UNIVERSIDADE DE SÃO PAULO

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IFUSP/P-604

HOMOGENEOUS SPACE-TIMES OF GÖDEL TYPE IN HIGHER-DERIVATIVE GRAVITY

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Outubro/1986

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ABSTRACT. A general theorem concerning any Gödel-type solution of higher-derivative gravity field equations, which may be produced by any reasonable physical source with a constant energy-momentum tensor, is analysed. The resulting class of metrics depends on two parameters, one of which is related to the vorticity. A general class of solutions of Gödel-type space-time-homogeneous universes in the context of the higher-derivative theory is exhibited. This is the most general higher-derivative solution of such type of metric and includes all known solutions of Einstein's equations related to these geometries as a special case. A number of completely causal rotating models is also obtained. Some of them present the interesting feature of having no analogous in the framework of general relativity.

PACS. 04.60.+n Quantum theory of gravitation

PACS. 04.20.Jb Solutions to equations

PACS. 98.80.Dr Relativistic cosmology

1. INTRODUCTION

General relativity with higher-derivative terms has been considered[1-4] as a very attractive candidate for a theory of quantum gravity. The theory is defined by the action

$$I = \int d^4x\sqrt{-g}\,[\frac{R}{\varkappa} + \frac{\Lambda}{\varkappa} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + L_m] \,, \qquad (1.1)$$

where α and β are dimensionless coupling constants (in natural units), $\varkappa$ and Λ are the Einstein and cosmological constants, respectively, and L_m is the matter Lagrangian density. The corresponding field equations are given by

$$H_{\mu\nu} = - T_{\mu\nu} \,, \qquad (1.2)$$

$$H_{\mu\nu} = \frac{1}{\varkappa}\,(R_{\mu\nu} - \frac{1}{2}\,Rg_{\mu\nu}) + \frac{\Lambda}{\varkappa}\,g_{\mu\nu}$$

$$+ \alpha(-R^2g_{\mu\nu} + 4RR_{\mu\nu} - 4g_{\mu\nu}\Box R + 4\nabla_\nu\nabla_\mu R) \qquad (1.3)$$

$$+ \beta(-2\Box R_{\mu\nu} - R_{\rho\theta}R^{\rho\theta}g_{\mu\nu} + 4R_{\mu\rho\theta\nu}R^{\rho\theta} - g_{\mu\nu}\Box R + 2\nabla_\nu\nabla_\mu R),$$

with trace

$$T = \frac{R}{\varkappa} + \frac{4\Lambda}{\varkappa} + 4(3\alpha + \beta)\Box R. \qquad (1.4)$$

For the quantum field theorist this higher-derivative theory has the great advantage of being renormalizable by power counting[1], whereas, as it is well known, classical general relativity is clearly perturbatively nonrenormalizable by power counting in four dimensions[5,6]. In the pure classical framework, the aforementioned theory may be considered as a possible generalization of Einstein's general relativity, in the sense that it respects the geometrical nature of gravity as well as its gauge symmetry (invariance under general coordinate transformations). Recent work has shown[4,7-12] that the presence of a ghost responsible for a pseudo-nonunitarity of the theory, which was considered its achilles's heel, is no more a vulnerable point of it. The reason is that the ghost is unstable. In spite of the previously mentioned virtues, comparatively little is known about fourth-order gravity theory. Of course a better understanding of its behaviour is of vital interest to those working on quantum gravity, and in particular, quantum cosmology. Consequently, the investigation of cosmological models in the framework of higher-derivative gravity is well suited.

Here we wish to focalize the so-called Gödel-type universes[13]. These models are defined by the line element

$$ds^2 = [dt^2 + H(x)dy]^2 - D^2(x)dy^2 - dx^2 - dz^2, \quad (1.5)$$

and are such that in case

$$H = e^{mx}, \; D = \frac{e^{mx}}{\sqrt{2}}, \quad (1.6)$$

we recover Gödel's universe[14], which is a solution of Einstein's equations with an energy-momentum tensor given by

$$T_{\mu\nu} = \rho v_\mu v_\nu, \; v^\alpha = \delta^\alpha{}_0,$$

$$m^2 = -2\Lambda = \kappa\rho = 2\Omega^2, \quad (1.7)$$

where ρ is the constant density of matter, v^α is the fluid four-velocity, and Ω is the rate of rigid rotation of matter. Our choice for Gödel-type models is dictated, first of all, by their simplicity, which will allow us to accomplish the formidable task of finding exact solutions of higher-derivative gravity field equations, in the case of models that are homogeneous in space and time (ST-homogeneous). And secondly, because this analysis will give us the opportunity of answering a very interesting question, i. e., what happens to the causal pathologies of these universes when quantum corrections are introduced in the standard general relativity theory.

We organize the paper in the following way. In section 2, we present a general theorem concerning any Gödel-type solution of fourth-order gravity field equations with constant energy-momentum tensor. The resulting class of metrics is characterized by two parameters, one of which is related to the rotation of the matter relative to the compass of inertia. Of course, any reasonable physical source will put restrictions on these parameters through the higher-derivative equations. Taking into account the last consid eration, we show in section 3 that a geometry having as source a perfect fluid plus a massless scalar field and an electromagnetic field can fit the parameters of the ST-homogeneous Gödel-type universes. This is the most general higher-derivative solution concerning this type of metric and includes all known solutions of Einstein's equations related to such geometries as a special case. On the other hand, contrary to what generally happens in Einstein's theory, the restrictions on the parameters of the ST-homogeneous Gödel-type models, imposed by the sources through the higher-derivative equations, will provide us with a number of solutions which contain no closed time-like lines, i. e., that are completely causal. We will look into this subject in a comprehensive way in the last section.

2. A GENERAL THEOREM

In order to facilitate our calculations, we shall use a class of locally stationary observers represented by the vectors $e^{(A)}{}_{\alpha}$ defined by $\theta^A = e^{(A)}{}_{\alpha}dx^{\alpha}$, wherein the 1-forms θ^A are given by

$$\theta^0 = dt + H(x)dy,\ \theta^1 = dx,\ \theta^2 = D(x)dy,\ \theta^3 = dz. \qquad (2.1)$$

(Capital letters are tetrad indices and vary from 0 to 3 and greek indices are tensor indices). As a consequence, the vectors $e^{(A)}{}_{\alpha}$ assume the form

$$e^{(0)}{}_0 = e^{(1)}{}_1 = e^{(3)}{}_3 = 1,\ e^{(0)}{}_2 = H,\ e^{(2)}{}_2 = D, \qquad (2.2)$$

and the geometry (1.5) may be written as

$$ds^2 = \eta_{AB}\theta^A\theta^B \ , \qquad (2.3)$$

where $\eta_{AB} = \mathrm{diag}(+,-,-,-)$.

On the other hand, taking into account that $dx^{\alpha} = e^{\alpha}{}_{(A)}\theta^A$, we get immediately

$$e^0{}_{(0)} = e^1{}_{(1)} = e^3{}_{(3)} = 1,\ e^0{}_{(2)} = -\frac{H}{D}\ ,\ e^2{}_{(2)} = D^{-1}, \qquad (2.4)$$

We shall also need the Ricci coeficients of rotation defined by

$$\gamma^A{}_{BC} = -e^{(A)}{}_{\alpha;\beta}e^{\alpha}{}_{(B)}e^{\beta}{}_{(C)} \qquad (2.5)$$

[We use the comma for partial derivative, the semicolon for covariant derivative, and the bar for tetrad components of covariant derivatives. For instance, $R_{AB|C} = R_{AB;\alpha}e^{\alpha}{}_{(C)} = R_{AB,\alpha}e^{\alpha}{}_{(C)}$]. From (2.2) and (2.4) together with (2.5) we get the following nonvanishing components concerning these coeficients

$$\gamma^0{}_{12} = \gamma^1{}_{02} = \gamma^1{}_{20} = -\gamma^2{}_{01} = -\gamma^2{}_{10} = -\gamma^0{}_{21} = \frac{H'}{2D}\ ,$$

$$\gamma^2{}_{12} = -\gamma^1{}_{22} = \frac{D'}{D}\ , \qquad (2.6)$$

where the prime denotes differentiation with respect to x.

In the local inertial frame defined by $\theta^A = e^{(A)}{}_{\alpha} dx^{\alpha}$ the higher-derivative gravity field equations, Eqs. (1.2) and (1.3) , take the form

$$H_{AB} = - T_{AB} \,, \tag{2.7}$$

$$\begin{aligned} H_{AB} = {} & \frac{1}{\kappa}(R_{AB} - \frac{1}{2} R\eta_{AB}) + \frac{\Lambda}{\kappa}\eta_{AB} \\ & + \alpha[-R^2\eta_{AB} + 4RR_{AB} - 4\eta_{AB}\eta^{CD}(R_{|C|D} - \gamma^M{}_{CD}R_{|M}) \\ & + 4(R_{|A|B} - \gamma^M{}_{AB}R_{|M})] + \beta\{-R^{CD}R_{CD}\eta_{AB} + 4R_{ACDB}R^{CD} \\ & - \eta_{AB}\eta^{CD}(R_{|C|D} - \gamma^M{}_{CD}R_{|M}) + 2(R_{|A|B} - \gamma^M{}_{AB}R_{|M}) \\ & - 2\eta^{CD}[R_{AB|C|D} - (\gamma^M{}_{AC}R_{MB|D} + \gamma^M{}_{BC}R_{AM|D} \\ & + \gamma^M{}_{AC|D}R_{MB} + \gamma^M{}_{BC|D}R_{AM}) - \gamma^M{}_{AD}(R_{MB|C} - \gamma^N{}_{MC}R_{NB} \\ & - \gamma^N{}_{BC}R_{MN}) - \gamma^M{}_{BD}(R_{AM|C} - \gamma^N{}_{AC}R_{NM} - \gamma^N{}_{MC}R_{AN}) \\ & - \gamma^M{}_{CD}(R_{AB|M} - \gamma^N{}_{AM}R_{NB} - \gamma^N{}_{BM}R_{AN})]\} \,. \end{aligned} \tag{2.8}$$

We are ready now to demonstrate the following general result.

Theorem: Any Gödel-type solution of higher-derivative gravity field equations $H_{AB} = - T_{AB}$, having as source of the geometry any field with T_{AB} independent of the points of the space-time, is space-time-homogeneous up to a local Lorentz transformation.

Proof: The only surviving components of H_{AB} (Eq.(2.7)) related to Gödel - type metrics (Eq.(1.5)) are

$$\begin{aligned} H_{00} = {} & \frac{1}{\kappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 - \frac{R}{2}\right] + \frac{\Lambda}{\kappa} + \alpha[-R^2 - 2R(H'/D)^2 + \frac{4D'R'}{D} + 4R''] \\ & + \beta\left\{-\frac{15}{4}\left(\frac{H'}{D}\right)^4 - 3\frac{H'}{D}\left(\frac{H'}{D}\right)'' - \frac{3}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 - 3\frac{D'}{D}\frac{H'}{D}\left(\frac{H'}{D}\right)' \right. \\ & \left. + 6\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - 2\left(\frac{D''}{D}\right)^2 - 2\left(\frac{D''}{D}\right)'' - 2\frac{D'}{D}\left(\frac{D''}{D}\right)'\right\} , \end{aligned} \tag{2.9}$$

$$\begin{aligned} H_{11} = {} & \frac{1}{\kappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 + \frac{D''}{D} + \frac{R}{2}\right] - \frac{\Lambda}{\kappa} + \alpha\left[-R^2 - R(H'/D)^2 - \frac{4D'R'}{D}\right] \\ & + \beta\left\{-\frac{9}{4}\left(\frac{H'}{D}\right)^4 - 5\frac{D'}{D}\frac{H'}{D}\left(\frac{H'}{D}\right)' + 4\frac{D'}{D}\left(\frac{D''}{D}\right)' + \frac{1}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 \right. \\ & \left. - \frac{H'}{D}\left(\frac{H'}{D}\right)'' + 4\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - 2\left(\frac{D''}{D}\right)^2\right\} , \end{aligned} \tag{2.10}$$

$$\begin{aligned} H_{22} = {} & \frac{1}{\kappa}\left[-\frac{1}{2}\left(\frac{H'}{D}\right)^2 + \frac{D''}{D} + \frac{R}{2}\right] - \frac{\Lambda}{\kappa} + \alpha[-R^2 - 4R'' - R(H'/D)^2] \\ & + \beta\left\{-\frac{9}{4}\left(\frac{H'}{D}\right)^4 - \frac{9}{2}\left[\left(\frac{H'}{D}\right)'\right]^2 + 4\left(\frac{D''}{D}\right)'' - 2\left(\frac{D''}{D}\right)^2 \right. \\ & \left. - \frac{H'}{D}\frac{D'}{D}\left(\frac{H'}{D}\right)' - 5\frac{H'}{D}\left(\frac{H'}{D}\right)'' + 4\left(\frac{H'}{D}\right)^2\frac{D''}{D}\right\} , \end{aligned} \tag{2.11}$$

$$H_{33} = \frac{1}{\kappa}\frac{R}{2} - \frac{\Lambda}{\kappa} + \alpha[R^2 - 4R'' - 4\frac{D'}{D}R'] + \beta\left\{\frac{3}{4}\left(\frac{H'}{D}\right)^4 + 2\left(\frac{D''}{D}\right)^2\right.$$

$$- \frac{3}{2} \left[\left(\frac{H'}{D}\right)'\right]^2 - 2\,\frac{D''}{D}\left(\frac{H'}{D}\right)^2 - \frac{H'}{D}\left(\frac{H'}{D}\right)'' + 2\left(\frac{D''}{D}\right)'' - \frac{D'}{D}\,\frac{H'}{D}\left(\frac{H'}{D}\right)'$$

$$+ 2\,\frac{D'}{D}\left(\frac{D''}{D}\right)'\Bigg\}\,, \qquad (2.12)$$

$$H_{02} = -\frac{1}{2\kappa}\left(\frac{H'}{D}\right)' + \alpha\left[-2\left(\frac{H'}{D}\right)'R - \frac{2H'R'}{D}\right] + \beta\Bigg\{-\left(\frac{H'}{D}\right)'''$$

$$-9\left(\frac{H'}{D}\right)^2\left(\frac{H'}{D}\right)' + 3\left(\frac{H'}{D}\right)'\frac{D''}{D} + 4\,\frac{H'}{D}\left(\frac{D''}{D}\right)' + \left(\frac{H'}{D}\right)'\left(\frac{D'}{D}\right)^2$$

$$-\frac{D'}{D}\left(\frac{H'}{D}\right)''\Bigg\}\;; \qquad (2.13)$$

where

$$R = \frac{1}{2}\left(\frac{H'}{D}\right)^2 - 2\,\frac{D''}{D}\;; \qquad (2.14)$$

The assumption of T_{AB} constant, however, implies that H_{AB} is constant too. On the other hand, it is not difficult to see from the above equations that H_{AB} is constant in case we have

$$\frac{H'}{D} = \text{const} \equiv 2\Omega,\; \frac{D''}{D} = \text{const} \equiv m^2\;. \qquad (2.15)$$

These are precisely the necessary and sufficient conditions to a Gödel-type metric be space-time-homogeneous[15,16].QED.

Thus the whole class of solutions with T_{AB} constant is characterized by the two independent parameters m and Ω. It is not difficult to show that the last parameter is related to the vorticity.

In fact, in the local frame considered, the rotation may be written as

$$\omega_{AB} = \frac{1}{2}\,[(\gamma^0{}_{BA} - \gamma^0{}_{AB}) + (\gamma^0{}_{AB}\delta^0{}_B - \gamma^0{}_{B0}\delta^0{}_A)]\,, \qquad (2.16)$$

for a velocity field given by

$$v^A = e^{(A)}{}_0 = \delta^A{}_0\;. \qquad (2.17)$$

As a consequence, the vorticity assumes the form

$$\omega_{12} = -\frac{1}{2}\,\frac{H'}{D}\;. \qquad (2.18)$$

It follows then that the vorticity vector $\omega^A = \frac{1}{2}\,\epsilon^{ABCD}\omega_{BC}v_D$ is given by

$$\omega^A = (0,0,0,\Omega),$$

where $2\Omega = H'/D$. (2.19)

3. A CLASS OF HIGHER-DERIVATIVE GÖDEL-TYPE SOLUTIONS

It is reasonable to question, ab initio, what material content we may consider as source of our geometry, in order to obtain the most general higher-derivative Gödel-type solution, i.e., a solution which includes all known solutions of Einstein's equations related to such geometries. The answer is straightforward if we appeal to a recent work of Rebouças and Tiomno[16]. There, they exhibit a remarkable class of exact solutions of Einstein-Maxwell-scalar field equation which is the most general solution of a Gödel-type ST-homogeneous metric. So we consider a rotating universe $(\Omega \neq 0)$ for which the material content is a perfect fluid of density ρ and pressure p plus a source-free electromagnetic field F_{AB} and a massless scalar field S. Consequently, the energy-momentum tensor in the tetrad frame becomes

$$T_{AB} = \rho v_A v_B - p(\eta_{AB} - v_A v_B) + T^{(S)}_{AB} + T^{(EM)}_{AB} , \qquad (3.1)$$

where

$$T^{(EM)}_{AB} = \frac{1}{4} F_{CD} F^{CD} \eta_{AB} - F_{AM} F_B{}^M ,$$

$$T^{(S)}_{AB} = S_{|A} S_{|B} - \frac{1}{2} \eta_{AB} S_{|M} S_{|N} \eta^{MN} , \qquad (3.2)$$

and v^A is given by Eq. (2.17).

The Maxwell equations concerning the source-free electromagnetic field are given by

$$F^{AB}{}_{|B} + \gamma^A{}_{MB} F^{MB} + \gamma^B{}_{MB} F^{AM} = 0,$$

$$F_{[AB|C]} + 2F_{M[C} \gamma^M{}_{AB]} = 0 , \qquad (3.3)$$

whereas the zero-mass scalar field equation is as follows

$$\eta^{AB} S_{|A|B} - \gamma^M{}_{AB} \eta^{AB} S_{|M} = 0 . \qquad (3.4)$$

The brackets denote total antisymmetrization.

On the other hand, the fact we are requiring space-time homogeneity of the Gödel-type models implies that T_{AB} is constant(cf. theorem in the last section). We remark also that we have a preferred direction in our universe determined by the rotation. Taking into account the above considerations we can seek solutions of (3.3) and (3.4), respectively, related to our model. Let us first consider the electromegnetic field. Since it is not a pure test field but also acts as source of the curvature, it must then be compatible with the space-time symmetries. As a consequence, we are led to take both $\vec{E}$ and $\vec{B}$ along the direction of rotation. Thus, the only nonvanishing components of F_{AB} are

$$F_{30} = - F_{03} = E(z) ,$$

$$F_{12} = - F_{21} = B(z) , \qquad (3.5)$$

Using (2.6) and (3.5), equations (3.3) reduce respectively to

$$E_{,3} + \frac{H'}{D} B = 0 ,$$

$$B_{,3} - \frac{H'}{D} E = 0 . \qquad (3.6)$$

But, since $H'/D = 2\Omega$ (ST-homogeneity), the general solution of Eqs. (3.6) can be written as

$$E = E_0 \cos[2\Omega(z - z_0)] \ ,$$

$$B = E_0 \sin[2\Omega(z - z_0)] \ , \qquad (3.7)$$

where E_0 and z_0 are constants. In the case of the massless scalar field it is trivial to show that if we take

$$S = az + b \ , \qquad (3.8)$$

where a and b are constants, we can satisfy Eq. (3.4) as well as the space-time symmetries.

Now, the non-null components of H_{AB} for the ST-homogeneous Gö del-type metric are

$$H_{00} = \frac{1}{\varkappa}[-3\Omega^2 + m^2] + \frac{\Lambda}{\varkappa} + \alpha[-20\Omega^4 - 4m^4 + 24\Omega^2 m^2] + \beta[-60\Omega^4 + 24m^2\Omega^2 - 2m^4] \ , \qquad (3.9)$$

$$H_{11} = H_{22} = \frac{1}{\varkappa}[-\Omega^2] - \frac{\Lambda}{\varkappa} + \alpha[-12\Omega^4 - 4m^4 + 16\Omega^2 m^2] + \beta[-36\Omega^4 + 16m^2\Omega^2 - 2m^4] \ , \qquad (3.10)$$

$$H_{33} = \frac{1}{\varkappa}[\Omega^2 - m^2] - \frac{\Lambda}{\varkappa} + \alpha[4\Omega^4 + 4m^4 - 8\Omega^2 m^2] + \beta[12\Omega^4 + 2m^4 - 8m^2\Omega^2] \ . \qquad (3.11)$$

As a result, the higher-derivative gravity field equations reduce to the following set of three equations

$$\rho = \frac{E_0^2}{2} - \frac{3}{2}a^2 - \frac{\Lambda}{\varkappa} - 2m^4(2\alpha + \beta) + 4\Omega^4(\alpha + 3\beta) + \Omega^2/\varkappa , \qquad (3.12)$$

$$p = -\frac{E_0^2}{2} + \frac{a^2}{2} + \frac{\Lambda}{\varkappa} + 2m^4(2\alpha + \beta) + 12\Omega^4(\alpha + 3\beta) - 16\Omega^2 m^2(\alpha + \beta) + \Omega^2/\varkappa \ , \qquad (3.13)$$

$$\frac{m^2}{\varkappa} = 16\Omega^4(\alpha + 3\beta) + 4m^4(2\alpha + \beta) - 24m^2\Omega^2(\alpha + \beta) + 2\frac{\Omega^2}{\varkappa} - E_0^2 + a^2 \ . \qquad (3.14)$$

The positivity of energy and pressure is guaranteed if the cosmological constant satisfies the relation

$$-12\Omega^4(\alpha + 3\beta) - 2m^4(2\alpha + \beta) + 16m^2\Omega^2(\alpha + \beta) + \frac{E_0^2}{2} - \frac{\Omega^2}{2} - \frac{a^2}{2} \leq \frac{\Lambda}{\varkappa} \leq 4\Omega^4(\alpha + 3\beta) - 2m^4(2\alpha + \beta) + \frac{E_0^2}{2} - \frac{3}{2}a^2 + \frac{\Omega^2}{\varkappa} \ , \qquad (3.15)$$

which implies that

$$8\Omega^4(\alpha + 3\beta) + \Omega^2\left[\frac{1}{\varkappa} - 8m^2(\alpha + \beta)\right] - \frac{a^2}{2} \geq 0 \ , \qquad (3.16)$$

the equality having as consequence

$$\frac{\Lambda}{\varkappa} = \frac{E_0^2}{2} - \frac{2\Omega^2}{\varkappa} - 20\Omega^4(\alpha + 3\beta) + 24\, m^2\Omega^2(\alpha + \beta) - 2m^4(2\alpha + \beta) \tag{3.17}$$

Equations (3.13) and (3.14) imply in an equation of state $p = \gamma\rho$ for the cosmic fluid, wherein γ is a constant. The Lichnerowicz condition, $0 \leq \gamma \leq 1$, will be ensured if

$$\frac{\Lambda}{\varkappa} \leq \frac{E_0^2}{2} - a^2 - 2m^4(2\alpha + \beta) - 4\alpha^4(\alpha + 3\beta) + 8\Omega^2 m^2(\alpha + \beta) \ , \tag{3.18}$$

which is consistent with (3,15) and (3.16).

In the integration of Eqs. (3.11)-(3.14) three cases arise , according as m^2 is >, < or = 0. In order to make easier the comparison of our results with those of the literature, we express our solutions in cylindrical coordinates. Of course, Gödel - type metric in cylindrical coordinates, i.e.,

$$ds^2 = [dt + H(r)d\Phi]^2 - D^2(r)d\Phi^2 - dr^2 - dz^2 \ , \tag{3.19}$$

is precisely of the form (1.5). Gödel universe corresponds to

$$H(r) = \frac{2\sqrt{2}}{m} \sinh^2(\frac{mr}{2}) \ ,$$

$$D(r) = \frac{\sinh(mr)}{m} \ , \tag{3.20}$$

where Φ is an angular coordinate. We also call attention to the fact that the theorem of section 2 is valid mutatis mutandis.

In case I

$$[16\Omega^4(\alpha+3\beta)+4m^4(2\alpha+\beta)-24m^2\Omega^2(\alpha+\beta)+ \frac{2\Omega^2}{\varkappa} - E_0^2 + a^2 = m^2 > 0]$$

we obtain

$$ds^2 = [dt + \frac{4\Omega}{m^2} \sinh^2 \left(\frac{mr}{2}\right) d\Phi]^2 - \frac{1}{m^2} \sinh^2(mr)d\Phi^2 - dr^2 - dz^2. \tag{3.21}$$

Here Φ is to be regarded as an angular coordinate. In fact, Eqs. (3.21) satisfy Maitra's conditions for regularity near the origin [17], i.e.,

$$H = r^2 \times \text{const}, \ D = r.$$

We also have that the relation

$$\frac{\Omega^2}{\varkappa} > \frac{E_0^2}{4} - 8\Omega^4(\alpha+3\beta) + 10\Omega^2 m^2(\alpha+\beta) - m^4(2\alpha+\beta) \tag{3.22}$$

holds.

Case II

$$[16\Omega^4(\alpha+3\beta) + 4n^4(2\alpha+\beta) + 24n^2\Omega^2(\alpha+\beta) + \frac{2\Omega^2}{\varkappa} - E_0^2 + a^2$$

$$= - n^2 < 0, \ m^2 \equiv - n^2 < 0]$$

corresponds to the following metric:

$$ds^2 = [dt + \frac{4\Omega}{n^2} \sin^2 \left(\frac{nr}{2}\right) d\Phi]^2 - \frac{\sin^2 nr}{n^2} d\Phi^2 - dr^2 - dz^2. \tag{3.23}$$

The relation

$$E_0^2 > 2a^2 + 8n^2\Omega^2(\alpha+\beta) + 4n^4(2\alpha+\beta) \tag{3.24}$$

holds. Eq. (3.23) is an analytical extension of Eq. (3.21) with $m \to in$. We remark that our coordinates are true cylindrical coordinates, i.e., they satisfy Maitra's conditions.

The remaining case, $m^2 = 0$, may be considered as a limit of the first ($m^2 \to 0$) and the second ($n^2 \to 0$) cases , respectively. The metric is given by

$$ds^2 = [dt + \Omega r^2 d\Phi] - r^2 d\Phi^2 - dr^2 - dz^2. \tag{3.25}$$

In this case the following relation holds

$$\frac{4\Omega^2}{\kappa} + 32\Omega^4(\alpha+\beta) \geqq E_0^2 = \frac{2\Omega^2}{\kappa} + a^2 + 16\Omega^4(\alpha+3\beta) \geqq 2a^2. \tag{3.26}$$

We have thus succeeded in deriving the most general higher-derivative solution concerning ST-homogeneous Gödel-type universes. As we have antecipated, our solutions are such that we can recover from them all known solutions of Einstein's equations concerning such geometries. Indeed, as $\alpha,\beta \to 0$, we obtain Rebouças

and Tiomno solution[16], which includes all known solutions of Einstein's equations related to these geometries.[For instance, when $\alpha,\beta,E_0,a \to 0$, we get Gödel solution[14] with $m^2 = 2\Omega^2$(cf.Eq.(3.20)). If $\alpha,\beta,a,m \to 0$ we recover Som-Raychaudhuri metric[18]. Banerjee--Banerji[19] as well as Rebouças[20] solutions are obtained when $\alpha,\beta,a \to 0$, noting that the first one concerns to a charged fluid and thus the electromagnetic field is different from that of the second one, but both have the same $T^{(EM)}_{AB}$, and so on[21-24].]We also remark that Riemannian Gödel-type ST-homogeneous metrics with the same value of m^2 and Ω are isometric[16].

We have analysed so far the fourth-order gravity solutions from a classical point of view. In this sense, the parameters α and β are quite arbitrary. However, in the framework of quantum field theory, the situation is rather different. In fact, the higher-derivative theory contains two mass scales[12-14,25], associated with the spin-0 and spin-2 particles present in the linearized theory. They are given, respectively, by

$$m_0^2 = 1/2\kappa(3\alpha+\beta), \tag{3.27}$$

and

$$m_2^2 = -1/\kappa\beta\ . \tag{3.28}$$

The spin-0 particle has significance even in the nonlinear sector[26].

Thus, nontachyionic spin-0 and spin-2 particles, require $(3\alpha+\beta)$ to be positive and β to be negative, respectively. Consequently, these restrictions on the parameters α and β must be included in our solutions.

4. ROTATING GÖDEL-TYPE UNIVERSE WITHOUT VIOLATION OF CAUSALITY IN HIGHER-DERIVATIVE GRAVITY

It is interesting to consider the question of closed timelike lines in our solutions. To accomplish this we write Eq. (2.38) in the form

$$ds^2 = dt^2 + 2Hd\phi dt - Ld\phi^2 - dr^2 - dz^2, \tag{4.1}$$

where

$$L(r) = D^2 - H^2 . \tag{4.2}$$

Clearly, if $L(r)$ becomes negative at $r_1 < r < r_2$, then the curve defined by r,t,z = const is a closed timelike trajectory. The existence of such curves poses a difficult problem related to the possibility of violation of the well-established causality principle.

In our case, when $m^2 > 0$, Eq. (3.21) leads to

$$L(r) = \frac{4}{m^2}\sinh^2\left(\frac{mr}{2}\right)\left[1 - \left(\frac{4\Omega^2}{m^2} - 1\right)\sinh^2\left(\frac{mr}{2}\right)\right] , \tag{4.3}$$

Consequently, unless

$$m^2 \geqq 4\Omega^2, \tag{4.4}$$

$L(r)$ will become negative for

$$\sinh^2\left(\frac{mr_1}{2}\right) > 1\Big/\left(\frac{4\Omega^2}{m^2} - 1\right) . \tag{4.5}$$

Thus, the limiting case in which the noncausal region will disappear corresponds to $m^2 = 4\Omega^2$. On the other hand, a straightforward calculation gives the following relation in the case of our solutions with $m^2 > 0$:

$$\frac{4\Omega^2}{\varkappa} \geqq \frac{m^2}{\varkappa} - 32\Omega^4(\alpha+3\beta) + 40m^2\Omega^2(\alpha+\beta) - 4m^4(2\alpha+\beta) . \tag{4.6}$$

Undoubtedly, the solution $m^2 = 4\Omega^2$ is compatible with the preceding inequality. It follows then from (3.14) and (3.16) that

$$\frac{2\Omega^2}{\varkappa} = 16(3\alpha+\beta)\Omega^4 - E_0^2 + a^2 , \tag{4.7}$$

$$-16(3\alpha+\beta)\Omega^4 + \frac{2\Omega^2}{\varkappa} - a^2 \geqq 0 . \tag{4.8}$$

Consequently,

$$E_0^2 = 0,\ \frac{2\Omega^2}{\varkappa} = a^2 + 16(3\alpha+\beta)\Omega^4. \tag{4.9}$$

Now, from Eq. (3.17) we get

$$\frac{\Lambda}{\varkappa} = 4(3\alpha+\beta)\Omega^4 - \frac{2\Omega^2}{\varkappa} , \tag{4.10}$$

and from Eqs. (3.12) and (3.13)

$$\rho = p = 0 \,. \tag{4.11}$$

Admitting that $16\varkappa^2 a^2(3\alpha+\beta) < 1$ and taking into account that $(3\alpha+\beta)$ must be positive in order to avoid the tachyonic spin-0 particle, we obtain from Eq. (4.9) the following values concerning Ω^2:

$$\Omega^2_{(c)} = [1 - \sqrt{1 - 16\varkappa^2 a^2(3\alpha+\beta)}\,]/16(3\alpha+\beta)\varkappa \,, \tag{4.12}$$

$$\Omega^2_{(q)} = [1 + \sqrt{1 - 16\varkappa^2 a^2(3\alpha+\beta)}\,]/16(3\alpha+\beta)\varkappa \,. \tag{4.13}$$

When $(3\alpha+\beta) \to 0$, $\Omega^2_{(c)} \to \frac{a^2\varkappa}{2}$, and we recover Rebouças and Tiomno solution[16], which is the only known exact Gödel-type solution of Einstein's equations describing a completely causal space-time homogeneous rotating universe.

We have thus succeeded in finding two completely causal rotating solutions.We should like to mention that the solution concerning $\Omega^2_{(q)}$ has no classical analogous, and it is, as far as we know, the first known exact solution of higher-derivative gravity field equations with this characteristc. "Classical" here means "from the point of view of general relativity". On the other hand, it is not difficult to show that in case $m \leqq 0$ we can not have completely causal solutions.

Last but not least, it is interesting to question if the causal pathologies of these universes can be avoided in the absence of the scalar field. The answer is yes. Indeed, our previous results provide us with the completely causal rotating solution, in case $a^2 = 0$ and $m^2 > 0$:

$$\Omega^2 = \frac{1}{8(3\alpha+\beta)\varkappa} = \frac{m^2}{4} \,, \quad \Lambda = -\frac{3}{2}\,\Omega^2 \,, \quad \rho = p = 0 \,. \tag{4.14}$$

We point out that the above solution has no similar in the framework of general relativity.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge financial support from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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