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FACTORIZABLE S-MATRIX FOR $\frac{SO(D)}{SO(2) \otimes SO(D-2)}$ NON-LINEAR σ MODELS WITH FERMIONS

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ABSTRACT

We compute the exact S matrix for the non-linear sigma model with symmetry $SO(D)/SO(2)\otimes SO(D-2)$ coupled to fermions in a minimal or supersymmetric way. The model has some relevance in string theory with non-zero external curvature.

It has been shown[1] that D-dimensional scale invariance is compatible with a new term to be added to the Nambu-Goto string action, depending on the extrinsic curvature of the world sheet swept by the string in space-time

$$S = \frac{1}{2} T \int d^2\xi \sqrt{g} + \frac{1}{2f} \int d^2\xi (K_{\alpha}^{ib})^2 \sqrt{g} \quad . \tag{1}$$

It can be shown that the second term can be rewritten as follows. Define a normal to the string world-sheet, n_i^{μ}, such that

$$n_i^{\mu} n_j^{\mu} = \delta_{ij} \tag{2a}$$

$$n_i^{\mu} \partial_a x^{\mu} = 0 \tag{2b}$$

and a gauge field A_a^{ik} defined by

$$D_a n_i^{\mu} \equiv \partial_a n_i^{\mu} + A_a^{ik} n_k^{\mu} = - K_a^{ib} \partial_b x^{\mu} \quad . \tag{3}$$

With the above definitions we have the equality

$$\int \sqrt{g}\ K_b^{ia} K_a^{ib} d^2\xi = \int \sqrt{g}\ g^{ab} D_a n_i^{\mu} D_b n_i^{\mu} d^2\xi \tag{4}$$

and the action in the r.h.s. defines a NLSM on a symmetric space

SO(D)/SO(2) ⊗ SO(D-2). The theory has been discussed by several authors[2][3]. In particular[3], a sigma model defined on the manifold SO(D)/SO(2) ⊗ SO(D-2) on a flat two-dimensional metric is classically integrable, even if further constraints hold.

In two dimensions, the gravitational field implies constraints, but it is not physical, in the sense that there is no Einstein action in two dimensions[4]. Therefore, although they may appear as auxiliary fields (namely inside loops[5]), we shall ignore their presence, and study the usual non-linear sigma model defined on the symmetric manifold[6] SO(D)/SO(2) ⊗ SO(D-2). In general an exact factorizable S matrix exibiting a non-abelian gauge structure cannot appear in asymptotic states[7], but we shall define the model in such a way that only the abelian SO(2) is manifest.

The above mentioned classical integrability of the model manifests itself as higher local and non-local conservation laws. In the present case, quantum conservation laws are spoiled by anomalies[7][8]. However, if we couple the model either minimally or supersymmetrically to fermions, those anomalies cancel exactly against the Adler-Bardeen anomaly of the axial current, and higher conservation laws are restored in the quantum theory[9]. Therefore the S matrix is factorizable, and we present a candidate solution for it. In the following, whenever an equation has a label M(S) it refers to the minimal (supersymmetric) case.

The lagrangean densities are

$$L_M = \overline{D^\mu z_i} D_\mu z_i + i\bar\psi_i \not{D}\psi_i \tag{5M}$$

$$L_S = \overline{D^\mu z_i} D_\mu z_i + i\bar\psi_i \not{D}\psi_i + \frac{1}{4}\left[(\bar\psi_i\psi_i)^2 - (\bar\psi_i\gamma_5\psi_i)^2\right] \tag{5S}$$

where $i = 1, \ldots, D$

and $D_\mu = \partial_\mu + A_\mu$

with the constraints

$$\bar z_i z_i = 2 \tag{5aM,S}$$

$$\bar z_i \psi_i = \bar\psi_i z_i = z_i \psi_i = \bar z_i \bar\psi_i = 0 \quad . \tag{5bS}$$

Higher conservation laws are defined in terms of the currents

$$J^\mu_{ij} = \frac{1}{2} z_i \overleftrightarrow{D}^\mu \bar z_j - \frac{1}{2} z_j \overleftrightarrow{D}^\mu \bar z_i \tag{6M}$$

$$J^\mu_{ij} = \frac{1}{2} z_i \overleftrightarrow{D}^\mu \bar z_j + \frac{i}{2}\bar\psi_j\gamma^\mu\psi_i - \frac{1}{2} z_j \overleftrightarrow{D}^\mu \bar z_i - \frac{i}{2}\bar\psi_i\gamma^\mu\psi_j \tag{6S}$$

$$I^\mu_{ij} = i\bar\psi\gamma^\mu\psi(z_i\bar z_j - \bar z_i z_j) \tag{7M}$$

$$I^\mu_{ij} = \frac{i}{2}\left(\bar\psi_j\gamma^\mu\psi_i - \bar\psi_i\gamma^\mu\psi_j + \bar\psi\gamma^\mu\psi(z_i\bar z_j - \bar z_i z_j)\right) \quad . \tag{7S}$$

They obey[5][9]

$$\partial_\mu(J_\nu+I_\nu)_{ij} - \partial_\nu(J_\mu+I_\mu)_{ij} + [J_\mu,J_\nu]_{ij} = N[J_\mu,J_\nu]_{ij} + Z_{[j}\overline{Z}_{i]}(\partial_\mu I_\nu-\partial_\nu I_\mu) \tag{8M}$$

$$\partial_\mu(J_\nu+I_\nu)_{ij} - \partial_\nu(J_\mu+I_\mu)_{ij} + [J_\mu,J_\nu]_{ij} = N[J_\mu,J_\nu]_{ij} + Z_{[i}\overline{Z}_{j]}(\partial_\mu I_\nu^{kk}-\partial_\nu I_\mu^{kk}) \quad . \tag{8S}$$

It has been shown that the r.h.s. of (8) vanishes[9], due to a cancellation between fermion and boson anomalies. This implies conservation of the non-local charge presented in ref. (3). It acts linearly on asymptotic states. Equality of the in and out charges implies strong constraints on the S matrix.

We write an ansatz for it

$$\langle Z_i(\vartheta_1')Z_j(\vartheta_2')|Z_k(\vartheta_1)Z_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_2')\,{}_{k\ell}U_{ij}(\vartheta) + \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}U_{ji}(\vartheta) \tag{9aM,S}$$

$$\langle Z_i(\vartheta_1')\overline{Z}_j(\vartheta_2')|Z_k(\vartheta_1)\overline{Z}_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{kj}U_{i\ell}(i\pi-\vartheta) + \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}R_{ji}(\vartheta) \tag{9bM,S}$$

$$\langle \psi_i(\vartheta_1')\psi_j(\vartheta_2')|\psi_k(\vartheta_1)\psi_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{k\ell}V_{ij}(\vartheta) - \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}V_{ji}(\vartheta) \tag{9cS}$$

$$\langle \psi_i(\vartheta_1')\overline{\psi}_j(\vartheta_2')|\psi_k(\vartheta_1)\overline{\psi}_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{kj}U_{i\ell}(i\pi-\vartheta) - \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}S_{ij}(\vartheta) \tag{9dS}$$

$$\langle Z_i(\vartheta_1')\psi_j(\vartheta_2')|Z_k(\vartheta_1)\psi_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{k\ell}C_{ij}(\vartheta) + \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{kj}D_{i\ell}(\vartheta) \tag{9eS}$$

$$\langle Z_i(\vartheta_1')\overline{\psi}_j(\vartheta_2')|Z_k(\vartheta_1)\overline{\psi}_\ell(\vartheta_2')\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{kj}C_{i\ell}(i\pi-\vartheta) + \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}E_{ij}(\vartheta) \tag{9fS}$$

$$\langle \psi_i(\vartheta_1')\overline{\psi}_j(\vartheta_2')|Z_k(\vartheta_1)\overline{Z}_\ell(\vartheta_2)\rangle = \delta(\vartheta_1-\vartheta_1')\delta(\vartheta_2-\vartheta_2')\,{}_{kj}D_{i\ell}(i\pi-\vartheta) + \delta(\vartheta_1-\vartheta_2')\delta(\vartheta_2-\vartheta_1')\,{}_{k\ell}F_{ij}(\vartheta) \tag{9gS}$$

where $\vartheta = \vartheta_1 - \vartheta_2 \quad , \quad \vartheta_1 > \vartheta_2 \quad , \quad \vartheta_1' > \vartheta_2' \quad ,$

$P_i = m(\mathrm{sh}\,\vartheta_i \, , \mathrm{ch}\,\vartheta_i) \quad ,$ and

$${}_{k\ell}A_{ij}(\vartheta) = a_1(\vartheta)\delta_{ij}\delta_{k\ell} + a_2(\vartheta)\delta_{ki}\delta_{j\ell} + a_3(\vartheta)\delta_{i\ell}\delta_{jk} \quad .$$

Conservation of the non-local charge implies

$$x_1(\vartheta) = -\frac{2\pi i}{i\pi - \vartheta}\,\frac{x_2(\vartheta)}{D-2} \tag{10a}$$

$$x_3(\vartheta) = -\frac{2\pi i}{\vartheta}\,\frac{x_2}{D-2} \quad . \tag{10b}$$

$$y_1(\vartheta) = -\frac{2\pi i}{i\pi-\vartheta}\frac{y_3(\vartheta)}{D-2}$$

$$y_2(\vartheta) = -\frac{2\pi i}{\vartheta}\frac{y_3(\vartheta)}{D-2}$$

where x are the transmition amplitudes (u, v, c and d) and y reflection amplitudes (r, s, e and f).

These equations are not enough to determine the solution. We use the triangle relation in the minimal case, which imply

$$u_2(\vartheta)u_2(i\pi-(\vartheta+\vartheta'))\,r_3(\vartheta') = r_3(\vartheta)r_3(\vartheta+\vartheta')u_2(i\pi-\vartheta') + $$
$$+ u_2(i\pi-\vartheta)u_2(\vartheta+\vartheta')r_3(\vartheta') \tag{11aM}$$

$$u_2(\vartheta)r_3(\vartheta+\vartheta')u_2(\vartheta') = r_3(\vartheta)u_2(\vartheta+\vartheta')r_3(\vartheta') +$$
$$+ u_2(i\pi-\vartheta)r_3(\vartheta+\vartheta')u_2(i\pi-\vartheta') \tag{11bM}$$

whose solution is

$$u_2(\vartheta) = -\frac{\sin\mu(\vartheta-i\pi)}{\sin\mu i\pi}\,r_3(\vartheta) \quad . \tag{12M}$$

Compatibility with perturbative results, however, selects the solution

$$u_2(\vartheta) = \frac{i\pi-\vartheta}{i\pi}\,r_3(\vartheta) \quad . \tag{13M}$$

Unitarity is equivalent to

$$r_3(\vartheta)r_3(-\vartheta) = \frac{\vartheta^2}{\vartheta^2+\lambda^2}\,\frac{\pi^2}{\vartheta^2+\pi^2} \tag{14M}$$

$$\lambda = \frac{2\pi}{D-2} \quad .$$

The minimal solution is[10]

$$r_3(\vartheta) = \frac{1}{2}R(\vartheta)R(i\pi-\vartheta)U(\vartheta)U(i\pi-\vartheta) \tag{15aM}$$

$$R(\vartheta) = \frac{\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{\lambda}{2\pi}\right)\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{1}{2}\right)}{\Gamma\left(-\frac{i\vartheta}{2\pi}\right)\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{\lambda}{2\pi}+\frac{1}{2}\right)} \tag{15bM}$$

$$U(\vartheta) = \frac{\Gamma\left(\frac{1}{2}-\frac{i\vartheta}{2\pi}\right)}{\Gamma\left(1-\frac{i\vartheta}{2\pi}\right)} \tag{15cM}$$

without further poles.

For the sake of completeness, we give the solution with $\mu = ib \neq 0$

$$r_3(\vartheta) = \frac{\Gamma(b)}{\pi}\sin b\pi\,R(\vartheta)R(i\pi-\vartheta)U(\vartheta)U(i\pi-\vartheta) \tag{16aM}$$

$$R(\vartheta) = \frac{\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{\lambda}{2\pi}\right)\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{1}{2}\right)}{\Gamma\left(-\frac{i\vartheta}{2\pi}\right)\Gamma\left(-\frac{i\vartheta}{2\pi}+\frac{\lambda}{2\pi}+\frac{1}{2}\right)} \tag{16bM}$$

$$U(\vartheta) = \Gamma\left(1 + \frac{ib\vartheta}{\pi}\right) \prod_{\ell=1}^{\infty} \frac{S_\ell(\vartheta)}{S_\ell(0)} \tag{16cM}$$

$$S_\ell(\vartheta) = \frac{\Gamma\left(2\ell b + \frac{ib\vartheta}{\pi}\right)\Gamma\left(1 + 2\ell b + \frac{ib\vartheta}{\pi}\right)}{\Gamma\left((2\ell+1)b + \frac{ib\vartheta}{\pi}\right)\Gamma\left(1 + (2\ell-1)b + \frac{ib\vartheta}{\pi}\right)} . \tag{16dM}$$

In the supersymmetric case we found 15 equations of factorization. But analogously to the minimal case we have

$$u_2(\vartheta) = -\frac{\sin\mu(\vartheta-\nu)}{\sin\mu\vartheta}\, c_2(\vartheta) \tag{17aS}$$

$$v_2(\vartheta) = -\frac{\sin\mu(\vartheta+\nu)}{\sin\mu\vartheta}\, c_2(\vartheta) \tag{17bS}$$

$$d_3(\vartheta) = f_3(\vartheta) = \frac{\sin\mu\nu}{\sin\mu\vartheta}\, c_2(\vartheta) \tag{17cS}$$

$$r_3(\vartheta) = s_3(\vartheta) = \frac{\sin\mu\nu\ \sin\mu i\pi}{\sin\mu\vartheta\ \sin\mu(i\pi-\vartheta)}\, c_2(\vartheta) \tag{17dS}$$

$$c_2(\vartheta) = c_2(i\pi-\vartheta) = e_2(\vartheta) . \tag{17eS}$$

Analogously to CP^{D-1} and $O(D)$ models[11][10] obtained as particular limits we find

$$\mu = \frac{i}{2} \tag{18aS}$$

$$\nu = -\frac{2\pi i}{D-2} = -i\lambda \tag{18bS}$$

Finally, unitarity turns out to be

$$c_2(\vartheta)c_2(-\vartheta) = \frac{\vartheta^2}{\vartheta^2+\lambda^2}\ \frac{\mathrm{sh}^2\frac{1}{2}\vartheta}{\mathrm{sh}^2\frac{1}{2}\vartheta + \sin^2\frac{\lambda}{2}} \tag{19S}$$

with the minimal solution

$$C_2(\vartheta) = \prod_{\ell=0}^{\infty} \frac{\Gamma\left(\frac{\lambda}{2\pi} - \frac{i\vartheta}{2\pi} + \ell\right)\Gamma\left(1 + \frac{i\vartheta}{2} + \ell\right)}{\Gamma\left(1 + \frac{i\vartheta}{2\pi} + \frac{\lambda}{2\pi} + \ell\right)\Gamma\left(-\frac{i\vartheta}{2\pi} + \ell\right)} \prod_{\ell=1}^{\infty} \frac{\Gamma\left(-\frac{i\vartheta}{2\pi} - \frac{\lambda}{2\pi} + \ell\right)\Gamma\left(1 + \frac{i\vartheta}{2\pi} + \ell\right)}{\Gamma\left(\frac{1}{2} + \frac{i\vartheta}{2\pi} - \frac{\lambda}{2\pi} + \ell\right)\Gamma\left(-\frac{i\vartheta}{2\pi} + \ell\right)} . \tag{20S}$$

REFERENCES

(1) A.M. Polyakov, Nucl. Phys. B268 (1986) 406.

(2) T.L. Curtright, G.I. Ghandour, C.B. Thorn, C.K. Zachos, Phys. Rev. Lett. 57 (1986) 799, Phys. Rev. D.
E. Braaten, C.K. Zachos, Argonne prep. ANL-HEP-PR-86-114.
E. Braaten, R.D. Pisarski, S-MT ANL-HEP-PR-86-108.

(3) E. Abdalla, M.C.B. Abdalla, A. Lima-Santos, Int. Journal of Mod. Phys., to appear.

(4) S. Deser, B. Zumino, Phys. Lett. 65B (1976) 369.

(5) E. Abdalla, R.S. Jasinschi, Nucl. Phys. B232 (1984) 426.

(6) H. Eichenherr, M. Forger, Nucl. Phys. B155 (1979) 381.

(7) E. Abdalla, M. Forger, A. Lima-Santos, Nucl. Phys. B256 (1985) 145.

(8) E. Abdalla, M. Forger, M. Gomes, Nucl. Phys. B210 (1982) 181.

(9) E. Abdalla, M.C.B. Abdalla, M. Gomes, Phys. Rev. D25 (1982) 452, D27 (1983) 825.

(10) A.B. Zamolodchikov, Al.B. Zamolodchikov, Ann. Phys. 120 (1979) 253.
R. Shankar, E. Witten, Phys. Rev. D17 (1978) 2134.

(11) E. Abdalla, A. Lima-Santos, Phys. Rev. D29 (1984) 185.
R. Köberle, V. Kurak, Phys. Rev. Lett. 58 (1987).