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ON A PHASE TRANSITION OF A KOSTERLITZ-THOULESS-TYPE IN THE d=4, U(1)-LATTICE GAUGE THEORY

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ABSTRACT

We show that the d=4, U(1)-lattice gauge theory with the Villain action may be represented as a locally neutral gas of topological (plaquette) charges which interact via a logarithmically confining potential. Using this representation we then perform a renormalization group analysis as to show the existence of a phase transition of the Kosterlitz-Thouless-type. To that extent we present an improved hierarchical version of the model which displays (unlike the usual Migdal-Kadanoff approach) a stable line of gaussian fixed points at low temperatures, which should correspond to the usual deconfining region of these systems.

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The existence of a deconfining phase transition in the d=4 U(1)-lattice gauge theory was first proposed by Wilson [1] and rigorously shown by Guth [2], Fröhlich and Spencer [3], and C. King [4]. Concerning the order of the transition there is however a standing controversy [5,6]. In this letter we first show, using duality and solving some trivial cohomological equation, that the model can be written, at least if we use Villain action, as a gas of topological charges satisfying a local neutrality condition and interacting via a logarithmically growing potential. This makes the system look very much like the 2-dimensional (neutral) Coulomb gas. We then present an improved hierarchical version of the model where it is possible to exploit these features to show the existence of a stable line of gaussian fixed points for $0 < \beta_c < \beta < \infty$. It should be remarked that the standard Migdal-Kadanoff [7,8] recursion formulae have no stable fixed point other than the T=∞ one [9,10]. To avoid this difficulty a modified hierarchical model was developed in [11] in the study of the 2-d Coulomb gas. The methods used here are a natural extension of those in [11] for the problem at hand.

For simplicity we discuss only the Villain form of the action; we believe, though that the physical picture underlying our results remains true for the Wilson action. Under a duality transformation (see for instance [3], whose notation we follow) the partition function can be written as:

$$Z = \sum_{n:\delta n=0} e^{-\frac{1}{2\beta}(dn,dn)} \tag{1}$$

where the summation is made over all integer valued 1-forms, n, on $\mathbb{Z}^4$ satisfying the gauge fixing condition $\delta n = 0$.

Our notation is as follows. For a k-form, α, we define:

$$d\alpha(C_{k+1}) = \sum_{C_k \subset \partial C_{k+1}} \alpha(C_k) \tag{2}$$

where C_k are the oriented unit k-cells in $\mathbb{Z}^4$ ($k = 1$ links, $k = 2$ plaquettes, ...) and ∂C_k are the boundaries of C_k; and

$$\delta\alpha(C_{k-1}) = \sum_{C_k : \partial C_k \supset C_{k-1}} \alpha(C_k) \tag{3}$$

The d and δ operators are adjoint to each other in the scalar product:

$$(\alpha,\beta) = \sum_{C_k} \alpha(C_k)\beta(C_k) \tag{4}$$

i.e.:

$$(\alpha, d\gamma) = (\delta\alpha, \gamma) \tag{5}$$

where α, β are k-forms and γ is a (k-1)-form.

Notice that the lattice laplacean operator (second difference operator) Δ, satisfies:

$$-\Delta = d\delta + \delta d \tag{6}$$

Now, the gauge fixing condition $\delta n = 0$ can be integrated, i.e., there exists (by the Poincaré Lemma) a 2-form m (plaquette variable) such that:

$$n = \delta m \tag{7a}$$

The choice m is made unique by further requiring:

$$dm = 0 \tag{7b}$$

In terms of the m variables the partition function reads:

$$Z = \sum_{m:dm=0} e^{-\frac{1}{2\beta}(-\Delta m,-\Delta m)} \tag{8}$$

where used has been made of (6) and (7b).

The physical picture associated to the m-variables is better appreciated if we perform a sine-Gordon transformation [2,3], after which, (8) reads as:

$$Z = \sum_{q:dq=0} e^{-\frac{\beta}{2}(q,(-\Delta)^{-2}q)} \tag{9}$$

where the summation is performed over all integer valued 2-forms (plaquette charges!) satisfying the local neutrality condition $dq = 0$.

The relevance of expressing the system in terms of the q charges lies in that, in 4-dimension, the asymptotic behavior of their interaction is given by:

$$(-\Delta)^{-2}(x,y) \sim \frac{1}{(2\pi)^d}\int d^4p\, \frac{e^{ip(x-y)}}{p^4} \sim \frac{-1}{8\pi^2} \ln|x-y| \tag{10}$$

and therefore the m-fields are dimensionless as are the fundamental fields in the $d=2$ sine-Gordon field theory. We are therefore in a position to repeat the analysis in [11] for the hierarchical version of the model, which we now describe.

At each lattice site $x = (x_1,x_2,x_3,x_4) \in \mathbb{Z}^4$ we introduce a hypercube with faces parallel to the coordinate axis with a vertex at x and edge $\frac{1}{2}$, $\{y : x_i < y_i < x_i + \frac{1}{2}$, $i = 1, \dots, 4\}$. Notice that the hypercubes h_x and $h_{x'}$ do not intersect if $x \neq x'$. The (oriented) plaquettes of the hypercubes at x will be denoted by $p^i_{\mu\nu}(x)$ with $\mu,\nu = 1,\dots,4$, $\mu \neq \nu$, $p^i_{\mu\nu}(x) = -p^i_{\nu\mu}(x)$ and $i = 1, \dots, 4$. The index $\mu\nu$ labels the "plane" of the plaquette and i numbers, with some

prescription, different parallel plaquettes, in such a way that $p^i_{\mu\nu}(x)$ when translated by $a \in \mathbb{Z}^4$ is transformed into $p^i_{\mu\nu}(x+a)$.

A configuration of our system is determined by the real-valued 2-form m, which assigns to the plaquette $p^i_{\mu\nu}(x)$ the $m^i_{\mu\nu}(x)$, with the condition $m^i_{\mu\nu}(x) = -m^i_{\nu\mu}(x)$. The "neutrality" condition $dm = 0$ reads, for each $\frac{1}{2}$ unit-cube C inside the hypercube at x,

$$\sum_{p^i_{\mu\nu} \subset \partial C} m^i_{\mu\nu}(x) = 0 \tag{11}$$

where the summation is taken over all plaquettes $p^i_{\mu\nu}$ which are faces of C.

Now, following the ideas of Gawedzki and Kupiainen [12,13] as adapted in [11] we introduce the hierarchical correlation function for the exponential fields $\exp i(\alpha,m) = \exp\left\{i \sum_{\substack{\mu\nu,j\\ x}} \alpha^i_{\mu\nu}(x)\, m^j_{\mu\nu}(x)\right\}$, for an arbitrary 2-form α:

$$\left\langle e^{i(\alpha,m)}\right\rangle = \int d\mu_G(m)\, e^{i(\alpha,m)} = \begin{cases} e^{-\frac{\beta}{8\pi^2}\sum_{x,y}\sum_{\mu\nu,j} \alpha^j_{\mu\nu}(x)\, N_L(x,y)\, \alpha^j_{\mu\nu}(y)} & \text{if } d\alpha = 0 \\ 0 & \text{if } d\alpha \neq 0 \end{cases} \tag{12}$$

The measure $d\mu_C(m)$ is formally given by

$$\exp\left\{-\frac{1}{2}\sum_{x,y}\sum_{\mu\nu,i} m^i_{\mu\nu}(x)C^{-1}(x,y)m^i_{\mu\nu}(y)\right\} \text{ where } C(x,y) = -\frac{1}{8\pi^2}N_L(x,y)\ \ln L$$

plays the role of a Green's function for the "hierarchical $(-\Delta)^2$ ". Here $L > 1$ is an integer representing a scale parameter in the model and $N_L(x,y)$, the "hierarchical distance" between x and y, is the smallest positive integer N such that $[L^{-N}x] = [L^{-N}y]$, ($[Z]$ denotes the vector formed with the integer part of the components of $Z \in \mathbb{R}^4$), and so $N_L(0,0) = 1$.

As in [11,12,13] we check that the field $m^i_{\mu\nu}(x)$ admits the orthogonal decomposition:

$$m^i_{\mu\nu}(x) = M^i_{\mu\nu}([L^{-1}x]) + \xi_{\mu\nu}(L[L^{-1}x]) \tag{13}$$

where: a) the "block spin" variables $M^i_{\mu\nu}$ are gaussian with correlation functions determined by:

$$\left\langle e^{i(\alpha,M)}\right\rangle = \left\langle e^{i(\alpha,m)}\right\rangle \tag{14}$$

and b) the "fluctuation" fields, $\xi^i_{\mu\nu}$, are also gaussian and determined by:

$$\left\langle e^{i(\alpha,\xi)}\right\rangle = \int d\nu(\xi)\, e^{i(\alpha\,\xi)} = \begin{cases} \lfloor^{-\frac{\beta}{16\pi^2}(\alpha,\alpha)} & , d\alpha = 0 \\ 0 & , d\alpha \neq 0 \end{cases} \tag{15}$$

Notice that (15) implies that $\xi^i_{\mu\nu}(x)$ and $\xi^i_{\mu\nu}(y)$ are independent for $x \neq y$ and that the contribution of $\xi^i_{\mu\nu}(L[\frac{x}{L}])$ appearing in (13) is constant over x varying in a given block of side L.

The Villain model would be obtained by requiring the m variables to be integer. We will, as usual, consider sine-Gordon type of local perturbation of the gaussian measure $d\mu_C(m)$ by a multiplicative factor:

$$\Lambda(m) = \prod_x \lambda(\{m^i_{\mu\nu}(x)\}) \tag{16}$$

That is, our perturbation is local only with respect to the x variables with λ depending on all $m^i_{\mu\nu}$ variables inside a given hypercube. λ is a periodic in each $m^i_{\mu\nu}$ separately and we may expand in Fourier series:

$$\lambda(\{m^i_{\mu\nu}\}) = \sum_{\underset{\sim}{q}}{}' z_{\underset{\sim}{q}}\ e^{i\underset{\sim}{q}\cdot\underset{\sim}{m}} \tag{17a}$$

$$\underset{\sim}{q} = \{q^i_{\mu\nu}\} \quad , \quad \underset{\sim}{q}\cdot\underset{\sim}{m} = \sum_{\mu\nu,i} q^i_{\mu\nu} m^i_{\mu\nu} \tag{17b}$$

where $q^i_{\mu\nu}$ are integer and, for simplicity, the summation is taken only over "charge" configurations $\underset{\sim}{q}$: $dq = 0$; for convergence we require $z_{\underset{\sim}{q}} \in \ell^1$, i.e.:

$$\sum_{\underset{\sim}{q}} |z_{\underset{\sim}{q}}| < \infty \tag{18}$$

Because of the hierarchical scheme, the renormalization group transformation:

$$\lambda'(\underset{\sim}{M}) = \int d\nu(\underset{\sim}{\xi})\,\lambda(\underset{\sim}{M}+\underset{\sim}{\xi}) \tag{19a}$$

acts locally, and $\lambda' = \sum_{q}' z'_{q}\, e^{iq\cdot m}$ with:

$$z'_{q} = L^{-\frac{\beta}{16\pi^2} q\cdot q} \sum_{\substack{q_1,\ldots,q_{L^4} \\ \sum_{j=1}^{L^4} q_j = q}} z_{q_1}\cdots z_{q_{L^4}} \tag{19b}$$

Notice that the constrant $dq = 0$ is preserved under the Renormalization Group Transformation! It is easy to analyse the stability of the gaussian fixed point $\lambda_0 \equiv 1$, i.e., $(z_0)_q = \delta_{q,0} = \prod_{\substack{\mu\nu \\ i}} \delta_{q^i_{\mu\nu},0}$ (Kronecker δ). Linearizing the transformation $z \to z'$ around z_0, we obtain the linear operator A given by the diagonal matrix:

$$A_{qq'} = \begin{cases} \delta_{q,q'}\, L^{4-\frac{\beta}{16\pi^2} q\cdot q} & q \neq 0 \\ 0 & q = 0 \end{cases} \tag{20}$$

and therefore the fixed point is stable if and only if $\beta > \beta_c = 64\pi^2/q_0\cdot q_0$, where q_0 is the non-zero configuration of charges satisfying the constraint $dq = 0$ which minimize $q\cdot q$. In fact it is not difficult to show that the fixed point is globally attractive, i.e., $\lim_{n\to\infty} z^{(n)} = z_0$ if $\beta > \beta_c$. Full mathematical details will be presented elsewhere [14].

It is also possible, following the methods of [11], to introduce models corresponding to Z_N-lattice gauge theories and to show the existence of an intermediate phase associated to an interval $0 < \beta_c^{(1)} < \beta < \beta_c^{(2)} < \infty$ where the gaussian fixed point is stable for N sufficiently large.

If we exclude the possibility of many phase transition in the $d = 4$, $U(1)$ lattice gauge theory, the transition here described should be the usual deconfining phase transition observed in these models and therefore further analysis of our model should be helpful in understanding the open question concerning its order.

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