UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

IFUSP/P-641

DIRAC BRACKET QUANTIZATION OF BOSONIZED CHIRAL QCD_2

E. Abdalla
Instituto de Física, Universidade de São Paulo

K.D. Rothe
Depto. de Física, Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ, Brazil

Maio/1987

DIRAC BRACKET QUANTIZATION OF BOSONIZED CHIRAL QCD_2

E. Abdalla*

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brazil

K.D. Rothe**

Depto. de Física, Universidade Católica do
Rio de Janeiro, Rio de Janeiro, RJ, Brazil

ABSTRACT

We systematically derive the bosonized form of the chiral QCD_2 Lagrangean exhibiting explicitely the anomalous breaking of gauge invariance, and quantize it using Dirac's algorithm for constrained systems. As a side product we also discuss the Hamiltonean formalism for the principal sigma model, and derive the commutation relations of the chiral currents in both models.

*Work partially supported by CNPq.

**On leave of absence from the Institut für Theoretische Physik der Universität Heidelberg. Work supported by DAAD (West Germany) and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

1. INTRODUCTION

There has been much interest recently in the quantization of chiral gauge theories with anomalous breaking of gauge invariance [1]. Following Faddeev and Shatashvili's work [2] there have been recent proposals [3] for cancelling such anomalies via Wess-Zumino [4] terms. They lead to a non-anomalous gauge theory which coincides with the original one only in the gauge invariant sector. The physical relevance of this new theory appears questionable [5], especially in view of the fact, that gauge anomalies do not necessarily prevent a consistent quantization. This was first illustrated by Fackiw and Rajaraman [6] for the case of the "chiral Schwinger model". They exploited the lack of gauge invariance as guiding principle in the renormalization procedure, in order to demonstrate the existence of a non-trivial solution for a whole one parameter family of "chiral Schwinger models". A canonical quantization of the corresponding "Wess-Zumino" Lagrangean [7,8] via Dirac's algorithm [9] for constrained systems showed [7] that, depending on the value of the parameter, there existed two types of theories, involving either two or four constraints. The extension to the non-abelian case was first considered by Rajaraman [10], who based his discussion on Witten's work on the bosonization of free, flavor carrying fermions [11] and Coleman's type of arguments for coupling the gauge field to the fermionic currents.

In this paper we systematically study the quantization of chiral QCD_2, closely following the discussion of ref. [7]. A "first principle" derivation of the corresponding effective bosonic action is given in section 2. It involves a Wess-Zumino term. We first discuss the canonical quantization of the associated principal sigma model in section 3. We then turn to the quantization of the model of actual interest in section 4, where we compute the constraints and Dirac brackets. As in the abelian case [7] we shall have to distinguish between two possible types of theories, involving either two or four constraints. In the first case we compute the Dirac brackets. We conclude in section 5 with a discussion of the results.

2. THE EFFECTIVE BOSONIZED ACTION

In this section we derive the equivalent bosonic Lagrangean of chiral QCD_2 with left-handed coupling of the fermions, defined by

$$L_{ch} = -\frac{1}{4}\,\mathrm{tr}\,F_{\mu\nu}^2 + \bar{\psi}\left(i\partial\!\!\!/ + eA\!\!\!/\,\frac{(1-\gamma^5)}{2}\right)\psi \quad , \tag{2.1}$$

where $F_{\mu\nu}$ and A_μ are the Lie algebra valued chromoelectric field tensor and gauge potential [12] respectively

$$F_{\mu\nu} = \sum F_{\mu\nu}^a\, t^a$$

$$A_\mu = \sum A_\mu^a\, t^a \tag{2.2}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu] \quad .$$

The Lagrangean (2.1) is invariant under the local transformation,

$$\psi \to g_-\psi \quad , \quad \bar{\psi} \to \bar{\psi} g_- \tag{2.3a}$$

$$A_\mu \to A_\mu + \frac{i}{e}\, g\,\partial_\mu\, g^{-1} \tag{2.3b}$$

where

$$g_- = \exp\left(i\vartheta\,\frac{(1-\gamma^5)}{2}\right) \tag{2.4}$$

$$g = \exp i\vartheta$$

with ϑ a Lie-algebra valued field.

As is well known, the bosonic Lagrangean L, equivalent to L_{ch} above, is obtained by performing the fermionic integration in the partition function associated with L_{ch},

$$Z = \int DA^\mu\, D\psi\, D\bar{\psi}\; e^{i\int L_{ch}(A,\psi,\bar{\psi})} = \int DA^\mu\; e^{i\int L_{eff}(A)} \tag{2.5}$$

with

$$L_{eff} = -\frac{1}{4}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \Gamma^{(R)}[A] \tag{2.6}$$

$$i\Gamma^{(R)}[A] = \ell n\,\det\left(i\partial\!\!\!/ + ieA\!\!\!/\,\frac{(1-\gamma_5)}{2}\right) \tag{2.7}$$

and representing $\exp(i\Gamma^{(R)}[A])$ as a functional integral over a group valued Lorentz scalar field. To this end we consider $\Gamma^{(R)}[A]$ as defined by (2.7). For a regularization respecting the chiral structure of the coupling in (2.1), one obtain $\Gamma^{(R)}[A]$ from the corresponding functional of QCD_2,

$$i\Gamma[A] : \ell n\,\det(i\partial\!\!\!/ + eA\!\!\!/) \tag{2.8}$$

by simply setting $A_+ = 0$:

$$\Gamma^{(R)}[A] = \Gamma[A]\big|_{A_+=0} \tag{2.9}$$

Now, $\Gamma[A]$ has been calculated by various authors by either working in the "decoupling gauge" [13] or the light-cone gauge [14]. A gauge-invariant calculation $\Gamma[A]$ was given in ref. [15]. A manifest gauge-invariant form for $\Gamma[A]$ is obtained by observing that A_μ can always be decomposed in the form,

$$eA_\mu = \frac{i}{2}\,(g_{\mu\nu} + \epsilon_{\mu\nu})U\partial^\nu U^{-1} + \frac{i}{2}\,(g_{\mu\nu} - \epsilon_{\mu\nu})V\partial^\nu V^{-1} \tag{2.10}$$

where U and V are independent group valued fields which may

be parametrized by two Lie-algebra valued fields φ and Φ, as follows:

$$U = e^{i(\varphi+\Phi)} \quad , \quad V = e^{i(\varphi-\Phi)} \quad . \tag{2.11}$$

From (2.10) it follows that

$$eA_- = i\,U\,\partial_- U^{-1} \quad . \tag{2.12}$$

Now, $\Gamma[A]$ can be shown to be given by the functional [16]

$$\begin{aligned}\Gamma[A] &\equiv \tilde{\Gamma}[G] \\ &= -\frac{1}{8\pi}\int d^2x\,\mathrm{tr}(\partial_\mu G)(\partial^\mu G^{-1}) \\ &\quad - \frac{1}{4\pi}\int_0^1 dr \int d^2x\,\epsilon^{\mu\nu}\,\mathrm{tr}\left[(\tilde{G}^{-1}\partial_\mu\tilde{G})(\tilde{G}^{-1}\partial_\nu\tilde{G})(\tilde{G}^{-1}\partial_r\tilde{G})\right]\end{aligned} \tag{2.13}$$

where

$$G = V^{-1}U \tag{2.14}$$

and $\tilde{G}(x,r)$ is a smooth function of r interpolating between $G(x)$ and 1 :

$$\tilde{G}(x,r) = \tilde{V}^{-1}(x,r)\,\tilde{U}(x,r)$$

$$\tilde{V}(x,1) = V(x) \quad , \quad \tilde{V}(x,0) = 1 \qquad (2.15)$$

$$\tilde{U}(x,1) = U(x) \quad , \quad \tilde{U}(x,0) = 1 \quad .$$

Under the gauge transformation (2.3b), one has

$$U \to gU \quad , \quad V \to gV \quad . \qquad (2.16)$$

Hence expression (2.13) is manifestly gauge invariant. The same is not true for $\Gamma^{(R)}$ defined by (2.9), as becomes evident, by writing (2.9) as

$$\Gamma^{(R)}[A] = \tilde{\Gamma}[U] \qquad (2.17)$$

Eq. (2.17) shows that $\Gamma^{(R)}$ is not invariant under the transformation (2.16) reflecting the well known gauge-anomaly. We define a Wess-Zumino functional $WZ^{(R)}[A,g]$ by

$$WZ^{(R)}[A,g] = \tilde{\Gamma}[gU] - \tilde{\Gamma}[U] \qquad (2.18)$$

This definition is the analog of the Wess-Zumino functional introduced by Di Vecchia et al [17] for the case of QCD_2. Using the invariance of the Haar measure under gauge transformations, one easily finds

$$e^{i\Gamma^{(R)}[A]} = \text{const} \int [Dg]\, e^{-iWZ^{(R)}[A,g]} \quad . \qquad (2.19)$$

Now, $\tilde{\Gamma}[AB]$ has the remarkable property [18]

$$\tilde{\Gamma}[AB] = \tilde{\Gamma}[A] + \tilde{\Gamma}[B] - \frac{1}{4\pi}\int d^2x\, \mathrm{tr}(A^{-1}\partial_+ A)(B\,\partial_- B^{-1}) \quad . \qquad (2.20)$$

Using this property in (2.18), and recalling (2.12), we obtain,

$$WZ^{(R)}[A,g] = \tilde{\Gamma}[g] + \frac{ie}{4\pi}\int d^2x\, \mathrm{tr}(g^{-1}\partial_\mu g)(g_{\mu\nu} + \epsilon_{\mu\nu})A^\nu \quad . \qquad (2.21)$$

Combining all these results we finally obtain

$$Z = \int DA^\mu\, Dg\; e^{iS_{eff}[A,g]} \qquad (2.22)$$

with

$$S_{eff}[A,g] = \int d^2x \left(-\frac{1}{4}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{ae^2}{8\pi}A_\mu A^\mu\right)$$
$$-\tilde{\Gamma}[g] - \frac{ie}{4\pi}\int d^2x\, \mathrm{tr}\left[g^{-1}\partial^\mu g(g_{\mu\nu} + \epsilon_{\mu\nu})A^\nu\right] \quad . \qquad (2.23)$$

Following the argumentation of Jackiw and Rajaraman [6] we have included a term proportional to A_μ^2, with a an arbitrary constant, reflecting the ambiguity one has when calculating the fermionic determinant, due to the lack of "gauge invariance" as guiding principle. Note that the result (2.23), obtained here from first principles. agrees with that obtained by Rajaraman [10] using Coleman's principle of form invariance.

In order to develop a canonical Hamiltonian formalism for the action (2.23), we shall need the corresponding Lagrangean. To this end we observe that $-\tilde{\Gamma}[g]$ in (2.23) is the sum of the action of the principal non-linear σ-model

$$S_{P\sigma M} = \frac{1}{8\pi} \int d^2x \; \mathrm{tr}(\partial_\mu g^{-1})(\partial^\mu g) \tag{2.24}$$

and the Wess-Zumino term

$$S_{WZ} = -\frac{1}{4\pi} \int_0^1 dr \int d^2x \, \epsilon^{\mu\nu} \, \mathrm{tr}\left[(\tilde{g}\, \partial_\mu \tilde{g}^{-1})(\tilde{g}\, \partial_\nu \tilde{g}^{-1})(\tilde{g}\, \partial_r \tilde{g}^{-1})\right] . \tag{2.25}$$

Since S_{WZ} only contains first order time derivatives in the fields, it will not contribute to the Hamiltonian.

It will be convenient to imagine the r-integration as having been carried out, and to make the Ansatz

$$S_{WZ} = \frac{1}{4\pi} \int d^2x \; \mathrm{tr}\,(A(g)\partial_0 g) \tag{2.26}$$

with $A(g)$ some unknown matrix valued function of g. Fortunately

we shall not need to know $A(g)$ itself, but only the anti-symmetric tensor

$$F_{ij;k\ell} = \frac{\partial A_{ij}}{\partial g_{\ell k}} - \frac{\partial A_{k\ell}}{\partial g_{ji}} \; . \tag{2.27}$$

This tensor is explicitely calculable by comparing the variational derivative of S_{WZ} with respect to g as calculated from (2.25) and (2.26) respectively. From (2.25) one finds after some calculation [11]

$$\delta_g S_{WZ} = -\frac{1}{4\pi} \int d^2x \, \epsilon^{\mu\nu} \, \mathrm{tr}(g^{-1}\delta g)\partial_\mu (g^{-1}\partial_\nu g) \tag{2.28a}$$

whereas from (2.26) one obtains,

$$\delta_g S_{WZ} = \frac{1}{4\pi} \int d^2x \; F_{ij,k\ell}(x)\partial_0 g_{ji}\, \delta g_{\ell k} \; . \tag{2.28b}$$

Comparison of the two results (2.28) yields the <u>local</u> expression

$$F_{ij;k\ell}(x) = (\partial_1 g^{-1}_{i\ell}(x))\, g^{-1}_{kj}(x) - g^{-1}_{i\ell}(x)(\partial_1 g^{-1}_{kj}(x)) \; , \tag{2.29}$$

Hence, the theory to be quantized canonically is described by the Lagrangean density

$$L = L_{YM} + L_{P\sigma M} + L_{WZ} + L_I \tag{2.30}$$

where

$$L_{YM} = -\frac{1}{4}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{ae^2}{8\pi}\,A_\mu A^\mu \qquad (2.31a)$$

$$L_{P\sigma M} = \frac{1}{8\pi}\,\mathrm{tr}(\partial_\mu g^{-1})(\partial^\mu g) \qquad (2.31b)$$

$$L_{WZ} = \frac{1}{4\pi}\,\mathrm{tr}\,A(g)\,\partial_0 g \qquad (2.31c)$$

$$L_I = -\frac{ie}{4\pi}\,\mathrm{tr}\left[g^{-1}\partial^\mu g(g_{\mu\nu}+\epsilon_{\mu\nu})A^\nu\right] \quad . \qquad (2.31d)$$

As a preliminary step we shall proceed to first develop the Hamiltonian formalism for the principal non-linear sigma model (PσM).

3. CANONICAL QUANTIZATION OF PRINCIPAL σ-MODEL

Consider the Lagrangean density (2.31b). The momentum conjugate to g_{ij}, and the Hamiltonian are given respectively by [19]

$$\tilde{\Pi}_{ij} = \frac{1}{4\pi}\,\partial_0\, g^{-1}_{ji} \qquad (3.1)$$

$$H = \int dx^1 \left\{-2\pi\,\mathrm{tr}\,\tilde{\Pi}^T g \tilde{\Pi}^T g - \frac{1}{8\pi}\,\mathrm{tr}(g^{-1}\partial_1 g)(g^{-1}\partial_1 g)\right\} . \qquad (3.2)$$

The Hamilton equations of motion, written in terms of Poissons brackets, are

$$\partial_0\, q_{ij} = \left\{g_{ij}\,,H\right\} \qquad (3.3a)$$

$$\partial_0\,\tilde{\Pi}_{ij} = \left\{\tilde{\Pi}_{ij}\,,H\right\} \quad . \qquad (3.3b)$$

Using the properties

$$\left\{g_{ij}(x)\,,\tilde{\Pi}_{k\ell}(y)\right\} = \delta_{ik}\,\delta_{j\ell}\,\delta(x^1-y^1) \qquad (3.4a)$$

$$\left\{g^{-1}_{ij}(x)\,,\tilde{\Pi}_{k\ell}(y)\right\} = -\,g^{-1}_{ik}(x)\,g^{-1}_{\ell j}(x)\,\delta(x-y) \qquad (3.4b)$$

eq. (3.3a) just reduces to (3.1), while eq. (3.3b) becomes

$$\partial_0\tilde{\Pi}_{ij} = 4\pi(\tilde{\Pi}^T g\,\tilde{\Pi}^T)_{ji} - \frac{1}{4\pi}\left[\partial_1(g^{-1}\partial_1 g)\,g^{-1}\right]_{ji} \quad . \qquad (3.5)$$

Substitution of (3.1) into (3.5) then leads to the equation of motion,

$$\partial_\mu(g^{-1}\partial^\mu g) = 0 \qquad (3.6)$$

expressing the conservation of the Noether-current. Eq. (3.6) coincides with the Lagrange equations of motion following from (2.31b).

As is well known there exists another conserved current, $-\frac{i}{4\pi} g \partial_\mu g^{-1}$. Motivated by Witten's work [11] we define the currents

$$j_+ = -\frac{i}{4\pi} g^{-1} \partial_+ g = i \tilde{\Pi}^T g - \frac{i}{4\pi} g^{-1} \partial_1 g \tag{3.7a}$$

$$j_- = -\frac{i}{4\pi} g \partial_- g^{-1} = -i g \tilde{\Pi}^T + \frac{i}{4\pi} g \partial_1 g^{-1} \quad . \tag{3.7b}$$

It will be instructive to compare the Poisson brackets of the corresponding adjoint currents

$$j_\pm^a \equiv \mathrm{tr}(j_\pm t^a) \tag{3.8}$$

with the obtained for a theory of free fermions [11]. One finds

$$\{j_+^a(x), j_+^b(y)\}_t = f_{abc} j_+^c \delta(x-y) + \frac{1}{2\pi} \delta^{ab} \delta'(x-y) - \frac{i}{4\pi} f_{abc} \, \mathrm{tr}(g^{-1} \partial_1 g t^c) \, \delta(x-y) \tag{3.9a}$$

$$\{j_-^a(x), j_-^b(y)\}_t = f_{abc} j_-^c \delta(x-y) - \frac{1}{2\pi} \delta^{ab} \delta'(x-y) - \frac{i}{4\pi} f_{abc} \, \mathrm{tr}(g \partial_1 g^{-1} t^c) \, \delta(x-y) \tag{3.9b}$$

$$\{j_+^a(x), j_-^b(y)\}_t = \frac{1}{4\pi} \delta(x-y) \partial_1 \mathrm{tr}(t^a g^{-1} t^b g) \quad . \tag{3.9c}$$

We see that the Poisson bracket of j_+ and j_- does not vanish in the PSM, in contrast to case of a free fermion theory [11].

The transition to the quantum theory of the PSM is trivial, as there are no constraints involved, and is achieved by the usual substitution rule $\{A,B\} \rightarrow -i[A,B]$. This is not true for chiral QCD_2, as we shall see in the following.

4. CANONICAL QUANTIZATION OF CHIRAL QCD_2

The Lagrange equations corresponding to (2.30) are given by

$$\begin{aligned} (g^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\mu (g^{-1} \partial_\nu g) &= -ie(g^{\mu\nu} + \epsilon^{\mu\nu}) \nabla_\mu A_\nu \\ D_\mu^{ab} F_b^{\mu\nu} + \frac{ae^2}{4\pi} A^{a\nu} - \frac{ie}{4\pi} (g^{\nu\mu} - \epsilon^{\nu\mu}) \, \mathrm{tr}(g^{-1} \partial_\mu g t^a) &= 0 \end{aligned} \tag{4.1}$$

where we have used (2.29), and where ∇^μ is the covariant derivative defined by

$$\nabla^\mu = \partial^\mu + [g^{-1} \partial^\mu g, \quad] \quad . \tag{4.2}$$

The momenta conjugate to A^μ and g_{ij} are, respectively,

$$\Pi_0^a = 0 \tag{4.3a}$$

$$\Pi_1^a = F_{10}^a \tag{4.3b}$$

$$\Pi_{ij} = \frac{1}{4\pi}\partial_0 g_{ji}^{-1} + \frac{1}{4\pi} A_{ji}(g) - \frac{ie}{4\pi}(A_- g^{-1})_{ji} . \tag{4.3c}$$

Since, as we have already remarked in section 2, the Hamiltonian will not depend on the unknown function $A(g)$, it is convenient to define a new momentum variable by

$$\hat{\Pi}_{ij} = \Pi_{ij} - \frac{1}{4\pi} A_{ji}(g) . \tag{4.4}$$

It then follows from the Poissons bracket

$$\left\{g_{ij}(x), \Pi_{k\ell}(y)\right\}_t = \delta_{ik}\,\delta_{j\ell}\,\delta(x^1-y^1) \tag{4.5}$$

that $\hat{\Pi}_{ij}$ does not commute with itself,

$$\left\{g_{ij}(x), \hat{\Pi}_{k\ell}(y)\right\}_t = \delta_{ik}\,\delta_{j\ell}\,\delta(x^1-y^1) \tag{4.6a}$$

$$\left\{\hat{\Pi}_{ij}(x), \hat{\Pi}_{k\ell}(y)\right\}_t = -\frac{1}{4\pi} F_{ji;\ell k}(x)\,\delta(x^1-y^1) \tag{4.6b}$$

where $F_{ji;\ell k}$ is given by (2.27) and (2.29). Note that in the absence of the gauge field coupling, the difference between the principal sigma model and Witten's formulation [11] of the free fermion theory resides entirely in the non-vanishing Poisson

bracket (4.6b). The canonical Hamiltonean corresponding to (2.31) is found to be [20]

$$\begin{aligned} H = \int dx^1 \Big\{ & \frac{1}{2}\Pi_1^a \Pi_1^a + A_0^a D_1^{ab}\Pi_1^b - 2\pi\, \mathrm{tr}\, \hat{\Pi}^T g\, \hat{\Pi}^T g \\ & + \frac{1}{8\pi}\mathrm{tr}\,\partial_1 g\,\partial_1 g^{-1} - ie\,\mathrm{tr}\,\hat{\Pi}^T g A_- + \frac{ie}{4\pi}\mathrm{tr}\, g^{-1}\partial_1 g A_- \\ & + \frac{e^2}{8\pi}\mathrm{tr}\, A_-^2 - a\frac{e^2}{8\pi} A_+ A_- \Big\} \end{aligned} \tag{4.7}$$

which represents the non-abelian generalization of eq. (2.4b) in ref. [7].

Now, as eq. (4.3a) shows, we are dealing with a constrained system, which we shall quantize following Dirac's procedure [9]. The primary constraint is given by the weak equality

$$\Omega_1^a \approx 0 \tag{4.8a}$$

with

$$\Omega_1^a = \Pi_0^a \tag{4.8b}$$

The requirement that the constraint (3.8a) be conserved in time, leads to "Gauss's law" as a secondary constraint

$$\Omega_2^a \approx 0 \tag{4.9a}$$

with

$$\Omega_2^a = -D_1^{ab}\Pi_1^b + ie\,\mathrm{tr}(\hat{\Pi}^T g t^a) - \frac{ie}{4\pi}\mathrm{tr}(g^{-1}\partial_1 g t^a) + \frac{e^2}{4\pi}(a-1)A_0^a + \frac{e^2}{4\pi}A_1^a \quad . \tag{4.9b}$$

Since

$$\left\{\Omega_1^a(x),\Omega_2^b(y)\right\}_t = \frac{e^2}{4\pi}\delta^{ab}(1-a)\delta(x^1-y^1) \tag{4.10a}$$

we have - as in the abelian case [7] - a second-class system provided $a \neq 1$. We thus need to distinguish two cases:

Case $a \neq 1$

In order to compute the Dirac brackets we need to find the inverse of the matrix

$$Q_{\alpha a;\beta b} = \left\{\Omega_\alpha^a(x),\Omega_\beta^b(y)\right\} \tag{4.11}$$

with matrix elements given by

$$\left\{\Omega_1^a(x),\Omega_1^b(y)\right\} = 0$$

$$\left\{\Omega_2^a(x),\Omega_2^b(y)\right\} = e f_{abc}\left[\Omega_2^c + \frac{e^2}{4\pi}(A_1^c - (a-1)A_0^c)\right]\delta(x^1-y^1) \tag{4.10b}$$

and (4.10a). The inverse of (4.11) is given by

$$Q^{-1}(x,y) = \left[\frac{4\pi}{(1-a)e^2}\right]^2 \begin{pmatrix} ie\left[\Omega_2 + \frac{e^2}{4\pi}(A_1-(a-1)A_0)\right] & -\frac{1-a}{4\pi}e^2 \\ \frac{1-a}{4\pi}e^2 & 0 \end{pmatrix}\delta(x^1-y^1). \tag{4.12}$$

where $\Omega = \Omega^c t^c$, $t_{ab}^c = if_{acb}$, etc..

Using (4.12) one obtains in the usual way [9] the following (equal time) commutators:

$$\begin{aligned}
\left[A_a^1(x),\Pi_1^b(y)\right] &= i\,\delta^{ab}\,\delta(x^1-y^1) \\
\left[g_{ij}(x),\hat{\Pi}_{k\ell}(y)\right] &= i\,\delta_{ik}\,\delta_{j\ell}\,\delta(x^1-y^1) \\
\left[A_a^0(x),A_b^1(y)\right] &= \frac{4\pi i}{e^2(1-a)}D_1^{ab}(x)\,\delta(x^1-y^1) \\
\left[A_a^0(x),g_{ij}(y)\right] &= \frac{4\pi}{e(1-a)}(gt^a)_{ij}\,\delta(x^1-y^1) \\
\left[A_a^0(x),\Pi_1^b(y)\right] &= -\frac{i}{1-a}\left[\delta^{ab} - \frac{4\pi}{e}f_{abc}\,\Pi_1^c\right]\delta(x^1-y^1) \\
\left[A_a^0(x),\hat{\Pi}_{ij}(y)\right] &= +\frac{1}{(1-a)e}\Big[(t^a g^{-1}(y))_{ji}\,\delta'(x^1-y^1) + \\
&\qquad - 4\pi(t^a\hat{\Pi}^T)_{ji}\delta(x^1-y^1)\Big] \\
\left[\hat{\Pi}_{ij}(x),\hat{\Pi}_{k\ell}(y)\right] &= \frac{i}{4\pi}\left[g_{jk}^{-1}\partial_1 g_{\ell i}^{-1} - g_{\ell i}^{-1}\partial_1 g_{jk}^{-1}\right]\delta(x^1-y^1) \\
\left[A_0^a(x),A_0^b(y)\right] &= +\frac{i(4\pi)^2}{(1-a)^2e^3}f_{abc}\left\{(A_1^c - (a-1)A_0^c)\frac{e^2}{4\pi} + \Omega_2^c\right\}\delta(x^1-y^1).
\end{aligned} \tag{4.13}$$

Using these commutators it now a matter of patience to compute the corresponding commutators for the currents of chiral QCD_2:

$$J_+(x) = -\frac{i}{4\pi} g^{-1}\partial_+ g = i\hat{\Pi}^T g - \frac{i}{4\pi} g^{-1}\partial_1 g - \frac{e}{4\pi} A_-$$

$$J_-(x) = -\frac{i}{4\pi} g\partial_- g^{-1} = -ig\hat{\Pi}^T + \frac{i}{4\pi} g\partial_1 g^{-1} + \frac{e}{4\pi}(gA_- g^{-1}) \; . \tag{4.14}$$

The corresponding commutation relations are:

$$\left[J_+^a(x), J_+^b(y)\right] = -i\,\frac{1+a}{1-a} f^{abc}\left(J_+^c(x) + \frac{e}{4\pi} A_-^c(x)\right)\delta(x-y)$$

$$-\frac{i}{2\pi}\frac{a}{1-a}\delta^{ab}\,\delta'(x-y)$$

$$+\frac{i}{(1-a)^2 e} f^{abc}\left[\Omega_2^c(x) + \frac{e^2}{4\pi}\left(a A_1^c(x) + (1-a) A_-^c(x)\right)\right]\delta(x-y) \tag{4.15}$$

$$\left[J_+^a(x), J_-^b(y)\right] = \frac{1}{1-a} f^{acd}\,\mathrm{tr}(g^{-1}t^b g t^c)(x)\left(J_+^d(x)\right.$$

$$\left.-\frac{e}{4\pi}\frac{a}{1-a} A_1^d(x) - \frac{1}{1-a}\Omega_2^d(x)\right)\delta(x-y) \; . \tag{4.16}$$

It is interesting to notice that we do not recover from above the Kac Moody algebra obtained by Witten [11], in the limit $e \to 0$ and $a \to 0$. This is due to the non-perturbative structure with respect to the charge, of the commutation relations (4.13).

Case $a = 1$

For $a = 1$, the Poisson bracket (4.10a) vanishes, so we have to look for the existence of further constraints. To this end it is convenient to write the $a = 1$ Hamiltonean as

$$H = \int dx^1 \left\{\frac{1}{2}\Pi_1^a\Pi_1^a - 2\pi\,\mathrm{tr}(\hat{\Pi}^T g\hat{\Pi}^T g) + \frac{1}{8\pi}\mathrm{tr}(\partial_1 g\,\partial_1 g^{-1})\right.$$

$$\left. + A_1^a D_1^{ab}\Pi_1^b - A_-^b\Omega_2^b\right\} \; . \tag{4.17}$$

The requirement that

$$\partial_0\Omega_2 \approx 0 \tag{4.18a}$$

then leads to the new constraint $\Omega_3^a \approx 0$, with

$$\Omega_3^a = \frac{e^2}{4\pi}\Pi_1^a + \frac{e^3}{4\pi} f^{abc} A_0^b A_1^c + e f^{abc} A_0^b \Omega_2^c \tag{4.18b}$$

Although

$$\left\{\Omega_1^a(x), \Omega_3^b(y)\right\} = -f^{abc}\left(\frac{e^3}{4\pi} A_1^c + e\,\Omega_2^c\right) \tag{4.19}$$

the system is not second class, since the determinant of the corresponding Q-matrix (4.11) is found to vanish. Hence we need to carry the algorithm one step further, which leads to a fourth constraint $\Omega_4^a \approx 0$, with

$$\Omega_4^a = -\frac{e^2}{4\pi}(D_1\Pi_1)^a + \frac{e^2}{4\pi}A_-^a - \frac{e^3}{4\pi}f^{abc}A_0^b D_1^{cd}A_0^d$$

$$+\frac{2e^3}{4\pi}f^{abc}A_0^b\Pi_1^c - \frac{e^2}{4\pi}\Omega_2^a + e\,f^{abc}A_0^b\Omega_3^c \quad . \tag{4.20}$$

There are no further constraints.

5. CONCLUSION

We have presented the canonical quantization of chiral QCD_2 incorporating the anomalous breakdown of gauge invariance. Because of this anomaly, the Hamiltonean turned out to describe a second class constrained system. We computed the corresponding commutator algebra consistent with the constraints. It is interesting to note that this algebra is local, despite the presence of the Wess-Zumino term (2.25) in the action (2.23). From the current-commutators (4.15, 16) one sees, that the commutation relations do not reduce to the Kac Moody algebra obtained by Witten in the limit $a \to 0$ and $e \to 0$. This is a result of the non-perturbative character of the commutator algebra (4.13).

ACKNOWLEDGEMENT

One of us (K.D.R.) thanks the Physics Department of PUC/RJ and the Physics Institute of USP/SP for the kind hospitality extended to him, and expresses his gratitude to the CNPq-DAAD scientific exchange program, which made this collaboration possible.

REFERENCES

[1] R. Jackiw, in Relativity, Groups and Topology II, R. Stora and B. de Witt, eds. (Les Houches Summer School, 1983), North-Holland, Amsterdam, 1984.

[2] L.D. Faddeev and S.L. Shatashvili, Phys. Lett. 167B, 225 (1986).

[3] O. Babelon, F.A. Schaposnik and C.M. Viallet, Phys. Lett. B177, 385 (1986).
K. Harada and I. Tsutsui, Phys. Lett. B183, 311 (1986).

[4] J. Wess and B. Zumino, Phys. Lett. 37B, 95 (1971).

[5] C.A. Linhares, H.J. Rothe and K.O. Rothe, "On the Cancellation of Anomalies in Chiral Gauge Theories", Heidelberg preprint HD-THEP-86-20, to appear in Phys. Rev. D.

[6] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. 54, 1219 (1985).

[7] H.O. Girotti, H.J. Rothe and K.D. Rothe, Phys. Rev. D33, 514 (1986).

[8] R. Rajaraman, Phys. Lett. 154B, 305 (1985).

[9] P.A.M. Dirac, "Lectures on Quantum Mechanics" (Yeshiva University, New York, 1964).

[10] R. Rajaraman, Phys. Lett. 162B, 148 (1985).

[11] E. Witten, Commun. Math. Phys. 92, 455 (1984).

[12] Our conventions and notations are:

$\gamma^0 = \sigma_x\,,\ \gamma^1 = -i\sigma_y\,,\ \gamma^5 = \gamma^0\gamma^1 = \sigma_3\,;\ \epsilon_{01} = -\epsilon_{10} = 1$

$\left[t^a, t^b\right] = i\, f_{abc}\, t^c\,,\ \ \mathrm{tr}(t^a, t^b) = \delta^{ab}$

$A_\pm = A_0 \pm A_1\,,\ \ \partial_\pm = \partial_0 \pm \partial_1\,;\ \ A^\pm = A$

We have: $\epsilon_{\mu\nu}\gamma^\nu = \gamma_\mu\gamma^5$

[13] R.E. Gamboa-Saravi, F.A. Schaposnik and J.E. Solomin, Nucl. Phys. B185, 239 (1981).
O. Alvarez, Nucl. Phys. B238, 61 (1984).
R. Roskies, Festschrift for Feza Gursey's 60th birthday, Symmetries in Particle Physics, ed. I. Bars, A. Chodos and T.-H. Tze (Plenum, New York, London, 1984).
R.E. Gamboa-Saravi, M.A. Muschietti, F.A. Schaposnik and J.E. Solomin, Ann. of Phys. 157, 360 (1984).
L.C.C. Botelho and M.A.R. Monteiro, Phys. Rev. D30, 2242 (1984).

[14] A. Polyakov and P.B. Wiegman, Phys. Lett. 131B, 121 (1983).

[15] K.D.Rothe, Nucl. Phys. B269, 269 (1986).

[16] The gauge invariant result obtained in ref. [15] can be cast into the form of eq. (2.13); M. Forget, private communication.

[17] P. Di Vecchia, B. Durhuus and J.L. Petersen, Phys. Lett. 144B, 245 (1984).

[18] A.M. Polyakov and P.B. Wiegmann, Phys. Lett. 141B, 223 (1984).

[19] We use a "tilde" in order to avoid confusion in subsequent sections. The superscript T stand for transpose.

[20] Our normalization of e differs from that of ref. [7] by a factor of $2\sqrt{\pi}$.