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ABSTRACT

The limiting total excitation energy of ^{208}Pb is calculated using the nuclear grand partition function. A temperature dependent shell-model potential and an energy-dependent nucleon-nucleus optical potential are employed to generate the temperature-dependent bound single particle energies ε_K and the continuum level density $\rho(\varepsilon)$ respectively. Levinson's theorem is used to make the relevant change in $\rho(\varepsilon)$ due to the T-dependence of $\varepsilon_K(T)$. The calculated limiting total excitation energy is about one half of the one recently obtained by Dean and Mosel.

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The temperature dependence of nuclear properties has been extensively discussed in recent years[1-9]. This stems partly from the now available heavy-ion accelerators that can accelerate HI beams to intermediate energies. This makes possible the study of the nuclear equation of state in a region of excitation energies and densities close to what is believed to correspond to a phase transition region[8]. Much interest has also arisen from the potential possibility of extending temperature-dependent Hartree-Fock (TEMDHF) calculations to these excitation energies and temperatures.

An important quantity which enters in any nuclear fragmentation calculation is the limiting total excitation energy, above which nuclear matter is found fragmented into small pieces. Several conflicting estimates of this excitation energy have appeared in the literature. In particular the approach adopted by Dean and Mosel[9], seems to be, at least in principle, the more general one since it relies on a fundamental property of the scattering system, namely Levinson's theorem. However, the limiting fragmentation energy they calculated is rather high ($\sim$ 12 MeV), as compared to the one found in statistical calculations of nuclear fragmentation. The letter seems to be quite successful in pinning down the major characteristics of the phenomenon[10]. We shall present below arguments to show that the discrepancy lies in the use in Ref. 9), of a

temperature independent mean field employed in the calculation of the discrete part of the nuclear partition function.

The purpose of this letter is therefore the elucidation of the temperature varying mean field in the determination of the thermal properties of nuclei. This, of course, is an obvious fact within the TEMDHF theory. However, in the context of the calculation of Ref. 9), the temperature-dependence of the mean field is found to be a major qualitative and quantitative addition. In the following, we first present a short summary of the Dean-Mosel model followed by an account of the modifications we develop. The calculations are then performed with a temperature-dependent shell-model potential and an energy-dependent optical potential model, for the generalized mean field at negative and positive nucleon energies. The resulting limiting fragmentation temperature is found to be almost a factor of $\frac{1}{2}$ lower than the one obtained by Dean and Mosel, in accordance with empirical findings.

The starting point of our discussion is the grand potential which, when counting both discrete and continuum states, taker the form

$$\Omega_N = -T \sum_k \ln \left[1 + \exp(\alpha - \beta \varepsilon_k)\right] - T \int_0^\infty g_N(\varepsilon) \ln\left[1 + \exp(\alpha - \beta\varepsilon)\right] d\varepsilon \qquad (1)$$

where T is the temperature, α is the chemical potential (through which the number of nucleons in the nucleon is fixed) $\beta \equiv \frac{1}{T}$, ε_K the shell-model single particle energies, ε the positive continuum nucleon energy and $g_{nucl}(\varepsilon)$ the nuclear continuum level density. This last quantity is usually expressed in terms of the nucleon elastic scattering phase shift, $\delta_{\ell,j}$ as[12]

$$g_N(\varepsilon) = \frac{1}{\pi} \sum_{\ell,j} (2j+1) \frac{d\delta_{\ell,j}}{d\varepsilon} \qquad (2)$$

The phase shifts $\delta_{\ell,j}$ are generated from a realistic energy-dependent optical potential. Clearly, to be consistent, the bound single-particle energies ε_K, should be generated from a temperature-dependent shell model interaction. This latter fact was not considered by Dean and Mosel, as they employed the T = 0 shell-model potential usually used in nuclear structure calculations.

The allowance of temperature-dependent bound single-particle energies implies qualitative changes in the derived thermodynamical quantities, such as the excitation energy and the entropy. The relations which fix $\alpha_n(T)$, $\alpha_p(T)$ and E, are

$$\frac{\partial}{\partial \alpha_n}\left[-\beta \Omega_{nucl.}\right] = N \qquad (3)$$

$$\frac{\partial}{\partial \alpha_p}\left[-\beta \Omega_{nucl.}\right] = Z \tag{4}$$

$$-\frac{\partial}{\partial \beta}\left[-\beta \Omega_{nucl.}\right] = N \tag{5}$$

From (3)-(5) we construct the entropy function

$$S = -\alpha_n N - \alpha_p Z + \beta E - \beta \Omega_{nucl.} \tag{6}$$

In order to evaluate the energy and entropy we need to specify our shell model and optical model potentials which generate $\varepsilon_K(\beta)$ and the phase shift (eq. (2)).

In the shell-model calculation we have employed for the nuclear interaction a Woods-Saxon form with temperature dependent depth and radius, namely

$$V_N(T) = V_0(T)\left\{1 + \exp\left[\frac{r - R_N(T)}{0.65}\right]\right\}^{-1} \tag{7}$$

and the usual Thomas form for the spin-orbit interaction. In order to construct the temperature dependence of the potential we proceed as follows. The nuclear radius is given by

$$R_N(T) = 1.24 A^{1/3}\, r_B(T) / r_B(0) \tag{8}$$

where $r_B(T)$ is the rms radius for the SIII interaction taken from table 3 of Ref. 4). If the central nuclear density ρ_0 is assumed to remain constant, then the potential depth $V_0(T)$ should decrease with temperature in order to conserve the nuclear number

$$V_0(T) = \frac{2A(0) - A(T)}{A(0)}\, V_0(0) \tag{9}$$

where

$$A(T) = \rho_0 \int_0^\infty \frac{r^2\, dr}{1 + \exp\left[\frac{r - R_N(T)}{0.65}\right]} \tag{10}$$

The neutron and proton depths at $T = 0$ are given by 63.01 MeV and 47.56 MeV in accordance with the values taken by Dean and Mosel[9]. The eigenvalues $\varepsilon_K(\beta)$ are then generated from these T-dependent potentials.

The continuum level density $g_{nucl}(\varepsilon)$ was constructed using Eq. (2) with the energy-dependent Wilmore and Hodgson optical potential[13]. The $\delta_{\ell,j}$ that appears in Eq. (2) is identified with the real part of the complex nuclear phase shifts.

At this point we remark that since the shell-model potential is temperature-dependent, the number of resulting eigenvalues that correspond to bound orbitals will also be T-dependent. This implies that there must be a corresponding change in $g_{nucl}(\varepsilon)$. This is so, since Levinson's theorem implies that the total number of states (both bound and continuum) in a potential does not change with the change in this potential. This conservation law states

$$N_B + \int_0^\infty g_{nucl.}(\varepsilon)\,d\varepsilon = 0 \equiv N_B + N_C \qquad (11)$$

Thus if N_B changes with T, N_C must do exactly the same, with opposite sign.

In Figs. 1 and 2 we present our calculation of the entropy per particle $\frac{S(T)}{A}$ and total energy per particle $\frac{E(T)}{A}$ as functions of the nuclear temperature.

We see clearly that the curve for $\frac{S(T)}{A}$ reaches a maximum at $T = 6$ MeV at which point $E(T)/A \simeq -16$ MeV namely, $E^*(T)/A$ attains the value 5.5 MeV, almost half of that obtained by DM.

This value of the limiting total excitation energy is slighly higher than the one found in statistical fragmentation calculations[11] ($E^*/A \approx 3$-4 MeV), but smaller than the one found by Dean and Mosel ($E^*/A \approx 8.2$ MeV).

In conclusion, we have calculated the limiting total excitation energy of a finite nucleus, using single particle levels obtained from a temperature-dependent shell-model potential (for the bound orbits) and an energy-dependent nucleon-nucleus optical potential for the continuum states. It is found that the inclusion of the temperature dependence in the SMP, not considered in Ref. 9), results in a value for the limiting total excitation energy which is 50% smaller than the one obtained by Dean and Mosel. Our result is much closer to the value of E^*/A found in recent fragmentation calculations.

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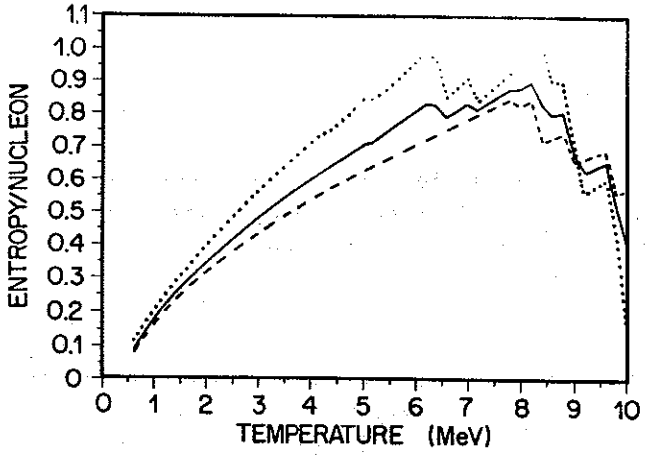
FIGURE CAPTIONS

FIGURE 1: Entropy per particle vs nuclear temperature (see text for details). Dashed curve, neutrons, dotted curve, protons and full curve nucleons.

FIGURE 2: Total energy per particle vs nuclear temperature (see figure 1 for explanation of curves).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1.0
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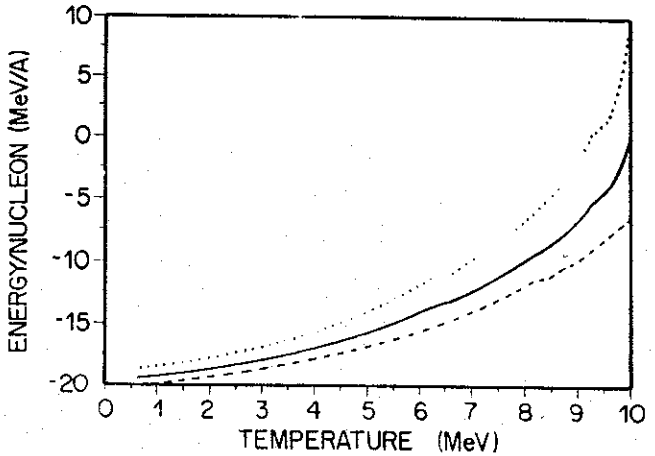
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10
TEMPERATURE (MeV)


ENERGY/NUCLEON (MeV/A)
10
5
0
-5
-10
-15
-20
0 1 2 3 4 5 6 7 8 9 10
TEMPERATURE (MeV)