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ANALYTIC STOCHASTIC REGULARIZATION AND GAUGE INVARIANCE

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Abstract

We prove that analytic stochastic regularization breaks gauge invariance. This is done by an explicit one loop calculation of the vacuum polarization tensor in scalar electrodynamics, which turns out not to be transversal. We also analyse the counterterm structure, Langevin equations and the construction of composite operators in the general framework of stochastic quantization.

Introduction

Due to many technical aspects, non abelian gauge theories are very difficult to dealt with. Non perturbatively, there are the Gribov ambiguities which prevent a clear gauge specification.(1) On the other hand, at the practical side, computer simulations, using Monte Carlo methods have unvealed a lot about the structure of the models on a lattice.(2) In spite of this success, crucial problems still persist whenever fermions are included. Even at the perturbative level the situation is not much better since, in many instances, it is not easy to find a gauge preserving regularization scheme.(3) Case examples are supersymmetric gauge theories where most of the popular schemes fail.

With such pletora of troubles we found very fortunate Parisi and Wu proposal of stochastic quantization as a mean to circumvent some of the above problems.(4) Gauge specification in the first place, is not necessary or, better saying, is automatically incorporated thus evading the Gribov ambiguities. Moreover, concerning gauge theories on a lattice, the introduction of a fifth variable (the Langevin time) permits a unique updating of the whole lattice data in each step, saving a huge amount of computer time.

After Parisi and Wu, some authors have proposed new regularization approaches based on the Langevin equation with a non white noise.(5,6) It is one of our purposes to show that, contrary to these hopes, such procedures in general break gauge invariance (see also (7)). This is verified through an explicit calculation of the current Green functions, employing a specific shape for

the regulator noise. It turns out that the current is not conserved which implies in the breakdown of the gauge symmetry. The non conservation of the current is due to, finite, non invariant terms induced by the regulator or/and infinite (as the regulator is removed) counterterms which would have to be included in the Lagrangian for a consistent renormalization program.

A possible failure of current conservation in the framework of analytic stochastic regularization has already been noticed.(5) Nevertheless, it was not clear what were the consequences for the gauge symmetry since this symmetry is apparently preserved in the Langevin equation. In ref.(4,6) it was claimed that the divergent part of the current Green functions were transversal. In this paper we discuss the appearance of both infinite and finite terms and the possible implications for gauge theories. These results are discussed in section II. Besides that that, in section III we analyse standard renormalization methods on the light of stochastic quantization.

II. Stochastic Quantization and Gauge Invariance

The basic element in stochastic quantization is the Langevin equation

$$\frac{\partial}{\partial t}\varphi_i(x,t) = -\frac{\delta}{\delta\varphi_i(x,t)} S[\varphi] + \eta_i(x,t) \tag{II.1}$$

where t is a fifth time variable and x represents a four dimensional space - time coordinate. S is the classical action and η a random field with gaussian probability, defining a Marcovian process. The two point correlation function of the field is given by

$$\langle \eta_i(x,t)\,\eta_j(y,t')\rangle = 2\,\delta_{ij}\,\delta(x-y)\,\delta(t-t') \tag{II.2}$$

and higher point Green functions are obtained with the help of Wick's decomposition. Using (II.1) and (II.2), averages can be computed through

$$\langle F[\varphi_i(t)]\rangle_\eta = \int \mathcal{D}\varphi_i\, F[\varphi_i(t)] \exp\left\{-\frac{1}{2}\int_0^\infty dt \int dx\, \eta_i\eta_i\right\} \tag{II.3}$$

The above Marcovian process can be related to the field theory specified by the action S as follows. The Green functions of the quantum field are given by the stationary limit of the equal (fifth) time averages of the random field φ, namely

$$\langle T\phi_{i_1}(x_1)\dots\phi_{i_N}(x_N)\rangle = \lim_{t\to\infty}\langle \varphi_{i_1}(x_1,t)\dots\varphi_{i_N}(x_N,t)\rangle_\eta \tag{II.4}$$

For perturbative purpose, it is convenient to split the action in two terms, a gaussian, quadratic in the fields, and an interaction term

$$S[\varphi] = \frac{1}{2}\varphi_i \mathcal{D}_{ij}\varphi_j + V(\varphi) \tag{II.5}$$

so that the Langevin equations can be rewritten as

$$\frac{\partial\varphi_i}{\partial t} + \mathcal{D}_{ij}\varphi_j = -\frac{\partial V}{\partial\varphi_i} + \eta_i \tag{II.6}$$

where

$$D_{ij} = \delta_{ij}(-\partial^2 + m^2)\ ,\ \text{for a scalar field} \tag{II.7a}$$

and $$D_{\mu\nu} = -\partial^2\delta_{\mu\nu} + \partial_\mu\partial_\nu\ \text{for a gauge field} \tag{II.7b}$$

As noted elsewhere, due to the time derivative on the left hand side of (II.6), the propagator

$$G_{ij} = [\partial_t + \mathcal{D}]^{-1}_{ij} \tag{II.8}$$

exists even for a gauge theory. Indeed we have

$$\tilde{G}_{ij}(k,t) = \delta_{ij}\,\theta(t)\exp\{-t(k^2+m^2)\} \tag{II.9}$$

for a scalar propagator and

$$G_{\mu\nu}(k,t) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\exp(-tk^2) + \frac{k_\mu k_\nu}{k^2} \tag{II.10}$$

for the propagator of a gauge field (the tildes denote Fourier transformation).

In (II.10), the presence of a longitudinal part is to be noted. It has been remarked that such term does not contribute to the Green functions of gauge invariant objects.(4) However, as we shall see later, such term becomes particularly dangereous if a non invariant regularization scheme is employed.

Equation (II.6) can be solved iteratively, giving

$$\varphi_i(x,t) = G_{ij} * \left(-\frac{\partial V}{\partial\varphi_j}\left[G*\left(-\frac{\partial V}{\partial\varphi}+\eta\right)\right] + \eta_j\right) \tag{II.11}$$

where the asterisk is to remember that the products must be taken in the convolution sense. Using (II.11) and (II.2) the N point Green functions of the φ field can be computed. In a given order of perturbation the following Feynman rules obtain[8]

1. Draw all topologically distinct diagrams.

2. Use a cross, +, to represent the contraction of two η's Thus a line connecting a pair of vertices can be either a crossed or an uncrossed line. The crossed lines are distributed in the graph so that

2.1 Every loop has at least one crossed line.

2.2 Two external vertices can not be connected by a continuous path of lines without crosses.

2.3 Any crossed line can be connected with an external line by a path without crosses.

The number of crossed lines in a graph is

$$\#\text{ of } + = \#\text{ of loops} + \#\text{ of external lines} - 1 \tag{II.12}$$

Observe also that the vertices at the ends of an uncrossed line are naturally ordered according the values of their fifth times. On the other hand, if all lines linking a pair of vertices are crossed lines, then the amplitude for the graph decomposes into a sum corresponding to the two possible (fifth) time orderings of the vertices.

3. To the lines are associated the propagators

Uncrossed line ------> $G(x,t)$

$$\text{Crossed line} \dashrightarrow D(x-x';t,t') = \int_0^t d\tau \int d^d y\, G(x-y,t-\tau)\, G(x'-y,t-\tau) \tag{II.13}$$

A look at the amplitudes constructed with the above rules shows that they in general diverge. It is true that graphs with more internal crossed lines have a better ultraviolet behaviour but it is always possible to find graphs at least as divergent as those in the usual formulation of field theory. Following refs.(5,6) we introduce a non white noise

$$\langle \eta(x,t)\,\eta(x',t')\rangle = 2\delta(x-x')\, f_\epsilon(t-t') \tag{II.14}$$

where

$$\lim_{\epsilon \to 0} f_\epsilon(t) = \delta(t) \tag{II.15}$$

To be concrete, we choose a particular form for f_ϵ

$$f_\epsilon(t) = \epsilon |t|^{\epsilon - 1} \tag{II.16}$$

The meaning of the above procedure is the introduction of a non Marcovian element in the process described by (II.2).

The Green functions regulated by the use of (II.14 - 16) are mereomorphic functions of ϵ with poles on the real axis. As in the case of analytic regularization, we could adopt different ϵ's for each η contraction.[3] Although arbitrary, this has some advantages over the use of a unique ϵ. In the sequence, we will verify that not even the most divergent gauge dependent counterterm cancels if only one ϵ is used.

Using (II.14-16), the crossed propagator must be replaced by

$$D^{\epsilon}(x-x';t,t') = \int_0^{t} d\tau \int_0^{t'} d\tau' \int d^4y \; G(x-y;t-\tau)\; G(x'-y;t'-\tau')\; f_{\epsilon}(\tau-\tau') \tag{II.17}$$

which for a scalar field gives

$$\tilde{D}^{\epsilon}(p) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \; \frac{e^{-i\omega(t_1-t_2)}}{(p^2+m^2)^2+\omega^2} \; \tilde{f}_{\epsilon}(\omega) \tag{II.18}$$

In the special case defined by (II.16), we obtain

$$\tilde{f}_{\epsilon}(\omega) = \epsilon\,\Gamma(\epsilon)\,|\omega|^{-\epsilon}\sqrt{\frac{2}{\pi}}\,\sin\left(\frac{\pi}{2}(1-\epsilon)\right) \tag{II.19}$$

Let us now concentrate ourselves on the discussion of gauge invariance in scalar electrodynamics, which is described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \phi^* D^{\mu} D_{\mu}\phi \quad ; \; D_{\mu} = \partial_{\mu} + ieA_{\mu} \tag{II.20}$$

The Langevin equations governing the evolution of the fields A_{μ}, ϕ and ϕ^* are(9)(10)

$$\begin{aligned} \dot{A}_{\mu} &= -\frac{\delta S}{\delta A_{\mu}} + \eta_{\mu} = -\partial_{\rho}F^{\rho}{}_{\mu} - i\phi^*\overleftrightarrow{D}_{\mu}\phi + \eta_{\mu} \\ \dot{\phi} &= -\frac{\delta S}{\delta \phi^*} + \eta = -D^2\phi + \eta \\ \phi &= -\frac{\delta S}{\delta\phi} + \eta^* = -D^2\phi^* + \eta^* \end{aligned} \tag{II.21}$$

with the random fields η_{μ}, η^* and η satisfying

$$\begin{aligned} \langle \eta_{\mu}(x,t)\,,\,\eta_{\nu}(x',t')\rangle &= 2\,\delta_{\mu\nu} f_{\epsilon}(t-t')\,\delta(x-x') \\ \langle \eta(x,t)\,,\,\eta^*(x',t')\rangle &= 2\, f_{\epsilon}(t-t')\,\delta(x-x') \\ \langle \eta(x,t)\,,\,\eta(x',t')\rangle_c &= 0 \end{aligned} \tag{II.22}$$

We are particularly interested in analysing the contributions to the photon polarization tensor $\pi_{\mu\nu}$. In lowest order of perturbation we found the graphs shown in fig I. Note that there is one graph contributing to fig I.a, four graphs

contributing to fig I.b – they correspond to different graphs with the same topology and having two crossed lines, one external and the other internal – and two graphs for the fig. I.c. We begin our analysis by making a preliminary calculation in two dimensions. Also for simplicity, in computing correlations functions, we suppose that the fifth times of the fields are all equal and very high. We then integrate over the fifth time of the internal vertices and keep only the dominant terms (i.e., only those surviving in the infinite fifth time limit). Thus, for the graph I.a we get

$$\mathrm{Fig\ I}a = \frac{1}{p^2}\int \frac{d^2k}{(2\pi)^2} \; \frac{(2k+p)_{\mu}\,(2k+p)_{\nu}}{(k^2+m^2)^{1+\epsilon}\left[(k+p)^2+m^2\right]^{1+\epsilon}\left[(k+p)^2+k^2+p^2+2m^2\right]} \tag{II.23}$$

This expression is very difficult to be evaluated. Although in two dimensions we could still obtain a closed form for it, we find more instructive to employ a different procedure which has the advantage of being generalizable to four dimensions. The basic observation is that $\pi_{\mu\nu}$ is analytic in m for m big enough. Then $\pi_{\mu\nu}$ can be expanded in powers of m^{-1} (or, equivalently, in powers of the external momenta) and the transversality property of $\pi_{\mu\nu}$ will be correct only if it is satisfyed in each order of the expansion. In the forthcoming calculation we will analyse the terms of the above mentioned expansion, up to the first one to be finite when the regularization is removed. For the graph of fig I.a we have

$$\mathrm{Fig\ I}a = \frac{\delta_{\mu\nu}}{8\pi\, p^2 m^2} \tag{II.24}$$

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For the graph of fig (I.b) the calculation is also straightforward but a little bit more extense. We get

$$\text{Fig I.b} = \frac{1}{(p^2)^2}\int \frac{d^2k}{(2\pi)^2} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2+m^2)^{1+\epsilon}\left[(k+p)^2+k^2+p^2+2m^2\right]}$$

$$= \frac{\delta_{\mu\nu}}{4\pi(p^2)^2}\left(\frac{1}{\epsilon}-1\right) - \frac{\delta_{\mu\nu}}{24\pi\, p^2 m^2} + \frac{p_\mu p_\nu}{24\pi (p^2)^2 m^2} \tag{II.25}$$

Finally, the graph of fig (I.c) gives

$$\text{Fig I.c} = \frac{-\delta_{\mu\nu}}{(p^2)^2}\int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2+1)^{1+\epsilon}}$$

$$= -\delta_{\mu\nu} / 4\pi (p^2)^2 \epsilon \tag{II.26}$$

Adding these contributions we note that the divergent pieces exactly cancel, leaving the result

$$\frac{1}{24\, p^2 m^2}\left(\delta_{\mu\nu} + \frac{2 p_\mu p_\nu}{p^2}\right) - \frac{\delta_{\mu\nu}}{2\pi (p^2)^2} \tag{II.27}$$

which is, evidently, non transversal. We could, of course, add non gauge invariant counterterms to remove the unwanted contributions. One possible choice is to add the counterterm

$$\frac{-1}{4\pi} A_\mu A^\mu + \frac{1}{16\pi m^2} A^\mu \partial^2 A_\mu \tag{II.28}$$

which will change (II.27) to

$$\frac{1}{24\, p^2 m^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{II.29}$$

After this warm up exercise, we turn our attention to the four dimensional case. The contributing diagrams are the same but we will have to proceed further in the momentum expansion, because the ultraviolet divergences are more severe. To have complete generality, we allow diferent ϵ's for each crossed line propagator. Our results are (for simplicity, we have set m = 1).

$$\text{Fig I.a} = \frac{1}{p^2}\int \frac{d^4k}{(2\pi)^4} \frac{(2k+p)_\mu (2k+p)_\nu}{\left[p^2+(p+k)^2+k^2+2\right]\left[k^2+1\right]^{1+\epsilon_1}\left[(k+p)^2+1\right]^{1+\epsilon_2}}$$

$$= \frac{\delta_{\mu\nu}}{(4\pi)^2 p^2}\left\{\frac{1}{2\epsilon_{12}} - \frac{3}{4} - \frac{3}{16}p^2\right\} + \frac{p_\mu p_\nu}{24(4\pi)^2 p^2} \quad ; \epsilon_{12} = \epsilon_1 + \epsilon_2 \tag{II.30}$$

$$\text{Fig I.b} = \frac{1}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{\left[(p+k)^2+1\right]^{1+\epsilon}} + \frac{1}{\left[k^2+1\right]^{1+\epsilon}}\right]\frac{(2k+p)_\mu (2k+p)_\nu}{\left[k^2+(k+p)^2+p^2+2\right]}$$

$$= \frac{-2\delta_{\mu\nu}}{(4\pi)^2 (p^2)^2 \epsilon} - \frac{\delta_{\mu\nu}}{6(4\pi)^2 p^2}\left[\frac{5}{\epsilon} - \frac{43}{6}\right] + \frac{p_\mu p_\nu}{3(p^2)^2 (4\pi)^2}\left[\frac{1}{\epsilon} - \frac{1}{3}\right] \tag{II.31}$$

$$\text{Fig I.c} = \frac{-2\delta_{\mu\nu}}{(p^2)^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+1)^{1+\epsilon}}$$

$$= \frac{2\delta_{\mu\nu}}{(4\pi)^2 (p^2)^2}\left(\frac{1}{\epsilon} - 1\right) \tag{II.32}$$

For equal ϵ's note that the term $\delta_{\mu\nu}/(4\pi)^2 p^2 \epsilon$ (a mass counterterm) cancels between (II.31) and (II.32). The remaining divergent terms are

$$\left\{\frac{\delta_{\mu\nu}}{(4\pi)^2 p^2}\left(\frac{1}{2\epsilon_{12}} - \frac{5}{6\epsilon}\right) + \frac{p_\mu p_\nu}{3(4\pi)^2 (p^2)^2 \epsilon}\right\} \tag{II.33}$$

If all ϵ are made equal this expression is not transversal. However with unequal ϵ we have more room to achieve transversality. In particular for $\epsilon_{12} = \epsilon$ we obtain

$$\frac{-1}{3(4\pi)^2 p^2 \epsilon}\left\{\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right\} \tag{II.34}$$

However, not all problems are solved since the finite terms

$$\frac{1}{9(4\pi)^2 p^2}\left\{4\delta_{\mu\nu} + 19\frac{p_\mu p_\nu}{p^2}\right\} \tag{II.35}$$

are still not transversal.

In the usual (i.e., non stochastic) formulation of gauge theories we have to add a gauge fixing term to the Lagrangian density. In a Lorentz gauge, this term is given by $(\partial_\mu A^\mu)^2/2\alpha$, and in general the Green functions are α dependent. Observables (including the S matrix), on the other hand, must be α independent. If a non gauge invariant renormalization scheme is employed then α independence can be achieved only at the expense of adding gauge dependent counterterms to the original Lagrangian.

The putative α dependence of the Green functions comes from graphs pictorically represented on fig II. The contributions of this type of diagrams vanish on shell only if the gauge field is coupled to a conserved current.*

In the framework of stochastic quantization, the above problem can be particularly dangerous since the longitudinal part of the A_μ field propagator has a piece proportional to the fifth time. Therefore, in all cases, it is mandatory that the gauge field be coupled to a conserved current.

* For a rigorous discussion of gauge invariance in QED, se ref. 13.

III. Renormalization

In this section we shall discuss some peculiarities of the renormalization in the formalism of stochastic quantization. In particular, we will show that the counterterms necessary to render the field theory finite can also be interpreted as counterterms in the stochastic theory, i.e., at finite fifth time. In spite of that, there may be differences (finite renormalizations) since the degree of divergence depends not only on the topology but also on the number of internal crossed lines. For simplicity but without loss of generality we choose the ϕ^4 model and scalar electrodynamics to base our considerations.

First of all, we notice that for finite, non zero times there will be a strong convergence factor, provided by the exponential in the stochastic propagators. As an especific example, consider the case of the photon self energy in scalar electrodynamics

$$\int \frac{d^4k}{(2\pi)^4} \frac{\exp\{-\tau[(k+p)^2+k^2+2m^2]\}}{k^2+m^2} \frac{\exp\{-(t-\tau)p^2\}}{p^2} \qquad \text{(III.1)}$$

which is finite, insofar τ is non vanishing. If we now integrate over the ficticious time then a divergence shows up. Indeed, we have

$$\int_0^t e^{-\tau x}\, dx = \frac{1-e^{-tx}}{x} \qquad \text{(III.2)}$$

and the first term gives origin to a (logarithimically) divergent integral. To exhibit this divergence in a more natural way, it is convenient also to Fourier transform with respect to the fifth time. Our notational convention is that the Fourier transform, $f(w)$, of $\tilde{f}(t)$ is given by

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} \frac{dt}{(2\pi)} e^{i\omega t} f(t) \qquad (III.3)$$

and we observe that w has dimension two in mass units.

With the above definition, the Fourier transform of the propagators are

Uncrossed: $\mathcal{G}(p,\omega) = \frac{1}{p^2+m^2+i\omega}$ (III.4)

Crossed: $\mathcal{D}(p,\omega) = \frac{1}{p^2+m^2+i\omega} \; \frac{1}{m^2+p^2-i\omega}$ (III.5)

As an illustration, in the following we will be specific to the four dimensional ϕ^4 theory. Using (III.4-5) and noting also that each loop contibutes with 6 to the power counting, we get that the degree of superficial divergence of a graph is

$$\delta(\gamma) = 6m - 2n - 2X \qquad (III.6)$$

where

m = # of loops

n = # of internal lines

X = # of crossed lines (internal)

Using now the relations

$$X = m + N_{unc.} - 1 \qquad (III.6a)$$

$$m = n - V + 1 \qquad (III.6b)$$

and $4V = 2n + N$ (III.6c)

with N, $N_{unc.}$ and V denoting the numbers of external lines, uncrossed external lines and vertices, we obtain

$$\delta(\gamma) = 6 - N_c - 3N_{unc.} \qquad (III.7)$$

where $N_{C.}$ is the number of crossed lines. We see that graphs with $N_{unc.} > 2$ are superficially convergent. Let examine now each case of possible divergence

1. $N_{unc.} = 2$, $N_{C.} = 0$. These are graphs contributing to the two point function and having no external crossed lines (see fig. III). It is easily verified that such divergences can be absorved in a multiplicative renormalization of the random field . At infinite fifth time the propagator associated to the crossed line will tend to the free propagator of the limiting field theory. Therefore this normaliztion is not an independent one but part of the wave function renomalization of field theory.

2. $N_{unc.} = 1$, $N_{C.} = 3$. This is the ususal logarithmic divergence of graphs with four external lines (see example in fig. IV). It corresponds to the (usual) four point vertex renormalization.

3. $N_{unc.} = 1$, $N_{C.} = 1$. This is the usual quadratic divergence of self energy graphs (see fig. V). It can be eliminated through a mass and wave function renormalizations.

In the stationary limit of field theory, all the external fifth times are made equal. This corresponds to join all external lines of each contributing graph in a new vertex, V_∞, and then integrate over the w variables of the additional loops. One can verify that no new divergences arises in this process. Actually, a simple calculation, similar to the one above gives

$$\delta(\gamma) = 4 - N_c - 3N_{unc.} - 3N \qquad (III.8)$$

80N where W denotes the number of internal lines of meeting at V_∞. Since $W > 1$ then, necessarily, $\delta(\gamma) < 0$. These conclusions agree with those in ref. (11), where a similar power counting was done directly in the space of the stochastic time coordinate.

From the above discussion, we conclude that the renormalization problem is essentially the same as in the usual formulation of field theory. The divergent parts can be removed by reparametrizing the original model. This can also be implemented by adopting a convenient subtraction scheme. Without committing ourselves to any particular scheme we want to add some remarks on the properties of the resulting theory. The first issue concerns the derivation of field equations. If, for example, we consider the bilinear $N\phi(-\partial^2+m^2)\phi$ – where the symbol N indicates a normal product prescription – then a basic step in the derivation is amputation of the line associated to the operator $(-\partial^2+m^2)\phi$ This is a trivial task since in momentum space $(-\partial^2+m^2)$ is just the inverse of the propagator and one can always arrange things so that this also happens in the regulated theory. So, it is possible to define bilinear normal products formally obeying the classical Eueler Lagrange equations. However, this result does not imply the absence of anomalies in Ward identities, since the Green functions of the current operator have an independent definition and, in principle, its divergence is unrelated to the above mentioned normal products. Indeed, Green functions of the object $\phi^* \overleftrightarrow{\partial^\mu} \phi$, regulated with the use of eq.(II. 16) do not satisfy current conservation because the regularized propagator is not the

inverse of $(-\partial^2+m^2)$. This observation is in complete accordance with our results of section II.

Another remark concerns the behaviour of the Green functions under renormalization group transformations, at finite fifth time. Since, as we have seen before, the elimination of all divergences can be accomplished by the usual wave, mass and charge renormalizations (the η field renormalization is, as we saw before, part of the wave function renormalization), then one should expect that the Green functions $G(x_1, \ldots, x_N)$ of the stochastic field ϕ would obey the renormalization group equation

$$\left[\mu^2\frac{\partial}{\partial\mu^2}+\beta\frac{\partial}{\partial g}+N\gamma\right]G^{(N)}=0 \qquad \text{(III.9)}$$

with the same β and γ as in the limit field theory. Equation (III.9) can be proved as follows. We introduce differential vertex operantions (DVO), corresponding to the different field monom ials of the Lagrangian of the model. Thus, in ϕ^4 we consider (to be more careful, we could dimensionally regularize our amplitudes)

$$\Delta_1 = \frac{1}{2}\int d^4x\ \phi^2 \qquad \Delta_3 = \frac{1}{4!}\int d^4x\ \phi^4$$

$$\Delta_2 = \frac{1}{2}\int d^4x\ \partial_\mu\phi\,\partial^\mu\phi \qquad \text{(III.10)}$$

which are defined by the same Feynman rules specifying the Green functions. Thus, Δ_1 means the insertion of a mass vertex in the graphs contributing to the Green functions. It is also convenient to intr ooduce a DVO, Δ_4, which counts the number of crossed lines in a graph. It is given by

$$\Delta_4 = \int d^4x\ \eta^2 \qquad \text{(III.11)}$$

As will be clear shortly, is not indepencdent of those in (III.9). Using these DVO's the action becomes formally

$$S[\phi] = (1+b)\Delta_2 + (m^2-a)\Delta_1 + (c-g)\Delta_3 + d\,\Delta_4 \qquad \text{(III.12)}$$

where a, b, c and d are counterterms. We adopt intermediate renormalization so that the propagator has a pole at $p^2 = -m^2$ but its residue and the other renormalization conditions are dependent on the value of a new mass parameter μ^2, a different renormalization spot.

The derivatives $\frac{\partial}{\partial g}$ and $\frac{\partial}{\partial \mu^2}$ have simple expressions in terms of the DVO's (III.10):

$$\frac{\partial}{\partial g} G^{(N)} = \left[-\frac{\partial a}{\partial g}\Delta_1 + \frac{\partial b}{\partial g}\Delta_2 + \frac{\partial (c-g)}{\partial g}\Delta_3 + \frac{\partial d}{\partial g}\Delta_4 \right] G^{(N)} \qquad \text{(III.13a)}$$

$$\frac{\partial}{\partial \mu^2} G^{(N)} = \left[-\frac{\partial a}{\partial \mu^2}\Delta_1 + \frac{\partial b}{\partial \mu^2}\Delta_2 + \frac{\partial c}{\partial \mu^2}\Delta_3 + \frac{\partial d}{\partial \mu^2}\Delta_4 \right] G^{(N)} \qquad \text{(III.13b)}$$

There is also a counting identity which is nothing but the integrated equation of motion

$$-N\,G^{(N)} = \left[(1+b)\Delta_2 + (m^2-a)\Delta_1 + (c-g)\Delta_3 + d\,\Delta_4 \right] G^{(N)} \qquad \text{(III.14)}$$

Besides the above identities, typical of the usual formulation of field theory, we have another equation which is a consequence of Δ_4 being an operation counting the number of crossed lines. Explicitly, from (III. 6a-c),

$$X = \frac{n}{2} - \frac{N}{4} + N_{unc.} \qquad \text{(III.15)}$$

Using now (III.13-15), we can easily establish the renormalization group equation (III.9). We replace (III.13-15) into (III.9) and eqaute to zero the coefficient of each DVO Δ_i, i=1,2,3. Two of these equations (namely those associated to the coefficients of Δ_1 and Δ_2) can used to fix β and γ. The

remaining one is then shown to be identically satisfyed by virtue of our on shell mass renormalization.

IV. Conclusions

In this paper we have discussed the use of analytic stochastic regularizxation in field theory. For gauge theories we shown that a difficulty arises since the gauge field is not coupled to a consereved current. As a consequence, if just one analytic regulator is used then transversality of the polarization tensor is broken by an infinite term. This conclusion is in accordance with a recent work showing the incompatibility of antalytic sotchastic regularization with Zwanziger gauge fixing.[7] In theories with just global invariances, the procedure is well defined presumably leading to sensible results.[11]

To define composite operators care must be taken since there are at least two "natural" but inequivalent ways of introducing normal products. The first possibility, which we have adopted in the derivation of the renormalization group, is supposed to be treated in the same way as an ordinary Lagrangian vertex. Another possibility, inequivalent to this one, is obtained by taking the product of fields at different points and then letting the points coincide. The difference between these two objects can be traced back to the flow of the fifth time. In the first case the fifth time flows into the special vertex (i.e., its time variable is higher than those of any other vertex nearby) whereas in the latter case the flow goes continuously through it.

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Figure Captions

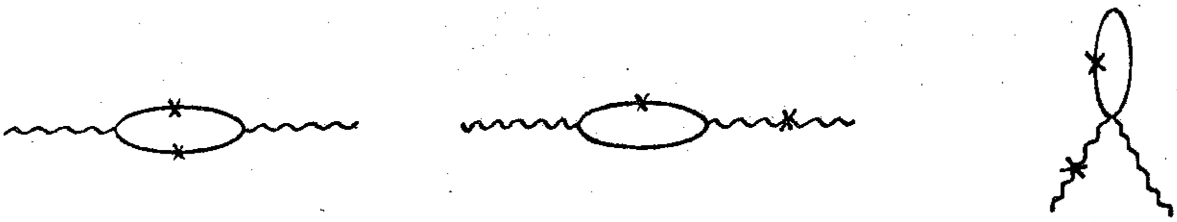
Fig. I One loop graphs contributing to the photon self energy in scalar electrodynamics.

Fig. II The non vanishing of this graph makes the Green functions gauge dependent. The special vertex V corresponds to the insertion of the operator

Fig.III Logarithimically divergent contribution to the renormalization of the random field

Fig. IV This graph contains part of the usual mass and wave function renormalization.

Fig. V Lowest order graph contribution to the charge renormalization in the framework of stochastic quantization.



(a) (b) (c)

Fig. I

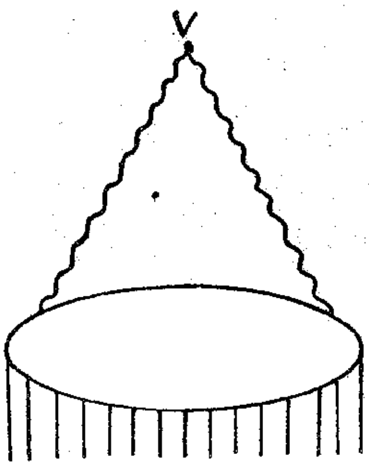


Fig. II

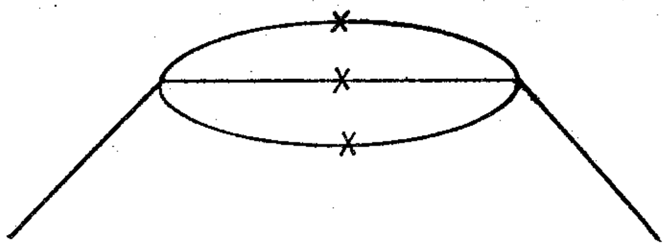


Fig. III

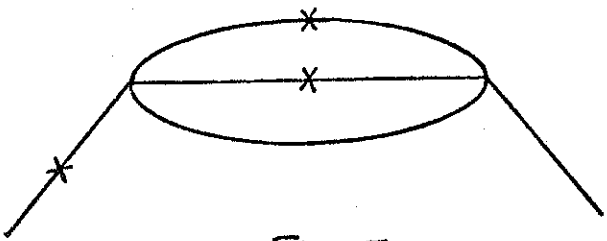


Fig. IV

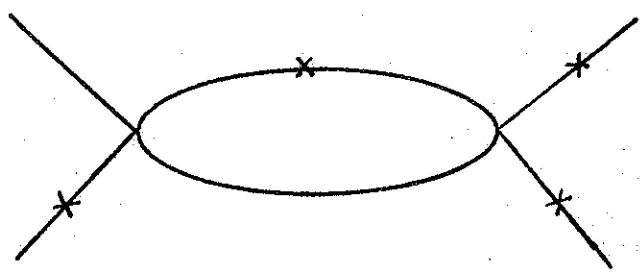


Fig. V