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ANTISCREENING EFFECTS OF CLASSICAL YANG-MILLS FIELDS

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ABSTRACT

We study a 2+1 dimensional Yang-Mills theory with a static color-charge density. Time independent solutions are found whose chromo-electric and chromo-magnetic fields fluctuate around constant values at large distances. The energy density characterizing these non-abelian configurations is higher than the one corresponding to the Coulomb solution, being constant in the asymptotic region.

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I. INTRODUCTION

Recently there have been many investigations of color dielectric models of confinement[1-4]. This approach gives an intuitive picture of the QCD vacuum as a dielectric medium, arising as a consequence of the quantum fluctuations of the Yang-Mills fields, whose antiscreening properties give rise to confinement. The present work has been motivated by the question of whether classical Yang-Mills fields could also support antiscreening configurations, which might be of importance to the properties of this medium. In the past years there have been several studies of classical Yang-Mills fields with static sources[5-8]. It was shown that for a wide variety of these sources, the corresponding configurations have a screening character with an energy which is lower than the static Coulomb energy. A related issue is whether or not there are other static solutions, and what is their energy in comparison to the solutions previously found.

In this paper we consider the static Yang-Mills theory with a color source density ρ^a in a two-dimensional space, where the analysis is comparatively simpler than in the three-dimensional case. This simplification occurs because with two space dimensions x and y, there is just one component of the chromo-magnetic field B^a given by:

$$B^a = \partial_x A^a_y - \partial_y A^a_x + g f^{abc} A^b_x A^c_y \tag{1}$$

where g denotes the effective coupling constant[9] which has dimensions of $(\text{length})^{-\frac{1}{2}}$. Due to the covariance of the equations of motion, we can work in a definite gauge which for convenience will be chosen to be a generalized axial gauge[10] characterized by the condition:

$$\Lambda_1 A_x^a + \Lambda_2 A_y^a = 0 \qquad (2)$$

where $\Lambda = (\Lambda_1, \Lambda_2)$ denotes an arbitrary bidimensional vector. In this case we see that because of the antisymmetry of the structure constants f^{abc}, the last term in (1) vanishes and the magnetic field reduces to the same form as in the abelian case. Of course, due to the non-linearity of the equations of motion, to be discussed in the next section, the fields $A_{x,y}^a$ are genuinely non-abelian. However, we shall show that the above form of the magnetic field will allow for an important simplification in the analysis of the field equations.

In order to find a truly non-abelian configuration it is essential to have a non-vanishing magnetic field. To see this consider for instance the gauge condition (2) with $\Lambda_2 = 0$. Then, if B^a vanishes we obtain from (1) the relation:

$$\partial_x A_y^a = 0 \qquad (3)$$

which implies that A_y^a could be at most a function of y only.

However, in the gauge $A_x^a = 0$ we can still perform an overall gauge transformation which is independent of x, without restricting the physical content of the theory. We can therefore use this freedom to choose without loss of generality $A_y^a = 0$, in which case the solution reduces to the abelian one where the configurations are characterized by non-vanishing gauge fields A_0^a which are parallel to the source density ρ^a.

The paper is organized as follows. In section II we study the corresponding Yang-Mills equations for spherically symmetric source densities. We have considered more specifically point-like sources in order to abstract the effects associated with their strength from others related to the actual form of the sources. These equations reduce, under rather general conditions, to a system of coupled non-linear differential equations for two functions of a radial distance variable. Assuming that the antiscreening effects are due to a large scale behavior of the Yang-Mills fields[4], we impose that at the origin where the source is located the chromo-electric field has the same strength as in the abelian case. Using this boundary condition we derive analytically the asymptotic behavior of the solution and show that its associated potential increases linearly at large distances from the source density. Further analysis has been accomplished by numerical methods, describing the behavior of the functions in the whole region of variation of the radial variable. In section III we discuss the configurations of the chromo-electric and chromo-magnetic fields which are associated with the behavior of the functions previously described. We shall see that at large distances from the source, the fields oscillate rapidly around constant mean values. The fact that the average chromo-electric field is constant asymptotically contrasts sharply with the abelian case where the electric field vanishes rapidly in this region. This behavior indicates a crucial antiscreening property of the classical Yang-Mills fields. Furthermore, the average chromo-magnetic field has in the asymptotic region a constant magnitude which is related to the one of the mean chromo-electric field. We show both analytically and numerically that at large distances from the source, the energy density increases asymptotically to a value characterizing the classical Yang-Mills configurations. Finally we briefly discuss the possible significance of these results.

II. FIELD EQUATIONS AND SPHERICALLY SYMMETRIC SOLUTIONS

Let us first recall the Yang-Mills equations with static sources:

$$(D_\mu F_{\mu\nu})^e = \pi \delta_{\nu 0} \rho^e \tag{4.a}$$

where $\mu = 0, 1, 2$ with the covariant derivative D^{ea}_μ and the

field tensor $F^a_{\mu\nu}$ being given respectively by:

$$D^{ea}_\mu = \partial_\mu \delta^{ea} + g f^{eac} A^c_\mu \tag{4.b}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \quad . \tag{4.c}$$

Consistency of these equations requires the right-hand side to be covariantly conserved, which in the present circumstance implies that:

$$f^{abe} A^b_0 \rho^e = 0 \quad . \tag{5}$$

The spherical symmetry will be realized in a non-abelian fashion, where explicit radial symmetry is absent in the expression of the gauge potentials A^a_μ, but any rotational non-invariance can be compensated by a gauge transformation[11]. In this way, any physical quantity constructed from the potentials A^a_μ will have a manifest radial symmetry, since in this case no compensating gauge transformations are possible. The above conditions constitute a set of functional equations for the determination of the functional form of the gauge potentials. To this end we will specify the gauge condition in (2) by choosing $\Lambda_1 = x$ and $\Lambda_2 = y$ which is particularly convenient for our purposes[12]:

$$x A^a_x + y A^a_y = 0 \quad . \tag{6}$$

Considering for simplicity the gauge group SU(2), it is straightforward to solve these functional equations. These imply that the most general radial formula will involve four functions which can be parametrized as follows:

$$A_i^a = \frac{\delta_{ai} r^2 - x_a x_i}{r^3} T(r) \qquad \text{for} \quad a = 1, 2 \tag{7.a}$$

$$A_i^3 = \frac{\epsilon_{3im} x_m}{r^2} S(r) \qquad \text{for} \quad a = 3 \tag{7.b}$$

$$A_0^a = \frac{x_a}{r} U(r) \qquad \text{for} \quad a = 1, 2 \tag{7.c}$$

$$A_0^3 = V(r) \qquad \text{for} \quad a = 3 \tag{7.d}$$

Furthermore, in discussing spherical symmetry the source density ρ^a can be described as:

$$\rho^a = \begin{cases} \frac{x_a}{r} \Delta(r) & \text{for} \quad a = 1, 2 \qquad (8.a) \\ \sigma(r) & \text{for} \quad a = 3 \qquad (8.b) \end{cases}$$

With these parametrizations, the static Yang-Mills equations (4) become equivalent to the following set:

$$V'' + \frac{1}{r} V' = -\pi\sigma + \frac{g^2 T^2 V}{r^2} \tag{9.a}$$

$$T'' - \frac{1}{r} T' = -g^2 T V^2 - g^2 r^2 T U^2 \tag{9.b}$$

$$U'' - \frac{2}{r^2} U = \frac{g^2 U (T^2 + S^2)}{r^2} - \frac{gUS}{r} - \pi\Delta \tag{9.c}$$

$$S'' - \frac{1}{r} S' = -g^2 U^2 S + g r U^2 \tag{9.d}$$

while the consistency condition (5) becomes:

$$\sigma(r) U(r) - \Delta(r) V(r) = 0 \quad . \tag{10}$$

Since the above general system of coupled non-linear differential equations is very complicated, we will specialize to the more transparent physical situation where ρ^a points in the third direction of the internal space. Thus with $\Delta = 0$, the condition (10) requires U to vanish for an arbitrarily extended source density. Furthermore, equation (9.c) will be then identically satisfied, while (9.d) implies that S becomes a free-field function. Since from (7.b) S is related to the gauge potential A_i^3, its decoupling from all other functions is a direct consequence of the property previously discussed in connection with the gauge condition (2), when all non-linear couplings involving $\epsilon^{abc} A_x^b A_y^c$ vanish. A_i^3 only contributes to the magnetic field B^3 and since it is a free field gauge potential, it must satisfy the same boundary conditions as in the abelian case. In our case, when only a static source distribution is present, these conditions imply that the magnetic field should vanish and hence we can set without loss of generality $S = 0$.

With these important simplifications, the complex system (9) greatly reduces to the following one involving only two functions $V(r)$ and $T(r)$:

$$\frac{1}{r}\frac{d}{dr}(rV') = -\pi\sigma + g^2 \frac{T^2 V}{r^2} \tag{11.a}$$

$$\frac{d}{dr}\left(\frac{T'}{r}\right) = -g^2 \frac{TV^2}{r} . \tag{11.b}$$

When the non-abelian coupling g vanishes this system essentially describes, as expected, the abelian situation which might be characterized by the potential $V(r)$ and the source density $\sigma(r)$. It will be convenient henceforth to measure all distances in units of length given by g^{-2}, so that we will set $g = 1$ with all quantities being dimensionless.

We will now consider more specifically the case of a point like source density localized at the origin with $\sigma(r) = \frac{Q\,\delta(r)}{\pi r}$, where Q represents the magnitude of the chromo-electric charge. As described in the Introduction, we impose the boundary condition that in the neighbourhood of the

origin the electric field is essentially the same as in the abelian case. This requires that:

$$V(r \to 0) = Q\,\ell n(\tfrac{1}{r}) \quad \text{and} \quad T(r \to 0) = 0 . \tag{12}$$

With these boundary conditions it is straightforward to solve iteratively at small distances the non-linear system given by (11). With $g = 1$, we obtain up to corrections of order r^6 the following behavior:

$$V(r) \simeq Q\,\ell n(\tfrac{1}{r}) + \frac{B_0^2 r^4}{64}\left[Q\,\ell n(\tfrac{1}{r}) - \tfrac{1}{2}\right] \tag{13.a}$$

$$T(r) \simeq \frac{B_0}{2} r^2 \left\{1 - \frac{r^2}{8}\left[Q^2 \ell n^2(r) - \frac{33}{25} Q\,\ell n(r) + \frac{181}{250}\right]\right\} \tag{13.b}$$

where B_0 is a constant which will turn out to represent the magnitude of the magnetic field at the origin.

Let us turn now to the determination of the asymptotic properties of the solution at large distances from the source density. In this case, up to corrections of order r^{-2}, it can be verified that the dominant behavior of the system (11) is described by the functions:

$$V(r) \simeq K r \tag{14.a}$$

$$T(r) \simeq \sqrt{2} \cos\left(\frac{K r^2}{2}\right) \tag{14.b}$$

where K denotes a constant which will be related in the next section to the values of Q and B_0. Here it is important to notice that the potential $V(r)$ grows linearly with the distance, behavior which will prove to be crucial for the antiscreening properties of the Yang-Mills fields in the asymptotic region.

Although we cannot solve the non-linear system of coupled equations (11) for the whole range of the radial parameter r, it is possible to understand qualitatively some of the important features of the exact solution. To this end we remark, with the help of equation (11.a), that when rV' increases with the distance V must be positive, whereas when rV' decreases V must become a negative function. These features are illustrated in Figure 1, which has been obtained numerically corresponding to the initial set $Q=1$, $B_0 = 1.2$.

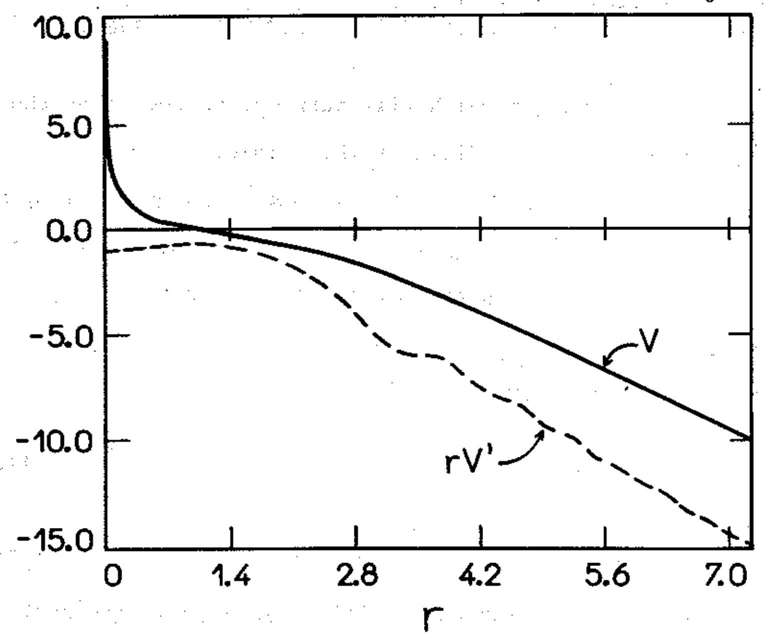

Fig. 1 - Profiles of the functions V and rV' for $Q=1$ and $B_0 = 1.2$. These show that already at moderate values of r, the potential V has almost reached its asymptotically linear behavior.

On the other hand if rV' increases so as to reach a zero at $r = \xi$, this point will correspond to a minimum of the potential $V(r)$ which will be positive in this case. These characteristics are shown in Figure 2, for the set $Q = 1$ and $B_0 = 9.2$.

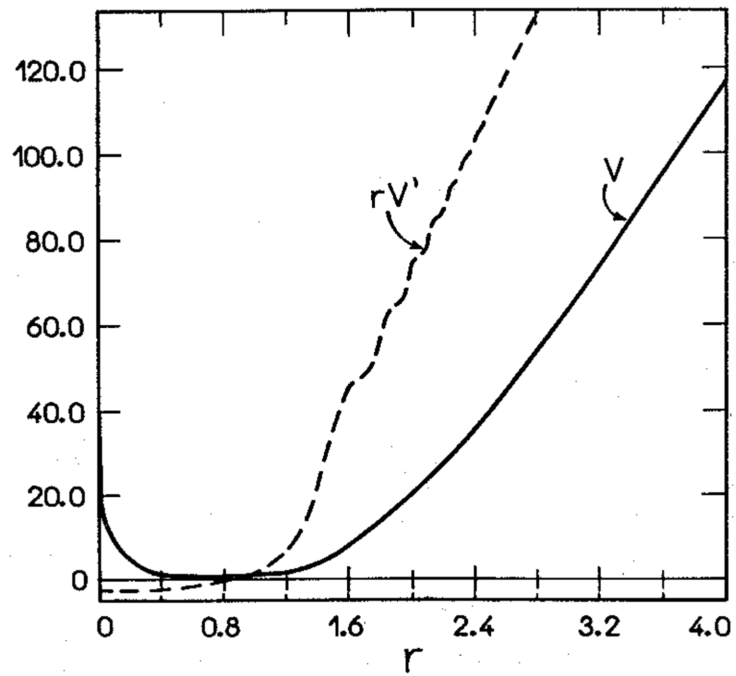

Fig. 2 - The numerical solution for the functions V and rV' corresponding to $Q=1$ and $B_0 = 9.2$. The linear profile of the potential $V(r)$ is manifest already at $r=4$.

Which one of the behaviors illustrated in these figures is actually realized depends critically on the possibility of V' to vanish. As can be seen by integrating (11.a), this condition requires that the following relation must hold:

$$Q = \int_0^{\xi} \frac{T^2 V}{r} \, dr \quad . \tag{15}$$

This equation can be satisfied provided the function T increases sufficiently rapidly, which requires from (13.b) that B_0 be greater than a critical value B_0^{cr}. We have investigated numerically eq. (15) for a variety of values Q and found that $B_0^{cr} \sim 5$. Our numerical analysis has also confirmed the oscillatory behavior of T given by (14.b), with an amplitude of order $\sqrt{2}$ in the asymptotic region.

III. ELECTROMAGNETIC YANG-MILLS FIELDS AND ENERGY RELATIONS

We will now consider the behavior of physical relevant quantities, like the chromo-electric and chromo-magnetic energy densities. To this end we note that the chromo-electric field $E_i^a = (D_i A_0)^a$ yields, with the help of equation (7), the following expression for the electric energy density:

$$E^2 \equiv E_i^a E_i^a = (V')^2 + \frac{T^2 V^2}{r^2} \quad . \tag{16}$$

Furthermore, using (1) and (7), the magnetic energy density

will be given by:

$$B^2 \equiv B^a B^a = \frac{(T')^2}{r^2} \quad . \tag{17}$$

We see from (13) that at short distances the behavior of these quantities is described by:

$$E^2 \simeq (V')^2 \simeq Q^2/r^2 \tag{18.a}$$

$$B^2 \simeq B_0^2 \tag{18.b}$$

i.e. the electric energy density is, according to our boundary conditions, like in the abelian case, while B_0 represents the magnitude of the magnetic field at the origin.

Turning now to the behavior of E^2 and B^2 at large distances, we find with the help of (14) that in this region we have:

$$E^2 \simeq K^2[2 + \cos(Kr^2)] \tag{19.a}$$

$$B^2 \simeq K^2[1 - \cos(Kr^2)] \tag{19.b}$$

We see that these densities fluctuate around constant values, in such a way that the average magnitude of E^2 is twice as big as the corresponding mean value of B^2:

$$\langle E^2 \rangle = 2 \langle B^2 \rangle = 2K^2 \quad . \tag{20}$$

We recall that from (14.a), K represents the slope of the potential $V(r)$ in the asymptotic region. As we have seen, for $B_0 < B_0^{cr}$ K is a negative quantity, whereas for $B_0 > B_0^{cr}$ it becomes positive. Furthermore, inasmuch as $|K|$ is related to the energy densities, it should be a gauge independent quantity which characterizes the Yang-Mills configurations. We have studied its behavior as a function of Q and B_0 and the result is presented in Figure 3.

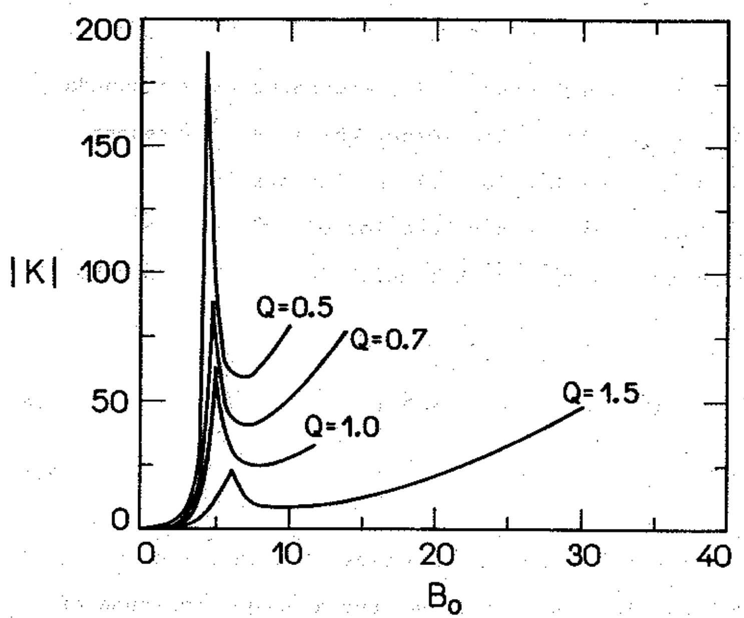

Fig. 3 - Profiles of the function $|K|$ for $Q = 0.5, 0.7, 1.0$ and 1.5. These curves display a critical point around $B_0^{cr} = 5$.

We can see a cusp-like behavior which is characteristic of phenomena with critical points, and that typical values of $|K|$ are rather large.

The period of oscillation of the electric and magnetic energy densities, which from (19) is of order $(|K|r)^{-1}$, is therefore very small in the asymptotic region. Furthermore, these oscillations cancel out when the total chromo-electromagnetic energy density is considered in this region:

$$\varepsilon_{as} = (E^2 + B^2)_{as} = 3K^2 \tag{21}$$

Further progress has been achieved by a numerical evaluation of the total density as a function of radial distance. The results have been displayed in Figure 4, which shows that at values of $r \simeq 30$, the energy density has almost reached its asymptotic value predicted by equation (21).

Except for the extremely rapid oscillations discussed above, the profile of the chromo-electric energy density is very similar. From (18.a) and (20) we can see that whereas E^2 has a Coulombic nature at the origin, its average value in the asymptotic region is constant. We believe that this behavior reflects a large-scale antiscreening phenomenon of classical Yang-Mills fields.

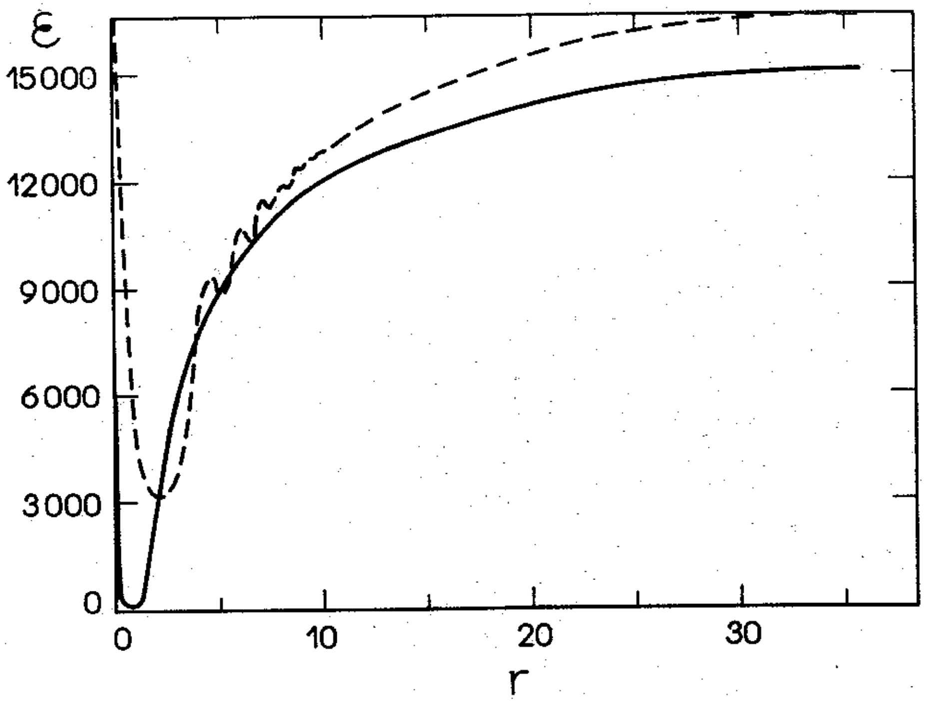

Fig. 4 - Behavior of the total energy density. The full line corresponds to $Q = 1$ and $B_0 = 9.2$. The broken line is associated with $Q = 0.5$, $B_0 = 0.5$ and has been augmented for clarity by a scale of 10^4.

Since ε_{as} is constant in the asymptotic region, we can estimate from (21) that the total energy will be of order $K^2 r^2$ at large distances r. We can actually evaluate exactly the corresponding part due to the magnetic energy, by considering the energy-momentum tensor:

$$\Theta_{\mu\nu} = F^a_{\mu\gamma} F^a_{\gamma\nu} + \frac{\delta_{\mu\nu}}{4} F^a_{\alpha\beta} F^a_{\alpha\beta} \tag{22}$$

which is not conserved owing to the presence of the external source density ρ^a:

$$\partial_\mu \Theta_{\mu\nu} = \pi \rho^a F^a_{0\nu} \tag{23}$$

Using the properties of $\Theta_{\mu\nu}$ and integrating by parts we obtain by a series of transformations that

$$\int B^2 \, dx dy = \pi \int dx dy \, \rho^a x_j E^a_j - \int_0^{2\pi} d\varphi \, x_i x_j \Theta_{ij} \tag{24}$$

where the last expression on the right-hand side corresponds to a surface term. Using our boundary conditions at the origin together with the parametrization (7), it is straightforward to show with the help of (16) and (17) that the total magnetic energy M within a radius r is given by:

$$M(r) = \pi \left\{ Q^2 - r^2 [V'(r)]^2 + T^2(r) V^2(r) + [T'(r)]^2 \right\}. \tag{25}$$

In the abelian case, when $T = 0$, we see from (18.a) that the magnetic energy vanishes, as expected. On the other hand, using the relations (14) which describe the asymptotic behavior of the non-abelian configurations we obtain:

$$M(r) = \pi \left[Q^2 + K^2 r^2 \right] . \tag{26}$$

The fact that the total energy becomes infinite as r grows indefinitely implies that a classical color charge cannot exist isolated in a medium with the characteristics described above. Crucial to this explanation is the possibility of an antiscreening behavior of the classical Yang-Mills fields at large distances. This phenomenon suggests in a natural way the further consideration of the important case corresponding to a dipole distribution with vanishing net color charge. We hope to report on this issue in a future communication.

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