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M.C. Nemes and **A.F.R. de Toledo Piza**
Instituto de Física, Universidade de São Paulo

J. da Providência
Laboratório de Física, Universidade de Coimbra
Coimbra, Portugal

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M.C. Nemes and **A.F.R. de Toledo Piza**
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brazil

J. da Providência
Laboratório de Física, Universidade de Coimbra
Coimbra, Portugal

ABSTRACT

We construct an orthogonal and complete set of solutions (in the sense of the Random Phase Approximation) to the linearized Vlasov equation for an infinite, homogeneous many-fermion system. Classical analogues of the RPA quasi-boson operators are obtained and compared to their quantum mechanical counterparts written in a classical phase-space representation. The initial value problem of determining the subsequent time evolution of a given disturbance of the (stable) equilibrium distribution is discussed.

I. INTRODUCTION

This paper reports on an investigation of the small amplitude stationary solutions of the classical Vlasov equation for an infinite system of particles obeying Fermi-Dirac statistics. We also indicate how the results can be extended to the richer framework provided by the full Wigner transform of the selfconsistent mean-field approximation(1). As is well known, the Vlasov equation can in fact be understood as the lowest order approximation (in powers of $\hbar$) to the quantum mean-field dynamics or, equivalently, as the long wavelength limit familiar from the Landau theory of Fermi liquids(2). Our interest in such matters stems from the fact that such stationary solutions appear, in particular, as nuclear matter counterparts of the familiar discrete vibrational excitation modes of finite nuclei as described in terms of the random phase approximation (RPA). This fact has been explored e.g. by Jennings and Jackson in a study of nuclear vibrations and their implications for the nuclear residual interaction(3). Besides this, a careful study of the structure of the stationary solutions can shed light on the basically dispersive damping mechanism known as Landau damping, also not just in the case of the infinite system, but for finite, collective many fermion systems such as the atomic nuclei as well.

Stationary wave solutions of the Vlasov equation were studied long time ago by van Kampen[4] in the context of plasma physics. We show that the modes involved in his method of solution coincide, except for normalization, with infinite system RPA modes. Taking advantge of this fact, we shall follow his work closely in order to treat the completeness problem for the set of stationary solutions in a constructive fashion. In particular, all results can be worked out analytically in the case of the degenerate Fermi gas, for which the equilibrium distribution reduces to a momentum step function. Though not analytical, due to the occurrence of integrals involving finite temperature Fermi distributions, the finite temperature case is in fact conceptually simpler, in the sense that, unlike in the degenerate case, no special treatment is required for zero-sound modes. The treatment given by van Kampen can actually be directly applied to this case. We therefore concentrate our discussion mainly on the degenerate Fermi gas.

In what follows, sections II, III and IV deal with the Vlasov equation. In the first of these the stationary modes, given in terms of associated canonical generators, are obtained, and in the second their ortogonality and completeness properties are explicitly worked out in complete analogy with familiar RPA results[5]. Section IV is devoted to the initial value problem in which the initial distortion of the Fermi system described by the Vlasov equation is specified. This is solved following closely the procedure of ref. (4) for the

degenerate case, in which the role of the zero sound modes is discussed. Finally, section V deals with the extension of these results for the case of the fully quantal, linearized mean-field dynamics and section VI is devoted to final comments and conclusions.

II. STATIONARY SMALL AMPLITUDE SOLUTIONS OF THE VLASOV EQUATION AND THEIR GENERATORS

We consider initially the mean-field dynamics of the phase-space distribution function $f(\vec{x},\vec{p},t)$ as given by the Vlasov equation

$$\frac{\partial f(\vec{x},\vec{p},t)}{\partial t} + \left\{ f(\vec{x},\vec{p},t),\, h(\vec{x},\vec{p}) \right\} = 0, \tag{II.1}$$

where the second term stands for the usual Poisson bracket

$$\{f, h\} = \frac{\partial f}{\partial \vec{x}} \cdot \frac{\partial h}{\partial \vec{p}} - \frac{\partial f}{\partial \vec{p}} \cdot \frac{\partial h}{\partial \vec{x}},$$

and assume that the potential energy part of h results from density averaging a two- and three-body momentum independent effective interaction, i.e.

$$h(\vec{x},\vec{p}) = h[f] = \frac{p^2}{2m} + \int v(\vec{x},\vec{x}')\, f(\vec{x}',\vec{p}',t)\, \frac{d\vec{x}'d\vec{p}'}{(2\pi\hbar)^3}$$

$$+\iint w(\vec{x},\vec{x}',\vec{x}'')\, f(\vec{x}',\vec{p}',t)\, f(\vec{x}'',\vec{p}'',t)\, \frac{d\vec{x}'d\vec{p}'}{(2\pi\hbar)^3}\, \frac{d\vec{x}''d\vec{p}''}{(2\pi\hbar)^3}\,. \qquad \text{(II.2)}$$

For simplicity, we shall also omit explicit reference to intrinsic degrees of freedom such as spin and isospin.

Even though an arbitrary function of h (subject to an appropriate selfconsistency condition) is a stationary solution of eq. (II.1), we restrict ourselves to translationally invariant stationary solutions of the form

$$f_0(\vec{p}) = \left[1 + \exp\{(h-\mu)/k_B T\}\right]^{-1}, \qquad \text{(II.3)}$$

where k_B is the Boltzmann constant, the chemical potential μ being fixed by

$$\int \frac{d\vec{p}}{(2\pi\hbar)^3}\, f_0(\vec{p}) = \rho_0\,,$$

this being the equilibrium density. In general we have $0 \leq f(\vec{x},\vec{p},t) \leq 1$, and the stationary solution (II.3) at $T = 0$ satisfies the pure state condition $f^2 = f$. It is worth noting at this point that the dynamical stability condition for f_0

$$E[f_0] \leq E[f]\,,$$

where $E[f]$ is the total mean energy associated with the distribution function f, and where f is canonically related to f_0 (see eq. (II.4) below) is fulfilled by eq. (II.3).

In order do deal with small amplitude fluctuations of the distribution function about f_0, we distort the equilibrium distribution through infinitesimal canonical transformations generated by (real) classical observables $S(\vec{x},\vec{p},t)$:

$$f(\vec{x},\vec{p},t) = f_0(\vec{p}) + f_1(\vec{x},\vec{p},t),$$

$$f_1(\vec{x},\vec{p},t) = \{f_0, S\}\,. \qquad \text{(II.4)}$$

When dealing with stationary excitations of infinite matter with translational invariance it is actually more convenient to Fourier analyse both the space and time dependence of the generators, and to introduce sharp frequency components $S_\omega(\vec{k},\vec{p})$ with good wavenumber $\vec{k}$ through

$$S(\vec{x},\vec{p},t) = \iint e^{i(\vec{k}\cdot\vec{x}-\omega t)}\, S_\omega(\vec{k},\vec{p})\, d\vec{k}\, d\omega \qquad \text{(II.5)}$$

Note that, unlike $S(\vec{x},\vec{p},t)$, the $S_\omega(\vec{k},\vec{p})$ are in general complex quantities. They are required to satisfy the reality condition

$$S^*_{-\omega}(-\vec{k},\vec{p}) = S_\omega(\vec{k},\vec{p}) \qquad \text{(II.6)}$$

so that, in fact, the Fourier component $S_\omega(\vec{k},\vec{p})$ corresponds to (e.g.) the positive frequency part of a real generator which must include also a negative frequency counterpart. These correspond thus to the creation and annihilation parts of the hermitean generator associated with particle-hole fluctuations of the stable ground state of standard time-dependent derivations of the RPA.

Direct substitution of eqs. (II.4) and (II.5) in eq. (II.1) yields in a straightforward way

$$\left\{\left(\frac{\vec{p}\cdot\vec{k}}{m}-\omega\right)S_\omega(\vec{k},\vec{p})-g\int\vec{k}\cdot\frac{\partial f_0}{\partial\vec{p}'}S_\omega(\vec{k},\vec{p}')\frac{d\vec{p}'}{p_F^3}\right\}\vec{k}\cdot\frac{\partial f_0}{\partial\vec{p}}=0. \tag{II.7}$$

Having in mind situations in which the residual interaction between fermions is of short range, together with the long wavelength limit, we have assumed contact two- and three-body potentials in eq. (II.2):

$$v = a\,\delta(\vec{x}-\vec{x}') \quad ; \quad w = b\,\delta(\vec{x}-\vec{x}')\delta(\vec{x}-\vec{x}'').$$

Furthermore, we introduced for convenience the dimensional constant $p_F^{-3} = (2m\mu)^{-3/2}$ so that g is expressed in energy units as

$$g = \frac{p_F^3}{(2\pi\hbar)^3}\,(a+2b\rho_0).$$

Eq. (II.7) shows that the generators $S_\omega(\vec{k},\vec{p})$ are only determined for values of $\vec{p}$ such that $\vec{k}\cdot\frac{\partial f_0}{\partial\vec{p}}$ does not vanish, and that $\vec{k}$ enters as a parameter. This implies, in particular, $p=p_F$ for the degenerate Fermi gas. Subject to this condition, the dimensionless solutions of eq. (II.7) have been written by van Kampen[4], in distribution form, as

$$S_\omega(\vec{k},\vec{p}) = \frac{g}{\hbar}\left[\frac{\mathcal{P}}{\frac{\vec{p}\cdot\vec{k}}{m}-\omega}+\lambda(\omega,\vec{k})\,\delta\!\left(\frac{\vec{p}\cdot\vec{k}}{m}-\omega\right)\right], \tag{II.8}$$

the function $\lambda(\omega,k)$ being subject to the subsidiary condition

$$\int\hbar\vec{k}\cdot\frac{\partial f_0}{\partial\vec{p}'}\,S_\omega(\vec{k},\vec{p}')\frac{d\vec{p}'}{p_F^3} = 1, \tag{II.9}$$

which also defines a particular normalization of the solutions. This condition, given the form (II.8) of $S_\omega(\vec{k},\vec{p})$, actually determines $\lambda(\omega,\vec{k})$ in all cases where the integral (II.9) over the δ-function part of eq. (II.8) does not vanish. In such cases, stationary solutions exist for a continuum of values of $\vec{k}$ and ω. When this integral vanishes, on the other hand, eq. (II.9) appears as a dispersion equation connecting $\vec{k}$ and ω.

If we use eq. (II.3), with $T\neq 0$, for f_0, eq. (II.9) is such that the integral over the δ-function part of $S_\omega(\vec{k},\vec{p})$ never vanishes. The degenerate case $(T=0)$, on the other hand, is special in the sense that the subsidiary condition gives

$$2 + s \ln\left|\frac{1-s}{1+s}\right| + \lambda(\omega,\vec{k})\, s\, \Theta(1-|s|) = -\frac{p_F^2}{2\pi m g} \qquad \text{(II.10)}$$

where $s = \frac{m\omega}{kp_F}$ is the usual Landau variable. When $|s| > 1$ this becomes the familiar Landau dispersion equation

$$2 + s \ln \frac{1-s}{1+s} = -\frac{p_F^2}{2\pi m g} \qquad \text{(II.11)}$$

which will have solutions $|s_L| > 1$ (zero sound) for $g > 0$ (repulsive residual interaction). Eq. (II.11) will also have solutions for $g < 0$ (attractive residual interaction), but in this case $|s| < 1$, thus implying $\lambda = 0$.

III. SCALAR PRODUCT, ORTHOGONALITY AND COMPLETENESS RELATIONS. THE DEGENERATE CASE

It is useful at this point to explore the correspondence of the $S_\omega(\vec{k},\vec{p})$ $(\omega > 0)$ with the creation part of the RPA modes

$$B_n^+ = \sum_{p\ell} \left[X_{p\ell}^{(n)} a_p^+ a_\ell + Y_{p\ell}^{(n)} a_p a_\ell^+ \right].$$

In the latter case, orthogonality relations can be proved in terms of the scalar product(6)

$$(B_m, B_n) = \mathrm{Tr}\left\{ [\rho_0, B_m^+]\, B_n \right\} = \mathrm{Tr}\left\{ \rho_0 [B_m^+, B_n] \right\}. \qquad \text{(III.1)}$$

We introduce, therefore, for the problem in hand a scalar product which is the classical counterpart of eq. (III.1), namely

$$(S_{\vec{k}\omega}, S_{\vec{k}'\omega'}) = \frac{\hbar}{i} \int f_0(\vec{p}) \left\{ S^*_{\vec{k}\omega}, S_{\vec{k}'\omega'} \right\} \frac{d\vec{x}\, d\vec{p}}{(2\pi\hbar)^3} \qquad \text{(III.2)}$$

where the spatial dependence of the generators is $\exp(i\vec{k}.\vec{x})$:

$$S_{\vec{k}\omega}(\vec{x},\vec{p}) = e^{i\vec{k}.\vec{x}}\, S_\omega(\vec{k},\vec{p}).$$

Eq. (III.2) can be evaluated analytically in a straightforward fashion for the solutions (II.8) in the case of the degenerate Fermi gas. In the particular case $g > 0$, which includes zero-sound modes with $|s_z| > 1$, we find

$$(S_{\vec{k}\omega}, S_{\vec{k}'\omega'}) = N(s) \frac{\omega}{|\omega|} \delta(\vec{k}-\vec{k}')\, \delta(\omega-\omega') \qquad \text{(III.3a)}$$

$$(S_{\vec{k}\omega_z}, S_{\vec{k}'\omega'_z}) = N_z(s_z) \frac{\omega_z}{|\omega_z|} \delta(\vec{k}-\vec{k}')\, \delta_{\omega_z\omega'_z} \qquad \text{(III.3b)}$$

and

$$(S_{\vec{k}\omega}, S_{\vec{k}'\omega_z}) = 0 \qquad \text{(III.3c)}$$

where the subscript z denotes zero-sound values, and the normalization factors N and N_z are given by

$$N(s) = 2\pi m p_F |s| \frac{\hbar}{(2\pi\hbar)^3} \left(\frac{g}{\hbar}\right)^2 \lambda^2(\vec{k},\omega) \qquad \text{(III.4a)}$$

$$N_z(s_z) = \frac{2\pi m p_F}{|\omega_z|} \frac{\hbar}{(2\pi\hbar)^3} \left(\frac{g}{\hbar}\right)^2 \left[s_z \ln\left|\frac{1-s_z}{1+s_z}\right| + \frac{2 s_z^2}{s_z^2-1} \right], \qquad \text{(III.4b)}$$

Eqs. (III.3a) and (III.3b) show explicitly in this case that negative frequency generators have negative norm, a familiar result from the RPA which stems from the stability of the equilibrium mean-field distribution. They also show that zero frequency modes have zero norm.

In a similar way, a formal path towards a completeness relation for the solutions (II.8) can be found by exploring their correspondence with discrete RPA modes B_n^+, B_n. To this effect, on the basis of the completeness relation for normalized modes[7]

$$\sum_n (B_n^+)_{\alpha\beta} \frac{\omega_n}{|\omega_n|} [B_n, \rho_o]_{\alpha'\beta'} = \delta_{\alpha\alpha'} \delta_{\beta\beta'}$$

where α, β are single-particle labels, we are led to

$$\frac{\hbar}{i} \int \overline{S}_{\vec{k}\omega}(\vec{x},\vec{p}) \frac{\omega}{|\omega|} \left\{ \overline{S}^*_{\vec{k}\omega}(\vec{x}',\vec{p}'), f_o(\vec{p}') \right\} d\vec{k}\, d\omega = (2\pi\hbar)^3 \delta(\vec{x}-\vec{x}')\delta(\vec{p}-\vec{p}') \qquad \text{(III.6)}$$

where again the $\overline{S}_{\vec{k}\omega}$ are properly normalized. This expression can in fact be formally derived from the orthogonality relations (III.3) multiplying them on th left by $\overline{S}_{\vec{k}\omega} \frac{\omega}{|\omega|}$ and integrating

over $\vec{k}$ and ω. Due to the involved $\vec{k}$ and ω dependence of the normalization factors (III.4) it is however not possible to check explicitely eq. (III.6) even in the case of the degenerate Fermi gas.

Furthermore, there is an important remark to be made in connection with eq. (III.6), and which applies also to the corresponding equation for the usual, discrete RPA modes: as it is evident from their derivation, these formal relations must be understood as applying within the subspace spanned by the stationary modes themselves. This means arbitrary particle-hole modes in the case of the usual RPA and arbitrary acceptable infinitesimal generators in the context of the linearized Vlasov equation. In the degenerate case, in particular, acceptable generators are only given at the Fermi surface, $\vec{p} = p_F\hat{p}$ (see eq. (II.7)). This feature arises, formally, as an artifact of the classical approximation to the dynamics together with the particular form of the equilibrium distribution for the degenerate fermions. It can be understood and justified, physically, in terms of the long wavelength approximation, which requires infinitesimally small wavevectors $\vec{k}$. The typical magnitude of $\vec{k}$, which plays the role of a momentum transfer, measures in fact the breadth of the active momentum shell, which harbors the relevant possible particle-hole excitations, which for the degenerate system is centered at the Fermi momentum.

IV. VAN KAMPEN'S CONSTRUCTIVE APPROACH TO COMPLETENESS

The preceding results reveal explicitly the identity (except for normalization) of the van Kampen modes (II.8) to RPA modes of the classical, small amplitude dynamics of extended many-fermion systems. In particular, eqs. (III.2) and (III.6) constitute the main tools involved in applications of the RPA. As examples, we can use these relations to construct linear response functions as Green's functions of the dynamical equation (II.7) and construct sum rules associated with given transition operators to the stationary modes[6]. It is nevertheless instructive and also useful to implement the expansion procedure adopted by van Kampen[4] in connection with Fermi systems. As was mentioned in the introduction, this is entirely trivial for equilibrium distributions f_0 with finite T (see eq. (II.3)), which fulfill the assumption of being everywhere different from zero[4]. In what follows, we therefore restrict ourselves once again to a brief discussion of the interesting special case of degenerate Fermi systems with $g > 0$.

Following ref. (4), we consider the initial value problem for an initially given distortion of the degenerate equilibrium distribution. This is expressed in terms of an infinitesimal generator $G(\vec{x},\vec{p})$ specified at the surface of the Fermi sphere, $\vec{p} = p_F \hat{p}$. The problem consists then in

obtaining the expansion of G in terms of the stationary modes $S_\omega(\vec{k},\vec{p})$, eq. (II.8). There is of course no loss of generality in considering in detail just a single Fourier component $G(\vec{k},\vec{p})\, e^{i\vec{k}\cdot\vec{x}}$ of the given generator, for which the required expansion reads

$$G(\vec{k},\vec{p}) = \int ds\, G(\vec{k},s)\, S_\omega(\vec{k},\vec{p})\,, \qquad \vec{p} = p_F\,\hat{p}\,. \tag{IV.1}$$

Note that the wavevector $\vec{k}$ enters only as a fixed parameter, so that we are allowed to define the expansion coefficients $G(\vec{k},s)$ by using the Landau variable s directly as integration variable. For a repulsive residual interaction, $g > 0$, the integration domain extends therefore from -1 to $+1$, <u>appart from contributions of the zero-sound modes</u> which have $|s_z| > 1$. In what follows we shall work under the assumption that these particular contributions have already been extracted from $G(\vec{k},\vec{p})$, so that we are left just with the integral over the continuous range of values of s.

Substitution of eq. (II.8) in eq. (IV.1) yields

$$G(\vec{k},p_F\hat{p}) = \frac{mg}{\hbar k p_F}\left[\lambda\, G(\vec{k},\hat{p}\cdot\hat{k}) + \mathcal{P}\int ds\, \frac{G(\vec{k},s)}{\hat{p}\cdot\hat{k}-s}\,\Theta(1-|s|)\right] \tag{IV.2}$$

where λ is determined by the subsidiary condition (II.9) which reads (cf. eq. (II.10))

$$\lambda s\,\Theta(1-|s|) + \mathcal{P}\int dx\, \frac{x\,\Theta(1-|x|)}{x-s} = -\frac{p_F^2}{2\pi m g} \qquad \text{(IV.3)}$$

As in ref. (4), the principal value integrals in eqs. (IV.2) and (IV.3) are handled with the help of the relation

$$\mathcal{P}\int dx\, \frac{F(x)}{x-s} = i\pi\, F_*(s) \equiv i\pi\left[F_+(s) - F_-(s)\right] \qquad \text{(IV.4)}$$

where $F_\pm(s)$ are the inverse Fourier transforms of the positive and negative frequency parts of F, respectively, so that

$$F(s) = F_+(s) + F_-(s) \qquad \text{(IV.5)}$$

and, F being a real function (cf. eq. (II.6)),

$$F_+(s) = F_-(s)^*.$$

In particular, for $F(x) = x\,\Theta(1-|x|)$ we get

$$F_+(s) = \frac{s}{2}\,\Theta(1-|s|) + \frac{1}{2\pi i}\left(2 + s\ln\left|\frac{1-s}{1+s}\right|\right). \qquad \text{(IV.6)}$$

With these tools eqs. (IV.2) and (IV.3) can be formally solved by first eliminating λ between them, then using eqs. (IV.4) and (IV.5) and separately equating positive and negative frequency inverse transforms on both sides of the resulting

equation. This procedure yields

$$G_\pm(\vec{k},\hat{p}\cdot\hat{k}) = -\frac{\hbar k p_F}{2\pi m g}\, \frac{\left[G(\vec{k},p_F\hat{p})\,F(\hat{p}\cdot\hat{k})\right]_\pm}{\dfrac{p_F^2}{4\pi^2 m g} \pm i\,F_\pm(\hat{p}\cdot\hat{k})} \qquad \text{(IV.7)}$$

where F, F_+ are defined as in eq. (IV.6). This solution can in fact be sustained provided its denominator does not vanish in the expected analyticity domain of G_+. In the case $g > 0$ that concerns us here, dangerous points would occur where the real part of the denominator vanishes, its imaginary part being also zero, i.e., for $|s| > 1$:

$$\frac{p_F^2}{4\pi^2 m g} + \frac{1}{2\pi}\left(2 + s\ln\left|\frac{1-s}{1+s}\right|\right) = 0\,.$$

These dangerous points are therefore just at those values of the Landau parameter which correspond to the zero-sound modes, and are therefore eliminated by the assumed projection of these modes out from the initial condition generator $G(\vec{k},\vec{p})$. Eq. (IV.7), added to the appropriate zero-sound components, gives thus the solution to the problem of expanding that generator.

The time evolution of the given initial distortion of the equilibrium distribution can now be directly constructed by taking into account the exponential time-dependent factors of the stationary modes. Non-stationarity will result from phase-mixing phenomena, and will hinge basically on the frequency distribution inherent to the initial distortion. In particular, initial distortions associated solely to the zero-sound for $g > 0$ and with given wavenumber will have a sharp frequency distribution and show no damping in this case.

V. LINEARIZED QUANTUM MEAN-FIELD DYNAMICS

The Vlasov equation (II.1) can be seen as the lowest (zero) order truncation, in powers of $\hbar$, of the Wigner transform of the quantal time-dependent mean-field dynamics(1). Under the same general assumptions which led to eq. (II.7), but keeping all powers of $\hbar$ when taking the Wigner transform of the quantum commutator we obtain

$$\left\{\left(\frac{\vec{p}\cdot\vec{k}}{m}-\omega\right)S_\omega(\vec{k},\vec{p})-\frac{g}{\hbar}\int\left[f_0\left(\vec{p}\,'+\frac{\hbar\vec{k}}{2}\right)-f_0\left(\vec{p}\,'-\frac{\hbar\vec{k}}{2}\right)\right]S_\omega(\vec{k},\vec{p}\,')\frac{d\vec{p}\,'}{p_F^3}\right\}\times$$

$$\times\frac{1}{\hbar}\left[f_0\left(\vec{p}+\frac{\hbar\vec{k}}{2}\right)-f_0\left(\vec{p}-\frac{\hbar\vec{k}}{2}\right)\right]=0 \qquad \text{(V.1)}$$

which immediately reduces to the former equation in the long wavelength limit. In eq. (V.1) the transferred momentum $\vec{k}$

still enters as a parameter, but the shifted equilibrium distributions duly allow for an active momentum shell of finite breadth given in terms of $\vec{k}$ in the degenerate case. The explicit relation (II.4) between the generators S and the corresponding fluctuation densities f_1, on the other hand, now appear as

$$f_1(\vec{x},\vec{p},t)=\frac{1}{\hbar}\iint\left[f_0\left(\vec{p}+\frac{\hbar\vec{k}}{2}\right)-f_0\left(\vec{p}-\frac{\hbar\vec{k}}{2}\right)\right]S_\omega(\vec{k},\vec{p})\,e^{i(\vec{k}\cdot\vec{x}-\omega t)}\,d\vec{k}\,d\omega \qquad \text{(V.2)}$$

which is just the familiar quantum commutator relation cast in Wigner form.

The solutions $S_\omega(\vec{k},\vec{p})$ of eq. (V.1) are still written in the form (II.8). The subsidiary condition (II.9) now reads, however

$$\int\left[f_0\left(\vec{p}\,'+\frac{\hbar\vec{k}}{2}\right)-f_0\left(\vec{p}\,'-\frac{\hbar\vec{k}}{2}\right)\right]S_\omega(\vec{k},\vec{p}\,')\frac{d\vec{p}\,'}{p_F^3}=1\,. \qquad \text{(V.3)}$$

For the special case of the degenerate system, this integral can be explicitely worked out yielding (cf. ref. (8))

$$\left\{1-\frac{1}{2y}\left[1-\left(s-\frac{y}{2}\right)^2\right]\ln\left|\frac{1+(s-\frac{y}{2})}{1-(s-\frac{y}{2})}\right|+\frac{1}{2y}\left[1-\left(s+\frac{y}{2}\right)^2\right]\ln\left|\frac{1+(s+\frac{y}{2})}{1-(s+\frac{y}{2})}\right|\right\}$$

$$-\lambda I(y,s)=\frac{p_F^2}{2\pi m g} \qquad \text{(V.4)}$$

where s is the Landau parameter, $y = \frac{\hbar k}{p_F}$ and the integral $I(y,s)$ over the δ-function part of S_ω is

$$I(y,s) = \frac{1}{2y}\left[\max\left(s^2, 1+ys-\frac{y^2}{4}\right) - \max\left(s^2, 1-ys-\frac{y^2}{4}\right)\right] \tag{V.5}$$

which reduces to $s\Theta(1-|s|)$ when $y \to 0$. In general, it is an odd, continuous, piecewise smooth function of s for given y, as shown schematically in figs. V.1 and V.2. Now the y-dependence of $I(y,s)$ implies that the spectrum of stationary modes can no longer be determined independently of the momentum transfer $\vec{k}$, as in the classical case. In fact, the domain of the y,s plane where $I(y,s) \neq 0$ is shown for the degenerate fermion system as the hatched area of fig. V.3. This domain replaces, in the quantum case, the corresponding classical domain which is the $-1 < s < 1$ stripe parallel to the y axis, and pictures the spectrum of stationary modes, except for additional possible ones, in regions where $I = 0$, appearing as solutions of the dispersion equation which then results from eq. (V.3).

The overflowing of the band of stationary solutions beyond the classical stripe, reaching values of s and y for which the consistency condition (V.4) will require $\lambda = 0$ will, in fact, be responsible for situations in which Landau damped zero-sound modes (with $g > 0$) do occur, as found from a different analysis in ref. (1). The interpretation of the damping found in this case involves again nothing but the phase mixing of a superposition of stationary solutions with different frequencies, as stated in ref. (4).

V. CONCLUSIONS AND FURTHER REMARKS

We have shown that a set of stationary linear modes of excitation satisfying orthogonality and completeness relations of the form characteristic of the random phase approximation can be constructed for infinite, homogeneous many-fermion systems even when one considers just the classical (Vlasov) limit of the quantum mean-field dynamics. The spectrum of excitations in the case of the classical equation of motion differs from the quantum spectrum in that its dependence on the momentum of the excitation enters only through the Landau parameter $s = m\omega/kp_F$, while separate dependence on k occurs in the quantum case. The classical results agree with the quantum results in the long wavelength limit.

The stationary modes satisfying RPA-like orthogonality and completeness relations coincide, except for normalization, with modes considered long ago by van Kampen[4]. His approach to the completeness problem, consisting in obtaining a solution of the singular integral equation defining the expansion coefficients of a given initial disturbance of the equilibrium distribution, can also be implemented even for the degenerate

Fermi system, provided due treatment is given to possible contributions of zero-sound modes.

The time evolution of the assumed initial disturbance will then evolve subject to the phase-mixing processes inherent to its expansion in the stationary modes. Thus, frequency distributions which are not absolutely sharp will give rise to dispersive damping phenomena (Landau damping). Given the correspondence of the stationary modes with the usual RPA modes of finite nuclei, a clear analog can be uniquely identified, in the latter context, to the Landau damping: it consists of an external excitation process which produces, in the finite nuclear system, a doorway state which fits within the particle-hole space of RPA excitations, but which overlaps with several RPA modes having different frequencies. The damping resulting therefrom is of course just part of the spreading width of the particular doorway which was fed by the external excitation mechanism.

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FIGURE CAPTIONS

Fig. V.1 - Qualitative plot of the function $I(y,s)$ as a function of s, for $y = 0$ and for a typical value of $y < 2$.

Fig. V.2 - Same as Fig. V.1 for $y > 2$.

Fig. V.3 - Domain in the y-s plane where $I(y,s)$ is different from zero.

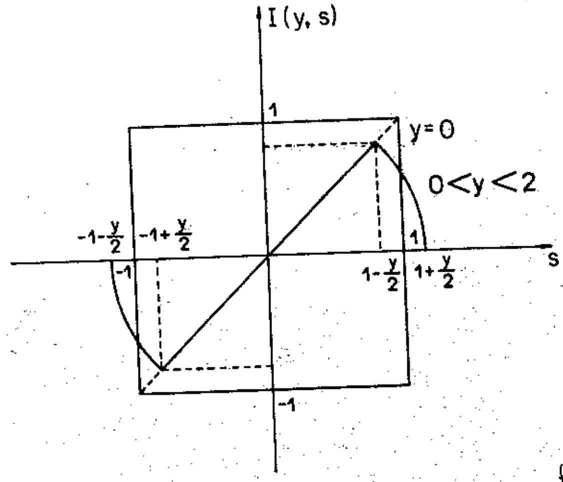

fig V.1

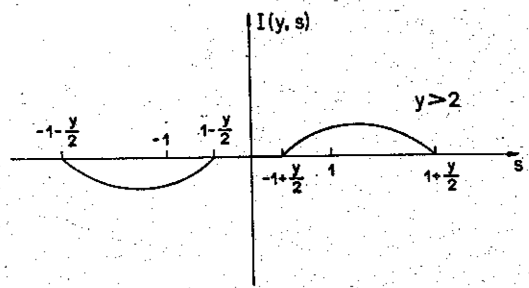

fig V.2

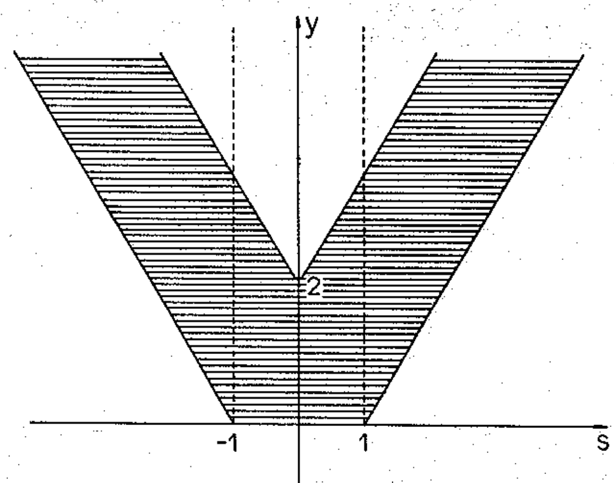

fig V-3