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THE SPINORIAL CHARACTER OF THE GENTILIONIC BARYON STATES

by

M. Cattani and N.C. Fernandes
Instituto de Física, Universidade de São Paulo

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Instituto de Física da Universidade de São Paulo,
C.P. 20516, 01498 São Paulo, SP, Brasil

"La matematica è l'alfabeto con il
quale Dio ha scritto l'Universo".

Galileo Galilei

SUMMARY

It is shown that the wavefunction, in an internal colour space of three quarks taken as gentileons, has a bi-spinorial character. Thus we verify that, in this context, our formalism differs drastically from parastatistics and fermionic theories of quarks. As a direct consequence of the spinorial character we see that the $SU(3)_{colour}$ representation can naturally be incorporated into the $S^{(3)}$ gentilionic symmetry and a selection rule for quark confinement is deduced. Comparing our results with Dirac's bi-spinorial formulation and with Prentki-d'Espagnat theory, striking resemblances are found.

1. INTRODUCTION

In a preceding paper(1) we have shown, assuming the quarks as gentileons, that the baryon wavefunctions are given by $\psi = \varphi . Y(\text{colour})$. The one-dimensional wavefunction $\varphi = (SU(6) \times O_3)_{\text{symmetric}}$ corresponds, according to the symmetric quark model of baryons, to a totally symmetric state, and the four-dimensional state $Y(\text{colour})$, corresponds to the intermediate representation of the symmetric group $S^{(3)}$. In order to preserve the $S^{(3)}$ symmetry, this wavefunction $Y(\text{colour})$, ought to depend on three new quantum states. They have been named red, blue and yellow only to be in agreement with the original idea of primary colours; they have not been assumed to be the well known $SU(3)_c$ eigenstates, where the subscript c means colour. We think that, to state general results and to avoid possible misunderstandings, it would be better if we have indicated, generically, these primary colours by α, β and γ, as we do in what follows. Indeed, in section 3, we will see that the $SU(3)_c$ eigenstates are a particular and natural representation for α, β and γ. In terms of α, β and γ, $Y(\text{colour})$ is given by[2]:

$$Y(\text{colour}) = Y(\alpha\beta\gamma) = Y(123) = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_1(123) \\ Y_2(123) \\ Y_3(123) \\ Y_4(123) \end{pmatrix} = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \tag{1.1}$$

where,

$$Y_1(123) = (|\alpha\beta\gamma\rangle + |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle - |\gamma\beta\alpha\rangle)/\sqrt{4} ,$$

$$Y_2(123) = (|\alpha\beta\gamma\rangle + 2|\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle + |\gamma\alpha\beta\rangle - 2|\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{12} ,$$

$$Y_3(123) = (-|\alpha\beta\gamma\rangle + 2|\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + 2|\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{12}$$

and $Y_4(123) = (|\alpha\beta\gamma\rangle - |\beta\alpha\gamma\rangle - |\gamma\alpha\beta\rangle + |\gamma\beta\alpha\rangle)/\sqrt{4}$. The colour state Y is decomposed into two parts, $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$, where $Y_+ = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ and $Y_- = \begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}$, corresponding to the duplication of states implied by the reducibility of our representation in the intermediate gentilionic states[1,2]. In section 2, we shall show that Y_+ and Y_- have a spinorial character, resulting a "bi-spinorial" character in Dirac's sense for $Y(\text{colour})$, according to section 4. The probability density function[2] for $Y(\text{colour})$ is given by the permutation invariant function $Y^+Y = |Y|^2 = (|Y_1|^2 + |Y_2|^2 + |Y_3|^2 + |Y_4|^2)/4$. The bi-spinorial character of $Y(123)$ is responsible for selection rules[1,2] predicting: (1) baryon number conservation, (2) gentileon confinement and (3) saturation. It is worthwhile to note that, in this context, our theory differs drastically from parastatistics[3-7] and fermionic theories of quarks. In the fermionic case, Y would be given by, $Y = (|\alpha\beta\gamma\rangle - |\alpha\gamma\beta\rangle - |\beta\alpha\gamma\rangle + |\gamma\alpha\beta\rangle + |\beta\gamma\alpha\rangle - |\gamma\beta\alpha\rangle)/\sqrt{6}$ and in parastatistics case Y would be written as, $Y = aY_1 + bY_2 + cY_3 + dY_4$, where a, b, c and d are arbitrary constants. For these last theories the wavefunction Y is one-dimensional, from which the selection rules (1), (2) and (3), above mentioned, cannot be deduced.

Our intention, in this paper, is to show explicitely the spinorial character of Y(colour) and to establish fundamental properties of the gentilionic system that can be deduced from this spinorial feature. Thus, in section 2, to show that Y_+ and Y_- are spinors we study the rotations of an equilateral triangle in an internal tri-dimensional Euclidean space E_3, that is called "colour space". In this colour space, three privileged colours α, β and γ occupy the vertices of the triangle. These are the primary colours α, β and γ.

In section 3, we obtain a very simple geometrical interpretation, in the colour space E_3, for the invariant $K^{[2,1]}_{(2,1)}$ associated with the gentilionic states[1]. From this analysis we conclude that the total colour quantity corresponding to the three states α, β and γ is given by a vectorial sum in the triangle plane and that it is a constant of motion, which is null. We also verify that the colour states α, β and γ can be naturally identified with the $SU(3)_c$ eigenstates blue, red and yellow. Thus, considering these primary colours of $SU(3)_c$ we see that, in the plane of the triangle, $K^{[2,1]}_{(2,1)}$ is represented by the operator $\tilde{M} = \tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3$, where $\tilde{q}_i = (\tilde{I}_3 + \tilde{Y}/2)_i$, $\tilde{I}_3$ is the colour isospin, $\tilde{Y}$ the colour hypercharge and the indices $i = 1, 2$ and 3 refer to the three quarks of the baryon. Considering the $SU(3)_c$ representation, we show that, only colour singlet states, for baryons and mesons,

can appear in the gentilionic approach. This last result is extremely important since it corresponds to a selection rule for quark confinement.

In section 4 we explore some connections of the gentilionic approach with some well established theories. A comparison of the gentilionic colourspinor with Dirac's bi-spinor formulation is given and an analogy lead us to substantiate our definition of particles and anti-particles. Also a revision of the main ideas of Prentki-d'Espagnat theory is presented and a sound interpretation of the classification scheme of particles is given in terms of the colour invariant $K^{[2,1]}_{(2,1)}$.

2. THE SPINORIAL CHARACTER OF Y(COLOUR)

In this section we present a detailed study of the symmetry properties of the state vector $Y(\text{colour}) = Y(123)$. We have emphasized the non-trivial case $N=3$ aiming to apply the theory to the description of $SU(3)$ models of strong interactions. Of course, it is possible to extend our results, concerning the structure of Y, for $N > 3$, at the expenses of unnecessary labour and non essential complications for our purposes. Since we intend to formulate our results in the quantum mechanical framework, let us recall that the symmetric group $S^{(3)}$, consisting of six permutation operators, necessarily imposes a unitary space with dimension 6 for its representation. Besides the two one-dimensional spaces, bosonic and fermionic, we have a four-rowed representation for the intermediate state expressed in the form,

$$\begin{pmatrix} Y'_+ \\ Y'_- \end{pmatrix} = \begin{pmatrix} \eta_j & 0 \\ 0 & \eta_j \end{pmatrix} \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \qquad (2.1)$$

where η_j $(j = 1,2,\ldots,6)$ are 2×2 matrices given by:

$$\eta_1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = I \quad ; \quad \eta_2 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \quad ;$$

$$\eta_3 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} \quad ; \quad \eta_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \qquad (2.2)$$

$$\eta_5 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \quad \text{and} \quad \eta_6 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} .$$

From the point of view of group representation theory, Eq. (2.1) immediately suggests the reducibility of the intermediate representation. Although these matters are discussed in detail in section 4, it should be observed now that, due to the separation of Y into two objects with two components, Y_+ and Y_-, an interpretation of these objects is claimed.

Let us show that it is possible to interpret the transformations of Y_+ and Y_- in terms of rotations of an equilateral triangle in a particular Euclidean space E_3. That is, we assume E_3 as a space where the colour states are defined by three orthogonal coordinates (X,Y,Z). Due to this assumption, this space will be named "colour space". It is also assumed that, in this colour space, the colours α, β and γ occupy the vertices of an equilateral triangle taken in the (X,Z) plane, as seen in Fig. 1. The unit vectors along the X, Y and Z axes are indicated, as usually, by $\vec{i}$, $\vec{j}$ and $\vec{k}$. In Fig. 1, the unit vectors $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ are given by, $\vec{m}_4 = -\vec{k}$, $\vec{m}_5 = -(\sqrt{3}/2)\,\vec{i} + (1/2)\,\vec{k}$ and $\vec{m}_6 = (\sqrt{3}/2)\,\vec{i} + (1/2)\,\vec{k}$, respectively.

(INSERT FIGURE 1)

We represent by $Y(123)$ the state whose particles 1, 2 and 3 occupy the vertices α, β and γ, respectively. Thus, we see that the true permutations, (312) and (231), are obtained from (123) under rotations by angles $\vartheta = \pm 2\pi/3$ around the unit vector $\vec{j}$. As one can easily verify, the matrices η_2 and η_3, that correspond to these permutations are represented by:

$$\eta_2 = -I/2 + i(\sqrt{3}/2)\,\sigma_y = \exp[i\,\vec{j}.\vec{\sigma}(\vartheta/2)] \quad \text{and}$$

$$\eta_3 = -I/2 - i(\sqrt{3}/2)\sigma_y = \exp[i\,\vec{j}.\vec{\sigma}(\vartheta/2)] \qquad (2.3)$$

where the σ_x, σ_y and σ_z are Pauli matrices.

Similarly, the transpositions (213), (132) and (321) are obtained under rotations by angles $\Phi = \pm\pi$ around the axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. The corresponding matrices are given by:

$$\begin{aligned} \eta_4 &= \sigma_z = i\exp[i\vec{m}_4 \cdot \vec{\sigma}(\Phi/2)] , \\ \eta_5 &= (\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_5 \cdot \vec{\sigma}(\Phi/2)] \quad \text{and} \\ \eta_6 &= (\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_6 \cdot \vec{\sigma}(\Phi/2)] . \end{aligned} \tag{2.4}$$

According to our preceding paper(1), there is an algebraic invariant, $K^{[2,1]}_{(2,1)}$, with a zero eigenvalue, associated with the $S^{(3)}$ gentilionic states. In analogy with continuous group, this invariant will be named "colour Casimir"(1). For permutations, that are represented by matrices with $\det = +1$, the invariant is given by $K_{rot} = \eta_1 + \eta_2 + \eta_3$. For transpositions, which matrices have $\det = -1$, it is defined by $K_{inv} = \eta_4 + \eta_5 + \eta_6$. Taking into account $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ and Eqs. (2.4) we see that, $K_{inv} = \eta_4 + \eta_5 + \eta_6 = (\vec{m}_4 + \vec{m}_5 + \vec{m}_6) \cdot \vec{\sigma} = 0$. This means that the invariant K_{inv} can be represented geometrically, in the plane (X,Z) of the colour space, by $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$, and that the equilateral triangle symmetry of the $S^{(3)}$ representation is an intrinsic property of $K_{inv} = 0$.

Eqs. (2.3) and (2.4) suggest a spinorial interpretation

for Y_+ and Y_-. Here, starting from a general standpoint, we show the correctness of this contention. It is well known that the non-relativistic spinor can be introduced in several ways(8). The interrelation of the various approaches is not obvious and can lead to misconceptions. In order to overcome the necessity of enumerating several approaches, let us stick on a geometrical image, recalling the very fundamental result on group isomorphism(9): $S^{(3)} \sim PSL_2(F_2)$, where $PSL_2(F_2)$ is the projective group associated with the special group SL_2 defined over a field F_2 with only two elements. Obviously, $PSL_2(F_2) \sim SL_2(F_2)/SL_2(F_2) \cap Z_2$, where the group in the denominator is the centre of SL_2 and corresponds to the central homotheties, since Z_2 is the intersection of the collineation group with SL_2.

If we consider the matrices (2.2) as representing transformations in a two-dimensional complex space characterized by homogeneous coordinates Y_1 and Y_2,

$$\begin{pmatrix} Y_1' \\ Y_2' \end{pmatrix} = \frac{1}{\rho}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \tag{2.5}$$

where ρ is an arbitrary complex constant and the latin letters substitute the coefficients taken from (2.2), it is clear that (2.2) constitute a homographic (or projective) group.

Making use of definition (2.5) we can see from

(2.2) that, apart from the identity n_1, the two matrices n_2 and n_3, which have $\det = +1$, are elliptic homographies with fixed points $\pm i$. If we translate these values for the variables of E_3, we see that n_2 and n_3 correspond to finite rotations around the $\vec{j}$ axis by an angle $\vartheta = \pm 2\pi/3$, agreeing thus with Eqs. (2.3). The remaining matrices n_4, n_5 and n_6 are elliptic involutions, with $\det = -1$. They correspond to space inversions in E_3, considered as rotations of $\pm\pi$ around the three axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. These matrices completely define the axis of inversion and the angle $\pm\pi$, as is seen from Eqs. (2.4). It is an elementary task to establish the correspondence, via stereographic projection, between the transformations in the two spaces, $Y_+(Y_-)$ and E_3.

A topological image can help us to see the 4π invariance of Y_+ and Y_-. If we consider the rotation angle $\vartheta(\Phi)$ as the variable describing an Euclidean disc, the covering space associated to this disc is a Moebius strip(10). Adjusting correctly the position of the triangles we can have a vivid picture of the rotation properties for each axis. This construction allow us to visualize the double covering of the transformation in E_3 and is a convincing demonstration of the spinorial link between E_3 and $Y_\pm$.

From this analysis we conclude that Y_+ and Y_- are spinors. As will be seen in section 4, the four-dimensional state function $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$ is a "bi-spinor" in Dirac's sense.

Since E_3 is a "colourspace", Y_+ and Y_-, in analogy with the isospinor in the isospace, will be named "colourspinors" (see more details in section 4).

As will be seen in next section, the vectorial representation, $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$, of the invariant K_{inv} will be extremely useful in connection with the $SU(3)_c$ model.

We observe that the same transformation properties of Y_+ and Y_- can be obtained if, instead of the equilateral triangle shown in Fig. 1, we consider the triangle drawn in Fig. 2.

(INSERT FIGURE 2)

In the vertices of the equilateral triangle of the Fig. 2 we have the colours $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. The unit vectors $\vec{m}_4^*$, $\vec{m}_5^*$ and $\vec{m}_6^*$ are given by, $\vec{m}_4^* = -\vec{m}_4$, $\vec{m}_5^* = -\vec{m}_5$ and $\vec{m}_6^* = -\vec{m}_6$. This means that, in this case, K_{inv} is represented geometrically by $\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0$. This two fold possibilities for depicting the triangle will be physically interpreted, in the next sections, in terms of the existence of colours and anti-colours.

3. THE $S^{(3)}$ SYMMETRY, THE $SU(3)_c$ AND QUARK CONFINEMENT

Useful physical interpretations relevant to properties of hadrons may be obtained from pursuing further the intimate relationship between certain geometric properties and the conservation laws of particles. In section 2, we have shown that it was possible to interpret the Y(colour) transformations in terms of rotations, in a colour space E_3, of only two equilateral triangles with vertices occupied by three privileged colours $\alpha(\bar{\alpha})$, $\beta(\bar{\beta})$ and $\gamma(\bar{\gamma})$. The Y must constitute symmetry adapted kets for $S^{(3)}$. In other words, their disposition in the plane of the triangle must agree with the imposition made by the colour Casimir. According to Fig. 1, these colours are defined by, $\alpha = \vec{m}_5 = (-\sqrt{3}/2, 1/2)$, $\beta = \vec{m}_6 = (\sqrt{3}/2, 1/2)$ and $\gamma = \vec{m}_4 = (0,-1)$, and according to Fig. 2, $\bar{\alpha} = \vec{m}_5^* = -\vec{m}_5$, $\bar{\beta} = \vec{m}_6^* = -\vec{m}_6$ and $\bar{\gamma} = \vec{m}_4^* = -\vec{m}_4$. The equilateral triangle symmetry of $S^{(3)}$ plays a fundamental role in E_3, allowing us to obtain a very simple and beautiful geometrical interpretation for the invariant $K_{inv} = 0$. Indeed, since the $S^{(3)}$ symmetry, according to section 2, implies that $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$ ($\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0$), we conclude that the total colour quantity of the baryon, pictured in E_3, is a constant of motion, which is null.

At this point it is instructive to compare our results with Gell-Mann's model for strong interactions. In his approach[11], the colour states (red, blue and yellow) are eigenstates of the colour hyperchange ($\tilde{Y}$) and of the colour isospin ($\tilde{I}_3$), both diagonal generators of the algebra of $SU(3)_c$. These eigenstates are given by: $|b\rangle = |-1/2, 1/3\rangle$, $|r\rangle = |1/2, 1/3\rangle$

and $|y\rangle = |0, -2/3\rangle$. Observing the vectorial character of the charge operator in the plane (isospin, hypercharge), the quark charge operator in the $SU(3)_{flavour} \otimes SU(3)_c$ representation is written as:

$$q = q_f + \tilde{q}_c = (I_3 + Y/2) + (a\,\tilde{I}_3 + b\,\tilde{Y}/2) \qquad (3.1)$$

where $q_f = I_3 + Y/2$ refers to flavour charge, $\tilde{q}_c = a\,\tilde{I}_3 + b\,\tilde{Y}/2$ refers to colour charge and a and b are arbitrary constants. Since it is not possible to determine, in the framework of that theory, the values of a and b, "ad hoc" values have been adopted for them. Gell-Mann used $a = b = 0$ and Han-Nambu used $a = b = -1$.

Taking into account that the SU(3) and $S^{(3)}$ fundamental symmetries are defined by equilateral triangles[11,12], it is quite apparent that the colour states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ would also be represented by eigenstates of $\tilde{I}_3$ and $\tilde{Y}$. Indeed, assuming that the axes X and Z (see Fig. 1) correspond to the axes $\tilde{I}_3$ and $\tilde{Y}$, respectively, and adopting the units along these axes as the side and the height of the triangle[12], we verify that $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ would be given by, $|\alpha\rangle = |b\rangle = |-1/2, 1/3\rangle$, $|\beta\rangle = |r\rangle = |1/2, 1/3\rangle$ and $|\gamma\rangle = |y\rangle = |0, -2/3\rangle$. If we have considered the states $|\bar{\alpha}\rangle$, $|\bar{\beta}\rangle$ and $|\bar{\gamma}\rangle$, seen in Fig. 2, we should verify that these states would correspond to the anti-colours $|\bar{r}\rangle$, $|\bar{b}\rangle$ and $|\bar{y}\rangle$ of the $\bar{3}$

colour representation.

Thus, if the colour states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ correspond to $|b\rangle$, $|r\rangle$ and $|y\rangle$, respectively, each unit vector $\vec{m}_j$ ($j = 4, 5$ and 6) is represented, in the plane $(\tilde{I}_3, \tilde{Y})$ by the operator $\tilde{q} = \tilde{I}_3 + \tilde{Y}/2$. This means that the invariant $\vec{M} = 0$ can be represented by the operator $\tilde{M} = \tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3$, where the indices 1, 2 and 3 refer to the three gentileons of the baryon. As can be easily verified, $\tilde{M}$ has zero eigenvalue for the states $Y(rby)$. According to the gentilionic theory[1,2], since two particles can occupy the same state, there could exist baryons described by $Y(nnm)$, where $n,m = r, b$ and y. However, for the states $Y(nnm)$ the expected value of $\tilde{M}$ differs from zero, that is, $\langle\tilde{M}\rangle \neq 0$. This last result contradicts the fact[1] that states with three different colours or states with two equal colours and one distinct are both associated with an invariant, which is null. Adopting the $SU(3)_c$ scheme, this implies that only colour singlet states $Y(rby)$ are consistently described in the gentilionic framework, and that the states $Y(nnm)$ must be excluded. With the $SU(3)_c$ choice, the baryon wavefunctions, in the gentilionic approach, will be given by $\psi = \varphi \cdot Y(rby)$, where the function φ is defined in section 1. In these conditions, in our formalism, one possibility is to define the individual quark charge operator as:

$$q = q_f + \lambda\tilde{q} = (I_3 + Y/2) + \lambda(\tilde{I}_3 + \tilde{Y}/2) \quad , \tag{3.2}$$

where λ is an arbitrary constant. Since with this definition, the total colour charge of the baryon is given by $\lambda\langle\tilde{M}\rangle$, the generalized Gell-Mann-Nishijima relation is automatically satisfied[1], independently of the λ value, because $\langle\tilde{M}\rangle = 0$ for the states $Y(rby)$. Putting $\lambda = -1$, we obtain integer quarks charges, according to Han-Nambu, and if $\lambda = 0$ we have the fractional charges adopted by Gell-Mann[11].

Another significant matter to be treated here are the mesons that, according to the gentilionic theory[1,13], are composed by a quark-antiquark pair. In view of the results of the present section, where we have seen that the 3 and $\bar{3}$ configurations of the $SU(3)_c$ can be naturally incorporated into the $S^{(3)}$ gentilionic symmetry, we conclude that mesons are also represented by colour singlet states.

A very important physical conclusion can be extracted from the fact that both baryons and mesons, in the gentilionic theory, are colour singlets: it implies that quarks must be permanently confined if they obey $S^{(3)}$ gentilionic symmetry. In contrast to previous works[1,2,13], where we have used only arguments involving dimensionality and symmetry properties of $Y(123)$ to justify the gentileon confinement, here, a selection rule for the confinement is obtained solely based on the exclusive existence of colour singlets.

It has been argued[14] that the confinement of quarks, despite some unusual properties, has a purely dynamical origin. Although being a reasonable hypothesis, it seems somewhat restrictive. Taking into account our previous papers[1,2,13] and the present work, we can infer that some kind of confinement mechanism must exist for gentileons. We do not know, at the moment, the exact mechanism. It could be produced by a very peculiar confining interaction potential between quarks, by an impermeable bag as proposed in the bag model, or something else. But any acceptable mechanism must be conceived under the imposition of agreeing exactly with the $S^{(3)}$ symmetry.

4. ANALOGIES AND COMMENTS

Generally speaking, the study of discrete symmetries is more difficult than the continuous symmetries. Nevertheless, by analogy and by extension of some conclusions, some very useful physical and mathematical insights can be gained.

One of the most interesting things about the properties of the gentilionic spinorial state Y(colour) is its resemblance with Dirac's spinor (or "bi-spinor"). According to Dirac's theory[15], the four-rowed quantity Φ, which is a reducible combination of an undotted and a dotted elementary spinor, is given by:

$$\Phi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \\ \chi_{\dot 1} \\ \chi_{\dot 2} \end{pmatrix} = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} , \quad \text{where} \quad \varphi = \begin{pmatrix} \varphi^1 \\ \varphi^2 \end{pmatrix}$$ is a contravariant undotted spinor and $\chi = \begin{pmatrix} \chi_{\dot 1} \\ \chi_{\dot 2} \end{pmatrix}$ is a covariant dotted spinor. From the geometrical point of view, φ can be interpreted as a spinor related to the north pole in a stereographic projection, whereas χ can be seen as a spinor related to the projection from the south pole[8]. The bi-spinor Φ transforms under the homogeneous Lorentz group as,

$$\Phi' = \begin{pmatrix} A & 0 \\ 0 & (A^+)^{-1} \end{pmatrix} \Phi ,$$ where $\underline{A}$ is a 2×2 matrix and $(A^+)^{-1}$ stands for its Hermitean conjugate. If $A = (A^+)^{-1}$, we have a duplication of states which corresponds to transformations in an orthogonal space R_3. Only in this case the above matrix commutes with the space inversion matrix and we get a reducible representation $\mathcal{D}^{\frac{1}{2}} \dotplus \mathcal{D}^{\frac{1}{2}}$. This duplication of states, as is well known[15], is connected with the double sign of the energy in elementary theory, and is connected with the occurrence of both negatively and positively charged particles with the same mass and spin in quantum field theory.

In the gentilionic approach, the duplication of states, Y_+ and Y_- , leads to the appearance of colours

(Fig. 1) and anticolours (Fig. 2), as was shown in section 2 and 3. Thus, within this framework, our two degenerate irreducible subspaces, Y_+ and Y_-, acquire a very important meaning: they admit the representation of both particles and anti-particles. This interpretation has very far reaching consequences: it justifies the possibility of using the 3 and $\bar{3}$ inequivalent representations of $SU(3)$ to study hadrons and anti-hadrons in the gentilionic approach.

Another purpose of this work is to suggest an analogy with the "geometrical" Prentki-d'Espagnat theory of isospace[15,16]. The main idea for classification of particles in this theory is to demand for strong interactions "invariance under both rotations and inversions in isospace". This amounts to saying that for these interactions there are two invariants: one associated with rotations, I_3, with $\det = +1$, and one associated with inversions, Y, with $\det = -1$. Using these two invariants, they have established a link between the conserved electric charge Q and isospace, showing that Q may be written as, $Q = I_3 + Y/2$. In order to classify the particles, they have constructed isoscalars, isopseudoscalars, etc.. The Λ, for instance, that is a neutral particle, is represented by a Dirac spinor-isoscalar: the eigenvalues of both I_3 and Y can only be zero.

Let us consider now our colourspace with Dirac's colourspinor $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$. Analogously to Prentki-d'Espagnat,

we have assumed that "in this space, the strong interactions are invariant under both rotations and inversions". According to section 2, this corresponds to the invariance under permutations and transpositions of the identical gentileons. In this colourspace we have only the "colourscalar" $|Y|^2 = Y^+Y = |Y_+|^2 + |Y_-|^2$. Thus, identifying our two invariants, K_{rot} and K_{inv}, defined in section 2, that are associated with the rotations and inversions, respectively, with I_3 and $Y/2$, we see that the baryon colour charge $\tilde{Q}$ would be given by, $\tilde{Q} = K_{rot} + K_{inv}$. Since both K_{rot} and K_{inv} have zero eigenvalue, we get the net colour charge of the baryon equal to zero, with no further assumptions.

We must note that, in our previous paper[1], the total colour charge operator $\tilde{Q}$ was assumed to having a eigenvalue $t/3$. It was shown to be zero. This agrees with our present results. However, we have decomposed $\underline{t}$ in terms of the colours red, blue and yellow: $t = t_r + t_b + t_y = 0$, for states with three different colours and $t = 2t_n + t_m = 0$, where n,m = red, blue and yellow, for states with two equal colours and one distinct. This decomposition, that lead us to conclude that $t_r = t_b = t_y = 0$, is meaningless if the $SU(3)_c$ representation is adopted. By the way, there is one point that remains to be analysed: the existence, in our gentilionic approach, of another kind of colour state representation, besides the $SU(3)_c$.

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FIGURE CAPTIONS

Fig. 1 - The equilateral triangle in the colour space (X,Y,Z) with vertices occupied by the colours α, β and γ.

Fig. 2 - The equilateral triangle in the colour space (X,Y,Z) with vertices occupied by the colours $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$.

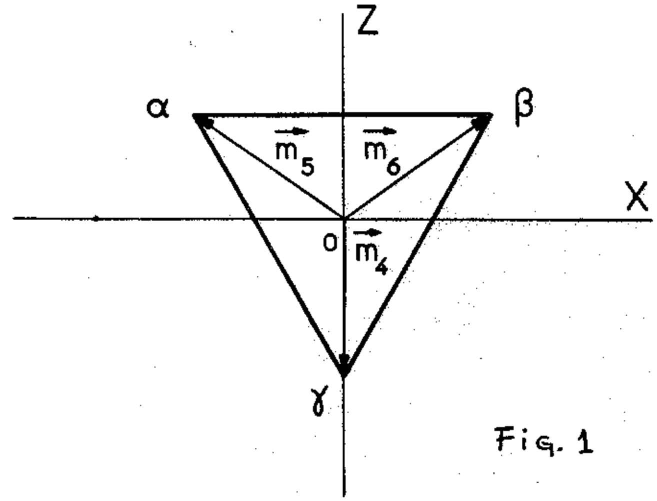

Fig. 1

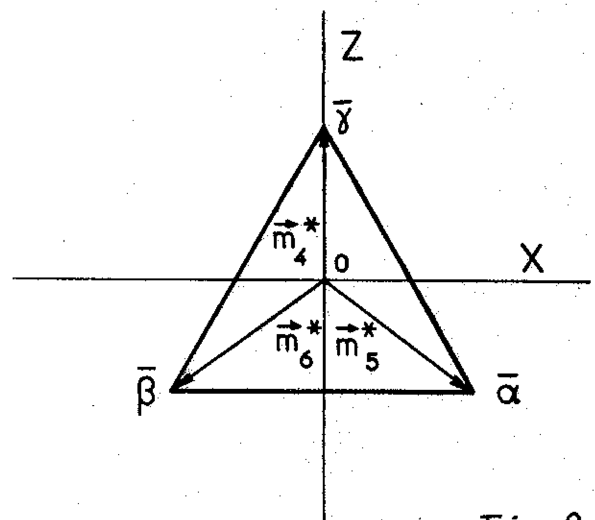

Fig. 2