0+1+3s^2)}{F_0(s^2-1) - F_0^2}\left(\frac{F_0}{F_0+1}\right)^2\left(\frac{\hbar\omega}{\epsilon_F}\right)^2 \qquad \text{(V.C.4)}$$

Substituting in this expression $\hbar\omega$ by the experimentally observed values and with $\epsilon_F \simeq 45$ MeV, we obtain for $F_0 = .2$ as in the previous subsection, widths which are about two orders of magnitude smaller than the observed ones. The calculated value for $\Gamma$ represents the bulk value for the damping. It suggests then, that the finiteness of nuclei should play an essential role in the damping of collective modes. Moreover, according to the results of the previous section, this should not be determined by parameters of transport coefficients of the nuclear fluid. Expressions for this damping involving other Landau parameters have been derived for liquid $^3He$[17]. However we do not expect qualitative changes in the result.

## CONCLUSIONS

In the present paper we have derived semiclassical kinetic equations, starting from a fully quantum microscopic dynamics for the nuclear one body density matrix. The basic ingredient which controlls the quality of the semiclassical approximation to the mean field (Hartree-Fock) dynamics is shown to be the spatial properties of the nuclear density. For the collision contribution the range of the two body potential may also play an important role as well as memory effects. It is therefore possible to obtain three different approximations by considering respectively low momentum transfer processes (which leads to a Fokker-Planck equation), arbitrary momentum transfers and no memory effects (which leads to the usual Boltzmann collision integral), and arbitrary momentum transfers and memory effects which turn out to be very important for low temperature collective excitations. Several applications have been discussed and an interesting open problem is the inclusion of the nuclear surface in a semiclassical calculation for the damping width of giant resonances. While this work was being written a paper came to our attention[18], which is a first attempt in this direction.



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