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SUMMARY

Assuming that quarks are gentileons we show that fundamental properties of composed hadrons are rigorously predicted from first principles. Stimulated by these results we propose in this paper a modified quantum chromodynamics where, instead of fermions, gentileons interact with gluons. It is shown that, when the Drell-Yan model is adopted, our approach and the usual QCD give identical predictions for the hadronic properties.

1. Introduction

During the last two decades, the theory of elementary particles has been developed in terms of the quark hypothesis of Gell-Mann-Zweig. The SU(3) (or the SU(6)) symmetry combined with several other group transformations has served as a guide to classify all known strongly interacting particles. Nevertheless, in spite of its great success in classifying the several multiplets found in nature, Gell-Mann's theory leaves some major questions open for investigation. Among these great problems we are interested in, we quote the statistical one which arises with the fermionic character of quarks and the two intriguing peculiar problems posed by the quark hypothesis: the quark confinement and the non-coalescence of hadrons. These are the most important problems in hadronic physics. Although being intensively studied, some aspects of these problems remain unsolved. With this in mind we have suggested, a few years ago[1-3], an alternative approach to the statistical problem of the quarks, assuming that they obey Gentile statistics instead of Fermi statistics.

In section 2 we present our main results about Gentile statistics and gentilionic systems[1].

In section 3 we make a summary of our preceding papers[1-3] where it is shown that, taking quarks as spin 1/2 gentileons, the following fundamental properties must be obeyed: quark confinement, non-coalescence of hadrons, baryon number conservation, existence of only color singlet hadrons and that the hadron color charge is a constant of motion equal to zero. These properties result from selection rules derived rigorously from basic principles of quantum mechanics.

In section 4 we propose an alternative quantum chromodynamics where, instead of fermions, gentileons interact with gluons. This approach will be referred as QCDG. It will be seen that, assuming the Drell-Yan model, the usual QCD and the QCDG will give identical predictions for the hadronic properties.

2. The Statistical Principle and the Gentileons

In a preceding work[1] we have shown rigorously, according to the postulates of quantum mechanics and to the principle of indistinguishability, that three kinds of particles could exist in nature: bosons, fermions and gentileons. These results can be synthesized in terms of the following statement (Statistical Principle): "Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively".

Bosonic and fermionic systems are described by one-dimensional totally symmetric and totally anti-symmetric wavefunctions, respectively. For bosons and fermions the creation and annihilation operators obey bi-linear commutation relations.

Gentilionic systems are described by multi-dimensional (spinorial like[3]) wavefunctions with mixed symmetries. Since they are represented by intermediate Young shapes, only three or more gentileons can form a system of indistinguishable particles. This means that two identical gentileons are <u>prohibited</u> to constitute a system of indistinguishable particles. This implies that gentileons cannot appear freely. Indeed, if this were possible, two free identical gentileons could constitute a two-particle system in an occasional collision. For gentileons the creation and annihilation operators obey multi-linear matricial commutation relations. Finally, due to very peculiar geometric properties[1] of the intermediate states, there appear <u>selection rules</u> confining the gentileons and prohibiting the coalescence of gentilionic systems. Two systems like [ggg] and [gggg], for instance, cannot coalesce into a composite system of indistinguishable particles [ggggggg]. Only bound states [ggg] - [gggg] could be possible. The gentileon confinement appears as a consequence of the selection rule which prohibits the decomposition of a system [ggg...gg] into [ggg...g] and [g].

In our above quoted papers[1,2] only systems of identical gentileons have been considered. Let us now consider systems composed of two different kinds of gentileons, g and G. Taking into account the statistical principle we must expect that systems like [gG], [gggG], [gggGGG] and so on, are allowed. On the other hand, systems like [ggG], [ggGG], [gggGG] ... are prohibited because [gg] and [GG] are not allowed[1]. Of course, the coalescence of mixed systems is also forbidden, as can be easily verified. It is important to note that the commutation relations for the creation and annihilation operators for g and G in [gG] must be bi-linear since the state-vector of the system is one-dimensional[1]. Thus, according to the special theory of relativity[4-6], they will be taken as commutative or anti-commutative for g(G) if the spin of g(G) is integer or half-integer, respectively.

The confinement and non-coalescence are intrinsic properties of gentileons as the total symmetrisation (anti-symmetrisation) is intrinsic to bosons (fermions), not depending on their physical interpretation. Thus, they could be assimilated to individual real particles or to dynamical

entities as quantum collective excitations. However, due to the selection rules imposed on the gentileons we think that they would be quite different from the usual particles and quantum collective states.

3. The Gentilionic Hadrons

Since gentileons are confined entities and their systems are non-coalescent it seems natural to think quarks as spin 1/2 gentileons[1,2]. With this hypothesis, the baryons [qqq], that are formed by three identical gentileons in color space, are represented by the wavefunctions $\psi = \varphi \cdot Y(\text{color})^{(2)}$. The one-dimensional wavefunction $\varphi = (SU(6) \times O_3)_{\text{symmetric}}$ corresponds, according to the symmetric quark model of baryons, to a totally symmetric state. The four-dimensional state Y(color) corresponds to the intermediate representation of the S_3 group. Assuming that the color states are the $SU(3)_{\text{color}}$ eigenstates, red, blue and yellow, Y(color) is written as[1-3]:

$$Y(\text{color}) = Y(123) = Y(bry) = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_1(123) \\ Y_2(123) \\ Y_3(123) \\ Y_4(123) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_+ \\ \\ Y_- \end{pmatrix}$$

where, $Y_+ = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, $Y_- = \begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}$,

$$Y_1(123) = (|bry\rangle + |rby\rangle - |ybr\rangle - |yrb\rangle)/\sqrt{4}\ ,$$
$$Y_2(123) = (|bry\rangle + 2|byr\rangle - |rby\rangle + |ybr\rangle - 2|ryb\rangle - |yrb\rangle)/\sqrt{12}$$
$$Y_3(123) = (-|bry\rangle + 2|byr\rangle - |rby\rangle - |ybr\rangle + 2|ryb\rangle - |yrb\rangle)/\sqrt{12}$$

and $Y_4(123) = (|bry\rangle - |rby\rangle - |ybr\rangle + |yrb\rangle)/\sqrt{4}$.

The spinorial character[1] of the state function Y(color) is responsible for selection rules predicting non-coalescence and quark confinement. It is worthwhile to note that, in this context, our theory differs drastically from parastatistics[7-11] and fermionic theories of quarks. In the fermionic case, Y(123) would be given by,

$$Y(123) = (|bry\rangle - |byr\rangle - |rby\rangle + |ybr\rangle + |ryb\rangle - |yrb\rangle)/\sqrt{6},$$

and in parastatistics case Y(123) would be written as, $Y(123) = a\,Y_1 + b\,Y_2 + c\,Y_3 + d\,Y_4$, where a, b, c and d are arbitrary constants. For these last theories the wavefunction Y(123) is one-dimensional, from which the selection rules above mentioned, cannot be deduced.

In a preceding paper[3] the state function Y(123) has been named "colorspinor" since we have shown that it is a bi-spinor in a 3-dimensional color space E_3. A particular, but very important, choice for this space is made when we assume the color states as the eigenstates of the $SU(3)_{\text{color}}$. In this case the axes X and Z of E correspond to the color isospin $\tilde{I}_3$ and to the color hypercharge $\tilde{Y}$, respectively. In these conditions it is easy to verify[3] that the color Casimir $K^{[2,1]}_{(2,1)} = 0$, which is the invariant of the S_3 algebra[2] associated with the gentilionic state, can be identified with the total baryon color charge operator $\tilde{Q}$. Then, by using the extended form of

Noether's theorem it can be proven[2,3] that the baryon color charge is a constant of motion equal to zero, that is, $\langle \tilde{Q} \rangle$ = constant = 0. This led us to a very important consequence when the colors are eigenstates of the $SU(3)_{color}$: the state function Y(123) must necessarily be composed by three different colors[3], that is, Y(color) = Y(bry). Then, the baryon wavefunctions are given by $\psi = \varphi \, . \, Y(bry)$.

In our approach[1,2] mesons are composed by a quark-antiquark pair $[q\bar{q}]$. According to the statistical principle (see section 2), systems like [q], [qq], $[qq\bar{q}]$ and $[qq\bar{q}\bar{q}]$, for instance, are prohibited (it could exist only bound states $[q\bar{q}]$ - $[q\bar{q}]$ of the mesons $[q\bar{q}]$).

Now let us make a summary of the fundamental properties that must be observed for composed hadrons if quarks are spin 1/2 gentileons: (1) quarks are confined, (2) baryons and mesons cannot coalesce, (3) baryonic number is conserved and (4) the hadron color charge is a constant of motion equal to zero. Incorporating the $SU(3)_{color}$ representation, both 3 and $\bar{3}$, into the gentilionic S_3 symmetry we see that the following additional conditions will appear: (5) the baryon wavefunctions will be given by $\psi = \varphi \, . \, Y(bry)$ and (6) only color singlet hadrons can exist.

The above mentioned hadronic properties have been predicted independently of the intrinsic nature of the gentileons; they could be particles, quantum collective excitations or something else. Consequently, no dynamical hypothesis, phenomenological or approximate arguments have been

used to prove them. They have been deduced from first principles: from the statistical principle or by using the symmetries of the S_3 intermediate representation. Thus, if quarks are gentileons, even though we uphold the intrinsic geometrical nature of confinement, we cannot exclude the possibility that there may be hidden or explicit a confining mechanism in the dynamical laws. After all, we cannot reduce all the concepts, which enter into a dynamical law, to geometrical notions. The confining mechanism could be produced by a very peculiar interaction between quarks, by an impermeable bag as proposed by the bag model or something else. At the moment these mechanisms are unknown. It is not our intention, in this paper, to study this problem.

We remark that, in the fermionic theory the baryon wavefunctions are given by $\psi = \varphi \, . \, Y_f(bry)$, where $\varphi = (SU(6) \times O_3)_{symmetric}$ and $Y_f(bry) = (SU(3)_{color})_{antisymmetric}$ corresponds to the totally antisymmetric fermionic color function. On the other hand, if quarks are taken as gentileons, $\psi = \varphi \, . \, Y(bry)$, where $Y(bry) = (SU(3)_{color})_{gentilionic}$. Thus, we see that in our theory the SU(3) continuous symmetry is maintained. Only the S_3 fermionic symmetry is substituted by the S_3 gentilionic symmetry, both discrete.

In spite of our stimulating general results, there remains the crucial problem of determining the intrinsic nature of the quarks and their dynamical properties. According to the current theoretical ideas, quarks are fermionic elementary particles. The mathematical formulation of the fermionic model, the QCD, is a successful modern field theory since it is able to explain many properties of the hadrons. In next section, taking quarks as spin 1/2 gentileons, a quantum chromodynamics is proposed where, instead of fermions, gentileons interact with gluons. This formalism (QCDG) will be compared with the usual QCD. It will be shown that adopting the Drell-Yan model the QCD and the QCDG will give identical predictions for the hadronic properties.

4. The Quantum Chromodynamics and the Drell-Yan Model

In this section a quantum chromodynamics using gentilionic quarks is proposed. This approach, indicated by QCDG, will be compared with the standard QCD. In this way, remembering that quarks have spin 1/2 and taking into account the SU(3) symmetry (color and flavor), the following Lagrangian density for gentilionic quarks interacting with gluons is suggested:

$$L = \sum_f [i\, q_a^+ \gamma^\mu \frac{\partial}{\partial x^\mu} q_a + g\, q_a^+ \gamma^\mu \left(\frac{\lambda_i}{2}\right)_{ab} A_\mu^i\, q_b - m_f\, q_a^+ q_a] +$$

$$- \frac{1}{4} \left(\frac{\partial A_\nu^i}{\partial x^\mu} - \frac{\partial A_\mu^i}{\partial x^\nu} + g\, f_{ijk} A_\mu^i A_\nu^k \right)^2 \qquad (4.1)$$

where the summation is over the flavors $f = u,d,s,c$ ... The summation over repeated indices a,b, ..., referring to color is understood. The $\lambda_i/2$ are the 3 x 3 matrix representation of the $SU(3)_{color}$ algebra generators, satisfying the commutation relations $[\lambda_i, \lambda_j] = i\, f_{ikj}\, \lambda_k/2$, where f_{ijk} are the SU(3) structure constants. The flavor symmetry is only oroken by the lack of degeneracy in the quark masses. Finally, the quark free fields q(x) are expanded in terms of positive and negative frequency solutions, $\varphi_{k+}(x)$ and $\varphi_{k-}(x)$, of Dirac's equation,

$$q(x) = \sum_k \{a_{k+}\, \varphi_{k+}(x) + a_{k-}^+\, \varphi_{k-}(x)\}$$

It is important to remark that, with the above assumptions, both theories, QCD and QCDG, will have the same gluons and the same Lagrangian densities. However, the creation and annihilation quark operators obey different commutation relations in these theories: in QCD they are bi-linear fermionic and in QCDG they are matricial gentilionic[1]. This difference will be analysed in what follows.

In QCD, quarks being fermions, a_α and a_α^+, obey the well known bi-linear relations, independently of the hadronic system:

$$[a_\alpha^+, a_\beta]_+ = \delta_{\alpha\beta}$$
$$[a_\alpha^+, a_\beta^+]_+ = [a_\alpha, a_\beta]_+ = 0 \qquad (4.2)$$

Considering now the gentilionic hadrons, let us see first the mesons $[q\bar{q}]$. According to section 2, the commutation relations for $\bar{q}$ and q are determined only by their spins. Since these are equal to 1/2, q and $\bar{q}$ can be taken as fermions from the algebraic point of view. Consequently, for processes involving only mesons, the QCDG and the QCD would give exactly the same predictions.

For baryons the quantum field calculations, in the general case, would be more complicated since the creation and annihilation operators obey gentilionic matricial relations[1]. However, a simplification is introduced when the color states

are taken as the eigenstates of the $SU(3)_{color}$, blue, red and yellow. In this case Y(color) must necessarily be composed by these three different colors[3], resulting for the baryon wavefunctions, $\psi = \varphi \cdot Y(ory)$. In these conditions, the quarks in [qqq] that, according to and Y(bry), have disponible an infinite number of quantum states, cannot assume the same color in the color space. In other words, two quarks in [qqq] cannot occupy the same quantum state. With this fermionic characteristic it is not difficult to verify[1] that the number of independent gentilionic commutation relations are reduced, remaining only a few ones:

$$[a_\alpha^+, a_\beta]_+ = \delta_{\alpha\beta},\ a_i\, a_i = a_i^+ a_i^+ = 0,$$
$$a_i^+ a_j^+ a_k^+ = G\binom{\alpha\beta\gamma}{ijk}\, a_\alpha^+ a_\beta^+ a_\gamma^+ \quad \text{and} \qquad (4.3)$$
$$a_i\, a_j\, a_k = G\binom{kji}{\gamma\beta\alpha}\, a_\alpha\, a_\beta\, a_\gamma\ ,$$

where the indices i, j and k can assume the values α, β and γ and G(...) are 4 x 4 matrices given elsewhere[2]. From the tri-linear relations we can obtain, for instance, the following transpositions, considered as bi-linear relation

$$a_\beta^+ a_\alpha^+ = G\binom{\alpha\beta\gamma}{\beta\alpha\gamma}\, a_\alpha^+ a_\beta^+$$
$$\text{and} \qquad (4.4)$$
$$a_\beta\, a_\alpha = G\binom{\gamma\alpha\beta}{\gamma\beta\alpha}\, a_\alpha\, a_\beta\ .$$

In spite of the great simplifications that have been introduced, it is evident from Eqs. (4.3) and (4.4) that gentilionic quarks cannot rigorously be taken as fermions since only the first three relations, $[a_\alpha^+ , a_\beta]_+ = \delta_{\alpha\beta}$, $a_i a_i = a_i^+ a_i^+ = 0$ are bi-linear fermionic. However, the bi-linear relations, $a_\beta a_\alpha$ and $a_\beta^+ a_\alpha^+$, and the tri-linear relations need to be employed only if we intend to take into account properties which are common to pairs of particles or to three particles in the [qqq] system. Thus, if we assume, in a first approximation, that in the baryonic processes only one quark participates, and the remaining two are spectators (Drell-Yan ,model[12]), the bi-linear and the tri-linear feature of the commutation relations will be irrelevant in cross section calculations. Under these circumstances only the bi-linear fermionic commutation relations need to be used and, consequently, gentileons can be taken as fermions from the algebraic point of view.

Thus, we see from the above analysis that, when the Drell-Yan model is valid, the QCD and the QCDG will give identical predictions for the hadronic properties. In both approaches the following additional conditions are assumed: (a) quark confinement, (b) non-coalescence of hadrons, (c) baryon number conservation, (d) only color singlet hadrons exist and (e) the hadron color charge is a constant of motion equal to zero. In spite of this we must note that the fermionic and the gentilionic theories are not equivalent. Indeed, in the QCDG these fundamental additional conditions appear naturally,

deduced rigourously from first principles, whereas in the QCD they are imposed "ad hoc".

In terms of cross section calculations, using the QCD and QCDG, probably it will be possible to decide if quarks are fermions or gentileons taking into account correlations between quarks in baryons. This, however, seems to be an extremely difficult task not only for gentileons but also for fermions[13].

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