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ABSTRACT

Helical winding current thresholds for the destruction of magnetic surfaces in large aspect-ratio tokamaks are estimated. The influence of magnetic islands created by external windings and by tearing instabilities on the disruptive instability is considered. The radial dependence of the perturbations and the change in the shear over the island widths are taken into account. Ergodization due to secondary islands caused by the coupling between the resonant helical fields and the toroidal curvature are also considered.

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INTRODUCTION

Disruptive instabilities limit the tokamak operation and their causes ares still unclear. There are experimental evidences that in several cases these instabilities are preceded by some tearing modes (Pulsator Team 1985, Cheetham 1985). Also, disruptive instabilities can be triggered by resonant magnetic fields created by helical windings (Karger et al 1975, Ellis et al 1981, Yoshida et al 1983). The effect of helical winding perturbations for a tokamak plasma were also investigated in the brazilian TBR-1 tokamak (Bender et al 1986).

In this article, we suppose that magnetic surface break-up caused by these resonances triggers the tokamak disruptions. We consider the plasma interaction with the limiter as observed in the Pulsator tokamak (Karger et al 1975) and the overlapping of magnetic islands created by external windings and by helical surface currents on rational magnetic surfaces as observed in the Toriut-4 tokamak (Yoshida et al 1983). The magnitudes of these surface currents can be deduced from experimental measurements of the poloidal magnetic field oscillations in tokamaks (Alladio et al 1985).

To calculate the size of the islands we consider the magnetic surfaces obtained from the differential equation

$$\vec{B}.\nabla\psi = 0 \quad , \tag{1}$$

where ψ is the stream function corresponding to a linear superposition of the unperturbed fields described by $\psi_0(r)$ with the resonant perturbation described by $\psi_1(r,u)$ (Morozov and Solovev 1966). Applying the Chirikov condition (Chirikov 1979) we estimate analytically helical winding current thresholds for disruptions in large aspect-ratio tokamaks with circular cross-sections. We calculate these thresholds taking into account the r-dependence of equilibrium and perturbed magnetic fields over the island widths. We consider also satellite magnetic islands due to toroidal corrections (Feneberg 1977).

In the section 2 we describe the equilibrium and in section 3 the resonant helical perturbations. In section 4 we discuss the magnetic surface destruction due to overlapping of magnetic islands with different helicities. In section 5 we consider the magnetic field line ergodization due to toroidal corrections. As an example, we apply our calculation to estimate the helical current thresholds for the brazilian TBR-1 tokamak (Tan et al 1986, Bender et al 1986).

2. MAGNETIC SURFACES IN LARGE ASPECT - RATIO TOKAMAKS

Plasma equilibrium in tokamaks is described by MHD equations. The differential equations for the magnetic field lines are

$$\frac{dr}{dz} = \frac{B_r}{B_z} \quad , \qquad r\,\frac{d\vartheta}{dz} = \frac{B_\vartheta}{B_z} \quad , \tag{2}$$

where r, ϑ and z are cylindrical coordinates. A cylindrically symmetric system with an helical perturbation (described in terms of the variables r and u) superimposed upon it possesses

magnetic surfaces given by the equation (1) where

$$\psi = \psi(r,u) \tag{3}$$

$$u = m\vartheta + kz \tag{4}$$

and m and k are constants. The helical magnetic field can be obtained from the stream function ψ by (Morozov and Solovev 1966)

$$r\,B_r = -\frac{\partial\psi}{\partial u} \tag{5}$$

$$m\,B_\vartheta + kr\,B_z = \frac{\partial\psi}{\partial r} \quad . \tag{6}$$

We consider a large aspect-ratio tokamak with circular cross-section, represented by a periodical cylinder with length $2\pi R$, and assume the tokamak scaling

$$\frac{B_{0_\vartheta}}{B_{0_z}} \approx -\frac{ka}{m} << 1 \qquad ; \qquad k = -\frac{n}{R} \tag{7}$$

where R and a are respectively the major and minor plasma radii and m and n are rational numbers. The unperturbed MHD-equilibrium with cylindrical symmetry is determined by B_{0_z} and the plasma current density $\vec{J}$:

$$\nabla B_{0_z} = 0 \tag{8}$$

$$\vec{J} = J_0\left(1 - \frac{r^2}{a^2}\right)^{\gamma} \hat{z} \quad . \tag{9}$$

where J_0 and γ are constants. In this case the unperturbed magnetic surfaces comprise a system of nested concentric cylinders, while the lines of force are helices with pitch characterized by the safety factor given by

$$q = \frac{r B_{o_z}}{R B_{o_\vartheta}} \quad . \tag{10}$$

At rational surfaces with

$$q(r_{m,n}) = \frac{m}{n} \tag{11}$$

the magnetic field lines close on themselves after m trips alogn the cylinder and n trips in the poloidal direction. Toroidal corrections to this cylindrical equilibrium are discusssed in section 5.

The equilibrium field can be expressed in terms of a zeroth-order stream function obtained from Eqs. (5) and (6),

$$\psi_o(r) = \int dr' \,(m B_{o_\vartheta} + kr' B_{o_z}) \quad , \tag{12}$$

where the component B_{o_ϑ} can be obtained by applying Ampere's law.

The helical perturbation on equilibrium is assumed to be created by resonant helical windings (Morozov and Solovev 1966) or due to a saturated tearing instability simulated by a perturbed current at rational surfaces (Fusmann et al 1981, Alladio et al 1985). The stream functions which describes the magnetic surfaces of these fields are

$$\psi_1(r,u) = f_m(r)\cos u \quad . \tag{13}$$

The ratio m/k caracterizes the perturbation helicities.

In this article we consider the linear superposition of the unperturbed equilibrium described by $\psi_o(r)$ with a resonant helical perturbation described by $\psi_1(r,u)$. Within this approximation, the stream function

$$\psi(r,u) = \psi_o(r) + \psi_1(r,u) \tag{14}$$

satisfies Eq. (1) (Morozov and Solovev 1966). This approximation is not valid for marginally stable states when the plasma response should not be neglected.

Magnetic field lines are confined to surfaces of constant ψ. This perturbation in resonance with the period of the equilibrium magnetic line creates m islands around the magnetic surface with $q = m/n$.

3. RESONANT HELICAL PERTURBATIONS

The magnetic field created by electrical currents I flowing in m pairs of helical windings, equally spaced, with radius b wounded on a circular cylinder (corresponding to a large aspect-ratio tokamak) exhibits helical symmetry. This field depends on the coordinates r and u. The ratio m/k characterizes the winding helicity.

We consider currents I flowing in opposite direction in adjacent windings. The irrotational helical field is conveniently described by means of a scalar potential expanded in a harmonic series in u_N ($u_N = N\vartheta + kz$, $N = (2p+1)m$ and p = 0,1,...). Near the axis ($r << R$) the field can be approximated by a single harmonic in u ($p=0$). From this we obtain, by using Eqs. (5) and (6), the stream function ψ_1. In this case the function f_m is given by (Morozov and Solovev 1966, Fernandes et al 1986)

$$f_m(r) \approx -\frac{m\mu_o I}{\pi}\left(\frac{r}{b}\right)^m \quad . \tag{15}$$

Fig. 1 illustrates the structures of the magnetic islands created by $m=2$ pairs of helical windings ($n=1$) with $I = 100\,A$. The diagram shows the intersections of the magnetic surfaces with a plane $z = const$. All magnitudes were adjusted to fit TBR-1 data. Thus $a = 0.08\,m$, $b = 0.11\,m$ and $R = 0.3\,m$. In this example we consider $q(0) = 1$, $q(a) = 3$ and $\gamma=2$.

The resonant poloidal magnetic field oscillations observed in tokamak plasmas, known as Mirnov oscillations, may be produced from helical current perturbations in the plasma (Fussmann et al 81). The exact distribution of these current perturbations is not known. Based on the experimental results and theoretical models of the resistive tearing mode instabilities, we consider a perturbation of the plasma current distributed poloidally and toroidally according to the measured m and n numbers of the relative phases of the Mirnov oscillations. The current perturbation is assumed to lie on the unperturbed rational magnetic surface with $q = m/n$. It corresponds to an helical sheet current following the field lines on this rational surface:

$$\vec{J}_{m,n} = j_{m,n}\,\delta(r-r_{m,n})\cos u(\hat{s}\cos\gamma + \hat{z}\,\mathrm{sen}\,\gamma) \quad , \tag{16}$$

where

$$\cot g\,\gamma = \frac{n\,r_{m,n}}{mR} \quad . \tag{17}$$

γ is the pitch angle of field lines on the unperturbed rational surface and $j_{m,n}$ depends on the amplitude of the measured Mirnov oscillation. The magnetic field produced by such a perturbation can be calculated and the stream function ψ_1 can be obtained from Eqs. (5) and (6); the function f_m for $r/R << 1$ is

$$f_m(r) \approx -\frac{\mu_o j_{m,n} r_{m,n}}{2}\left(\frac{r}{r_{m,n}}\right)^{\pm m} \tag{18}$$

for $r/r_{m,n} < 1$ (>1) we take $+m$ ($-m$) in the exponent.

Expanding ψ near $r = r_{m,n}$ we obtain a formula for small island half-widths $\Delta_{m,n}$

$$\Delta_{m,n} = 2\left[\frac{f(r_{m,n})}{\psi_o''(r_{m,n})}\right]^{\frac{1}{2}} \tag{19}$$

(the prime indicates a derivative with respect to r). To obtain half island widths $\Delta_{m,n}$, taking into account the radial dependence of the perturbation ψ_1 and the change in the safety factor q over the island width, we use the equation for the island separatrix

$$\psi_o(r) + \psi_1(r,u) = \psi_o(r_{m,n}) + \psi_1(r_{m,n},u_o) \quad , \tag{20}$$

where u_0 is the value of u at the x-point for which $r = r_{m,n}$. The accuracy of the formula (19) is illustrated by the comparison, in Figs. 2, 3 and 5 of helical current thresholds calculated using Eqs. (19) and (20). The agreeement is found to be good, specially for smaller islands.

4. MAGNETIC SURFACE DESTRUCTION

In this section we assume that the disruptive instability in tokamaks is caused by the destruction of magnetic surfaces and consider two distincts causes of disruptions: hard disruptions caused by plasma interaction with the limiter and soft disruptions due to interactions of magnetic islands with different helicities. In the first case the plasma is destroyed and in the second case it could recover.

There are experimental evidences that hard disruptive instabilities observed in tokamak discharges can be triggered by the interaction of $m=2$ magnetic islands created by resonant helical windings with the limiter at $r=a$ (Karger et al 1975). From Eqs. (19) or (20) we can calculate the helical winding current I in $m=2$ pairs of helical windings that causes the contact between the magnetic islands created at $r = r_{2,1}$ and the limiter. As an example, we show in the Fig. 2 the dependence of this current with $q(a)$ in the TBR-1 tokamak for discharges with $q(0) = 1$. This perturbation would trigger hard disruptions in this tokamak. With increasing $q(a)$ the resonance surface moves inward: therefore the width of the islands (approximately

proportional to the quantity $(I_{hel}/B_z)^{\frac{1}{2}}$) has to increase in order to get into contact with the limiter. This behavior agrees qualitatively with the Pulsator experimental data reported in Karger et al (1975).

We consider now the hypothesis that soft disruptive instabilities observed in tokamaks are caused by ergodic wandering of magnetic field lines. Magnetic surfaces break-up occurs due to the destruction of the system symmetry. As a symmetry breaking perturbation due to magnetic islands created by different resonant helical fields grows, magnetic surfaces are destroyed and a disruption may occur (Fussmann et al 1981). The degree of ergodic behavior depends upon the strength of perturbations created by external windings and by helical surface currents. We apply the Chirikov condition (Chirikov 1979), to estimate the helical winding current threshold for ergodization of the magnetic field lines. Two of magnetic islands with mode numbers (m,n) and (m',n') are ergodized when

$$S = \frac{\Delta_{m,n} - \Delta_{m',n'}}{|r_{m,n} - r_{m',n'}|} > 1 \qquad (20)$$

where S is known as stochastic parameter.

As an illustration we consider the ergodization due to equal currents I in two different pairs of (2,1) and (3,1) helical windings. In Fig. 3 we show the dependence of I with $q(a)$ for the TBR-1.

In Fig. 4 (curve A) we show the dependence of the $m=3$ helical winding current I with $q(a)$ for the overlap of $m=3$ induced islands and $m=2$ islands observed in TBR-1. The coefficient $j_{2,1}$ was obtained from the experimental Mirnov oscillation amplitude measured in this tokamak (Tan et al 1986). Adapting the model for tearing instabilities proposed by Miller (Miller 1985) to describe the magnetic oscillations observed in the TBR-1 we obtained the curve B in Fig. (4). The curves A and B were obtained for the same q(0) and q(a) values and the same values of the magnetic field in the magnetic axis. In the two models used we considered the same value for the perturbed magnetic field component $B_{1_r}(r_2)$, but the island widths are different because B_{o_z} is not uniform in Miller's model. The m=3 helical field (n=1) has been applied in the Toriut-4 tokamak (Yoshida et al 1983) to remove the critical m=2 island from the plasma periphery by the ergodization effect of the externally applied helical field. In such a way, the hard disruptive instability provocated by the increasing of the m=2 islands was avoided and a mild path into the $q(a) < 2$ regime was realized.

The helical current thresholds increase with q(a) although the distance between the rational magnetic surfaces with q=2 and q=3 remain almost the same. This happens because the island width decrease when the islands move inward, where the shear of magnetic field lines, represented by $\psi_o''(r_{m,n})$ in eq. (19), is smaller.

5. TOROIDAL EFFECT

When a single helical perturbation (m,n) is superimposed upon an equilibrium with toroidal symmetry both symmetries

are broken, and therefore the magnetic surfaces should be expected to disappear (Morozov and Solovev 1966). However for a small helical perturbation the major effect, on a large aspect ratio tokamak, is the appearence of (m±1) satellite magnetic islands on the rational magnetic surfaces with $q = (m \pm 1)/n$ (Fusmann et al 1981). The outward displacement of the magnetic surfaces proportional to r/R causes the magnetic islands to overlap for points on the outside of the magnetic axis for slightly smaller perturbation strengths and has a negligible effect on the threshold helical current amplitudes (Finn 1975).

In this section we take into account satellite islands due to the toroidal correction by multiplying the constant B_{o_z} by the factor $(1 + \frac{r}{R}\cos\vartheta)^{-1}$. Substituting the corrected term

$$B_{o_z} \rightarrow \frac{B_{o_z}}{1 + \frac{r}{R}\cos\vartheta} \tag{21}$$

into Eqs. (2) we obtain the coupling of the (m,n) terms with the m=1, n=0 terms of B_{o_z}. The Eqs. (2) become

$$\frac{dr}{dz} = \frac{B_{1_r}}{B_{o_z}(1 + \frac{r}{R}\cos\vartheta)^{-1} + B_{1_z}} = \frac{\left(\frac{r f_m'}{m^2 + k^2 r^2}\right)' \operatorname{sen} u}{B_{o_z}(1 + \frac{r}{R}\cos\vartheta)^{-1} + \frac{kr f_m'}{m^2 + k^2 r^2}\cos u} \tag{22}$$

and

$$r\,\frac{d\vartheta}{dz} = \frac{B_{o_\vartheta} + B_{1_\vartheta}}{B_{o_z} + B_{1_\vartheta}} = \frac{B_{o_\vartheta} + \frac{m f_m'}{m^2 + k^2 r^2}\cos u}{B_{o_z}(1 + \frac{r}{R}\cos\vartheta)^{-1} + \frac{kr f_m'}{m^2 + k^2 r^2}\cos u} . \tag{23}$$

We consider the form that Eqs. (22) and (23) take near $q = (m \pm 1)/n$. Expanding these expressions, neglecting terms of the order $(r/R)^2$ or higher and considering only the resonant terms we obtain

$$\frac{dr}{dz} \simeq \frac{r(r f_m')'}{2m^2 R B_{o_z}} \operatorname{sen} u' \quad , \tag{24}$$

and

$$r \frac{d\vartheta}{dz} \simeq \frac{r f_m'}{2m R B_{o_z}} \cos u' \quad , \tag{25}$$

where

$$u' = (m \pm 1)\vartheta + kz \quad . \tag{26}$$

From Eqs. (24), (25) and (26) we can find an integral of motion χ satisfying

$$\vec{B} . \nabla\chi \simeq 0 \quad . \tag{27}$$

This function χ is used in the same manner as the stream function ψ to calculate the width of the $m \pm 1$ islands at the magnetic surfaces with $q = (m \pm 1)/n$. We obtain

$$\chi(r,u') = \int (\frac{r}{a})^{\ell} \ (m' B_{o_\vartheta} + kr' B_{o_z}) dr' + \frac{a}{2R} (\frac{r}{a})^{\ell+1} f_m(r) \cos u' \tag{28}$$

where $\ell = -2$ for the surface currents while for the helical winding currents

$$\ell = 0 \ (-2) \quad \text{for} \quad r \sim r_{m+1,n} \quad (r \sim r_{m-1,n}) \quad .$$

The ratio between the widths of the primary and secondary islands is proportional to $(a/R)^{\frac{1}{2}}$. An example of winding currents required for ergodization due to the overlapping of $m=2$ and $m'=3$ islands is plotted in the Fig. (5) as a function of $q(a)$. In Fig. 6 we present the dependence of the stochastic parameter s with $q(a)$, in the TBR-1 discharges, obtained considering the interaction between the $m=2$ islands created by a current density $\vec{J}_{2,1}$ and the $m=3$ satellite islands due to the toroidal correction. Three different values for $q(0)$ have been chosen.

CONCLUSIONS

We consider that the magnetic surface destruction caused by resonant helical windings triggers the disruption instabilities in tokamaks. Within this hypothesis the $m=2$ helical winding current that causes the contact between the $m=2$ magnetic island and the tokamak limiter was calculated. The helical current threshold for magnetic surface break-up caused by overlapping of magnetic islands with two different helicities created by external windings and by tearing instabilities was estimated analytically. Ergodization caused by satellite magnetic islands on neighboring rational surfaces, due the action of toroidicity were also considered. The calculations presented can be applied to tokamak experiments with resonant helical windings. As an illustration the numerical applications were done for the brazilian TBR-1 tokamak.

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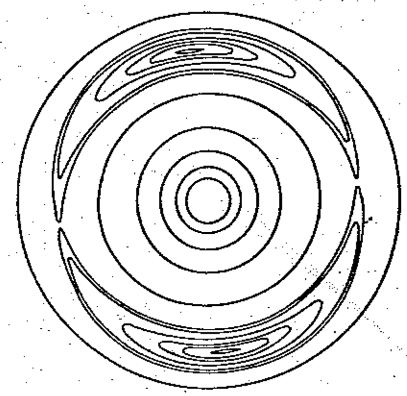

Fig. 1 - Island structures created by two pairs of helical windings ($m=2$, $n=1$, $q(a) = 3$, $\gamma=2$, $I_p = 10$ KA and $I = 100$ A).

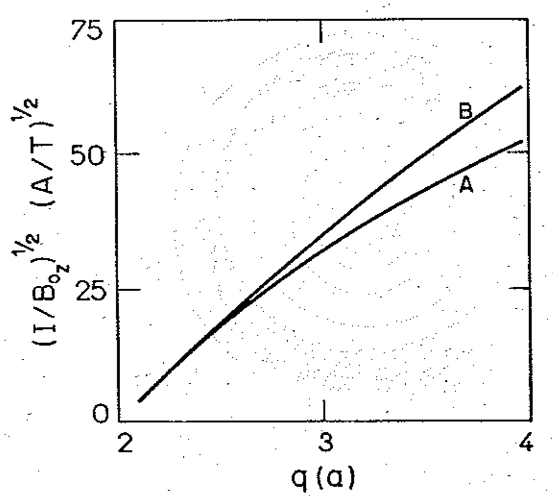

Fig. 2 - Helical winding current for the m=2 islands touch the limiter [q(0) = 1, I_p = 10 KA]. The curves A and B were obtained neglecting and taking into account the r-dependence of ψ over the island width.

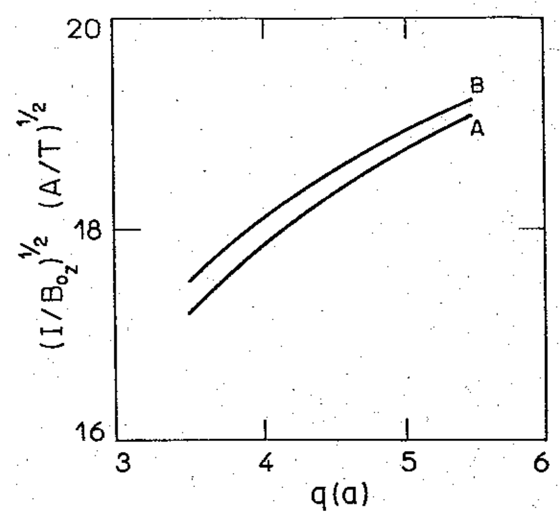

Fig. 3 - Helical current in two sets of helical windings for overlap of m=2 and m=3 resonances. The curves A and B were obtained neglecting and taking into account the r-dependence of ψ over the island width [q(0) = 1, I_p = 10 KA].

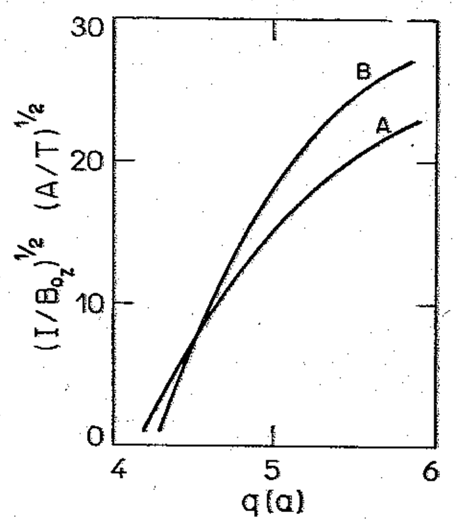

Fig. 4 - Helical winding current for overlap of m=3 induced islands and m=2 spontaneous islands observed in the TBR tokamak ($q(0) = 1$, $I_p = 10$ KA, n=1). The m=2 plasma resonance was described by the surface current density model (curve A) or by the Miller's model (curve B).

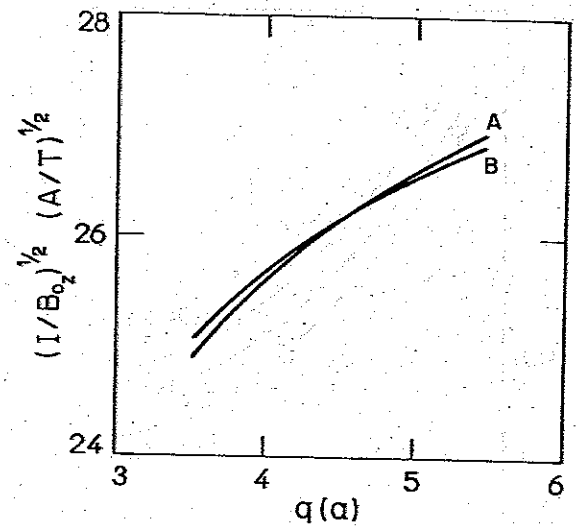

Fig. 5 - Helical winding current (m=2, n=1) for overlap of m=2 and m=3 resonances due to toroidal corrections ($q(0) = 1$, $I_p = 10$ KA). The curves A and B were obtained neglecting and taking into account the r-dependence of χ over the island width.

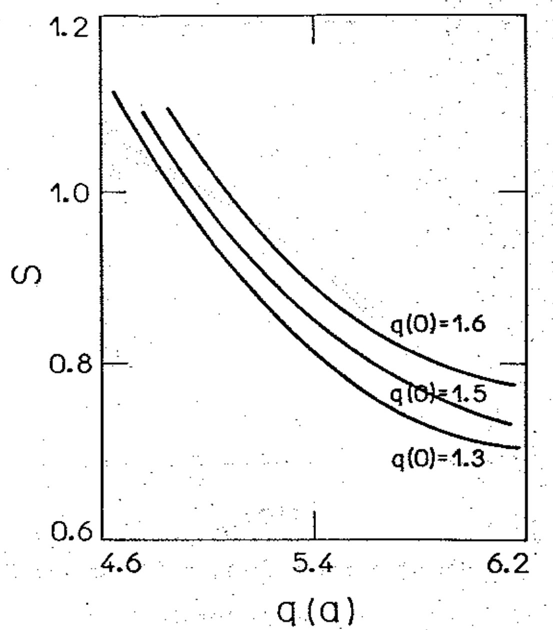

Fig. 6 - Stochastic parameter S, related to the interaction of the m=2 islands created by a helical surface density with the m=3 satellite islands, as a function of q(a) for various values of q(0) (B_{o_z} = .424 T).