UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-649

REFRACTION-DIFFRACTION INTERFERENCE IN HEAVY-ION ELASTIC ANGULAR DISTRIBUTIONS

M.P. Pato and **M.S. Hussein**
Instituto de Física, Universidade de São Paulo

Junho/1987

REFRACTION-DIFFRACTION INTERFERENCE IN HEAVY-ION ELASTIC ANGULAR DISTRIBUTIONS

M.P. Pato and M.S. Hussein*

Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, S.P., Brasil

ABSTRACT

A new representation of the refractive and diffractive parts of the elastic scattering amplitude is derived. Application is made to the elastic scattering of $^{13}C + ^{12}C$ at $E_{Lab} = 260.00$ MeV, recently discussed by several authors.

*Supported in part by the CNPq.

June/1987

Quite recently, Bohlen et al.[1] and Fricke and McVoy[2] have discussed an interesting diffraction-refraction interference phenomenon which seems to manifest itself in the farside component of the elastic scattering amplitude and cross-section of $^{12}C + ^{12}C$ at intermediate energies. The experimental data of Bohlen et al. (at $E = 20$ MeV/A) exhibit a rather shallow minimum at $\vartheta = 44^{o}$ and a broad maximum at $\vartheta \simeq 55^{o}$. The maximum results from a constructive interference between a refractive rainbow trajectory and a peripheral diffractive contribution. The refractive contribution represents deeply penetrating trajectories well inside the grazing distance of closest approach of ~ 6 fm.

The analysis of Ficke and McVoy[2] confirmed the existence of the phenomenon and investigated what types of systems might exhibit it more strongly. In particular they found empirically, within their optical potential analysis, that for the diffractive-refractive interference to occur, the radius of the imaginary part of the optical potential must be much larger ($\sim 30\%$) than that of its real part. The strength of imaginary part should be reasonably large.

In the present paper we further investigate this phenomenon. In particular, we present a potential-independent analysis based on absorption-modified uniform semiclassical approximation to the scattering amplitude with a conveniently parametrized elastic S-matrix element. This method, previously formally developed by Frahn and Gross[3], is extensively studied and further developed in Ref. 4). It enables a clear separation between diffraction and refractive contributions and therefore is ideal for studying the interference phenomenon of Bohlen et al.[1].

The starting point of our method is the usual Near/Far decomposition of the elastic scattering amplitude,

$$f(\theta) = f^{(+)}(\theta) + f^{(-)}(\theta) \tag{1}$$

$$f^{(\pm)}(\theta) = \frac{1}{ik\sqrt{2\pi \sin\theta}}\, e^{\pm i\pi/4} \sum_{\ell=0}^{\infty} (\ell+\tfrac{1}{2}) \left[|S(\ell)| e^{2i\delta(\ell)} \right] e^{\mp i(\ell+1/2)\theta} \tag{2}$$

The above decomposition is being widely used in the analysis of heavy-ion elastic scattering data. A recent review can be found in Ref. 5). When many partial waves are involved in the ℓ-sum above one may approximate it by an integral accordingly, after extending the lower limit of the integrals to $-\infty$ (quite safe since $|S_\ell| = 0$ for negative $\ell + \frac{1}{2} \equiv \lambda$), $I^{\pm} = f^{\pm}\sqrt{\sin\theta}$ become just Fourier transforms of $\sqrt{\lambda}\ |S(\lambda)|\ e^{2i\delta(\lambda)}$. We introduce another Fourier transform involving only $\sqrt{\lambda}\ e^{2i\delta(\lambda)}$ and call it $I_0^{(\pm)}(\theta)$

$$I_0^{(\pm)}(\theta) = \frac{e^{\pm i\pi/4}}{ik\sqrt{2\pi}} \int \lambda^{1/2} d\lambda\, e^{\mp i\lambda\theta}\, e^{2i\delta(\lambda)} \tag{3}$$

We now proceed to relate $I^{(\pm)}(\vartheta)$ to $I_0^{(\pm)}(\vartheta)$. This can be accomplished easily through a three-step procedure

Fourier transform $I^{(\pm)}(\vartheta)$, divide over $|S(\lambda)|$ and inverse Fourier transform. Calling the Fourier transform operator $\mathcal{F}$ and its inverse $\mathcal{F}^{-1}$, we have

$$\mathcal{F}^{-1}_{\lambda\to\theta}\, |S(\lambda)|^{-1}\, \mathcal{F}_{\theta\to\lambda}\, I^{(\pm)}(\theta) = I_0^{(\pm)}(\theta) \tag{4}$$

The operator $\mathcal{F}^{-1}|S(\lambda)|^{-1}\mathcal{F}$ is called a <u>pseudodifferential operator</u>[6]. It is easy to show that this operator can be written in the following form

$$\mathcal{F}^{-1}_{\lambda\to\theta}\, |S(\lambda)|^{-1}\, \mathcal{F}_{\theta\to\lambda} = \left|S\left(i\frac{d}{d\theta}\right)\right|^{-1} \tag{5}$$

where the right-hand side is to be understood as the modulus of $S(\lambda)$ taken with λ replaced by $i\,\frac{d}{d\vartheta}$.

We thus obtain the desired equation which relates the physical amplitudes $I^{(\pm)}(\vartheta)$ to the absorption-free amplitudes $I_0^{(\pm)}(\vartheta)$

$$\left|S\left(i\frac{d}{d\theta}\right)\right|^{-1} I^{(\pm)}(\theta) = I_0^{(\pm)}(\theta) \tag{6}$$

In heavy-ion scattering, $|S(\lambda)|$ is very close to a Fermi function

$$|S(\lambda)| = \left(1 + \exp\frac{\Lambda-\lambda}{\Delta}\right)^{-1} \tag{7}$$

Thus Eq. (6) becomes

$$\left[1+\exp\left[\left(\Lambda - i\frac{d}{d\theta}\right)/\Delta\right]\right] I^{\pm}(\theta) = I_0^{\pm}(\theta) \tag{8}$$

Eqs. (6) and/or (8) can be solved easily using the Green function method. To be specific we give the solution to Eq. (8),

$$I^{(\pm)}(\theta) = \frac{i}{2\pi} \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} d\theta' \, \frac{e^{i\Lambda(\theta'-\theta)} F(\Delta(\theta'-\theta))}{\theta'-\theta+i\varepsilon} \, I_0^{(\pm)}(\theta') \tag{9}$$

with $F(\Delta Z) \equiv \frac{\pi \Delta Z}{\sinh \pi \Delta Z}$, being the Fourier transform of $\frac{d}{d\ell}|S_\ell|$. We would like to emphasize here that Eq. (9) can be easily generalized to other parametrizations of $|S_\ell|$. The generality of Eq. (9) resides in the function F which can be simply taken as the Fourier transform of $\frac{d}{d\ell}|S_\ell|$.

We now decompose $I(\vartheta)$ into its diffractive, I_D and refractive I_R components

$$I^{(\pm)}(\theta) = I_D^{(\pm)}(\theta) + I_R^{(\pm)}(\theta) \tag{10}$$

where, from Eq. (9)[7],

$$I_D^{\pm}(\theta) = \frac{i}{2\pi} \frac{F(\Delta(\theta_\Lambda-\theta))}{\theta_\Lambda-\theta} \int_{-\infty}^{\infty} d\theta' e^{i\Lambda(\theta'-\theta)} I_0^{(\pm)}(\theta) \tag{11}$$

and

$$I_R^{\pm}(\theta) = \frac{i}{2\pi} \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} d\theta' e^{i\Lambda(\theta'-\theta)} \left[\frac{F(\Delta(\theta'-\theta))}{\theta'-\theta+i\varepsilon} - \frac{F(\Delta(\theta_\Lambda-\theta))}{\theta_\Lambda-\theta+i\varepsilon}\right] I_0^{\pm}(\theta') \tag{12}$$

The angle ϑ_Λ is determined from the condition

$$\frac{d}{d\theta'}\left(\Lambda\theta' + \Phi(\theta')\right)\Big|_{\vartheta'=\theta_\Lambda} = 0 \tag{13}$$

with $\Phi(\vartheta')$ being the phase of $I_0(\vartheta')$.

If $I_0(\vartheta')$ is dominated by a stationary phase, then ϑ_Λ is just the corresponding classical deflection function. However, Eq. (11) can be used in a more general sense. For instance if there are more than one s.p., I_D become a sum of their conbributions.

Notice that our refractive component $I_R^{\pm}$ is not the absorption-free amplitude $I_0(\vartheta)$ since it is modified by the factor inside the square brackets.

In what follows we shall present evidence that the phenomenon observed by Bohlen et al. is a result of the interference between I_R and I_D which would confirm the discussion of Fricke and McVoy[2]. Whether our I_R and I_D correspond exactly to theirs is an open question to be discussed later.

We have used the optical potential of Ref. 1

to generate the S-matrix elements that enter in our formulae. We have first checked that the reflection coefficient $|S_\ell|$ looks very much like a Fermi function, as in Eq. (7). We have further confirmed that the position of the nuclear rainbow (in ℓ) is about ten units of $\hbar$ inside of Λ (Eq. (7)). This fact clearly indicates that the inner branch ($\ell < \ell_r$) of the nuclear deflection function is unimportant as far as refractive effects are concerned. Accordingly we have used a symmetrical nuclear deflection function whose outer branch ($\ell > \ell_r$) approximates very well the optical potential generated deflection function. The underlying nuclear phase shift is thus taken to be of the form

$$\delta_N = d_o \left[1 + \exp\left(\frac{\lambda - \Lambda_1}{\Delta_1}\right)\right]^{-1}$$

$$|S_\ell| = \left[1 + \exp\left(\frac{\Lambda_2 - \lambda}{\Delta_2}\right)\right]^{-1} \tag{14}$$

where $d_0 = 4$, $\Lambda_1 = 26.5$, $\Delta_1 = 3.5$, $\Lambda_2 = 36$, $\Delta_2 = 3.5$.

The result of our calculation for $^{13}C + {}^{12}C$ at $E = 260$ MeV is shown in Fig. 2. The first part of the figure shows the usual Near/Far decomposiiton (plotted is $\ell n \frac{d\sigma}{d\vartheta}$ as done in Fricke and McVoy[2]). The second part of the figure shows the diffraction-refraction decomposition of the far side component. It is clear that for angles larger than 20° the far side contribution is predominantly refractive. The interference

between $f_D^{(-)}$ and $f_R^{(-)}$ is seen most conspicuously in the angular regions $15^\circ < \vartheta < 40^\circ$ and $\vartheta > 58^\circ$. The broad maxima at $\vartheta \sim 55$ seen in the data comes out in our calculation slightly shifted to $\vartheta \sim 62^\circ$. The minimum at 44° is not very pronounced in our calculation. We feel that this stems principally from our parametrization of S. A more detailed fitting procedure would certainly account for the data.

From our results above we conclude that our definition of f_D and f_R is quite reasonable and our procedure supplies a simpler and more direct mean of analyzing the data than those based on e.g. the Knoll-Schaeffer[4] method. Further, the theory we have developed, based on Eq. (6), supplies a powerful method to extend uniform semiclassical approximations which were employed for the evaluation of $f_0(\vartheta)$, to physical situations involving both refraction and absorption, such as encountered in nuclear physics. More detailed account of our theory with further applications will be published elsewhere[4].

REFERENCES

1) H.G. Bohlen, X.S. Chen, J.M. Cramer, P. Fröhlich, B. Gebauer, H. Lettau, A. Miczaika, W. von Oertzen, R. Ulrich and T. Wilpert, Z. Phys. A332, 241 (1985).

2) S.H. Fricke and K.W. McVoy, University of Wisconsin - Madison Preprint Mad/TH/86-29.

3) W.E. Frahn and D.H.E. Gross, Ann. of Phys. (N.Y.) 101, 520 (1976).

4) M.S. Hussein and M.P. Pato, in preparation.

5) M.S. Hussein and K.W. McVoy, Prog. in Part. and Nucl. Phys. 12, 103 (1984).

6) V.P. Maslov and M.V. Fedoriuk, "Semiclassical Approximation in Quantum Mechanics", D. Reidel Publishing Company, Dordrecht, Holland (1981).

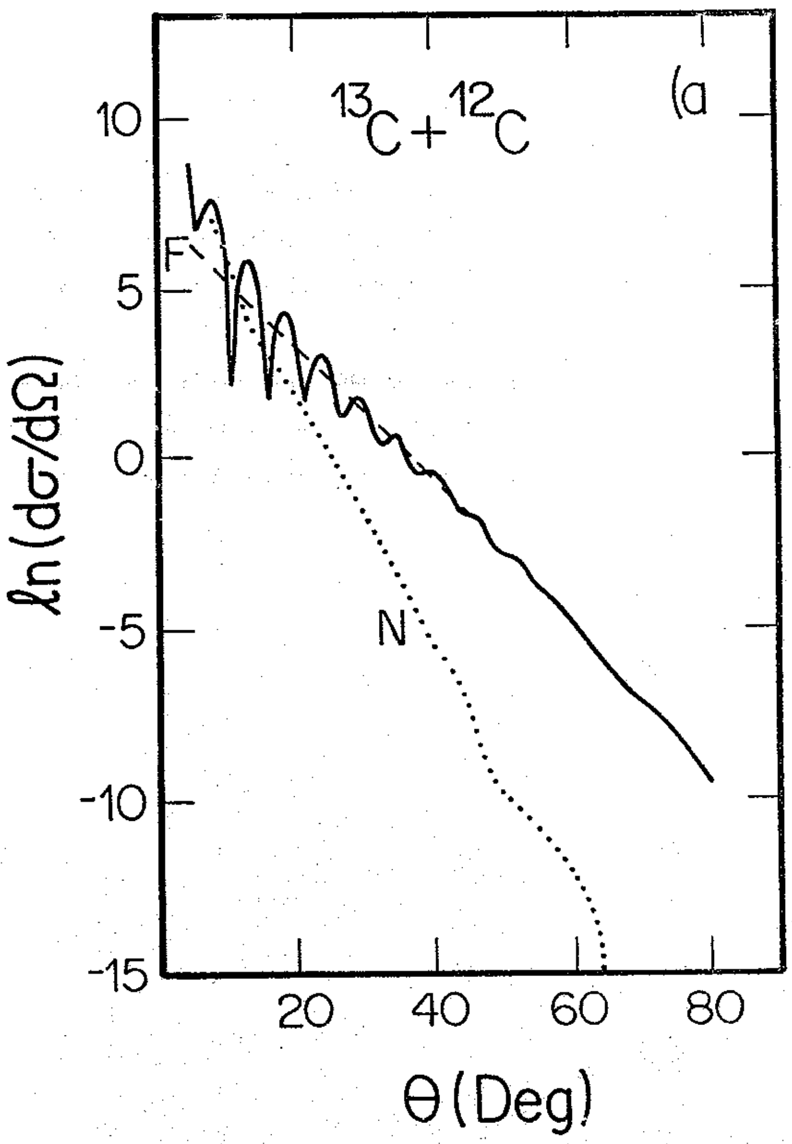
7) Our definition of $I_D^{\pm}$ is motivated by the physics that it should describe, namely scattering from the edge of the scatterer.

8) J. Knoll and R. Schaeffer, Ann. of Phys. (N.Y.) 97, 307 (1976).

FIGURE CAPTION

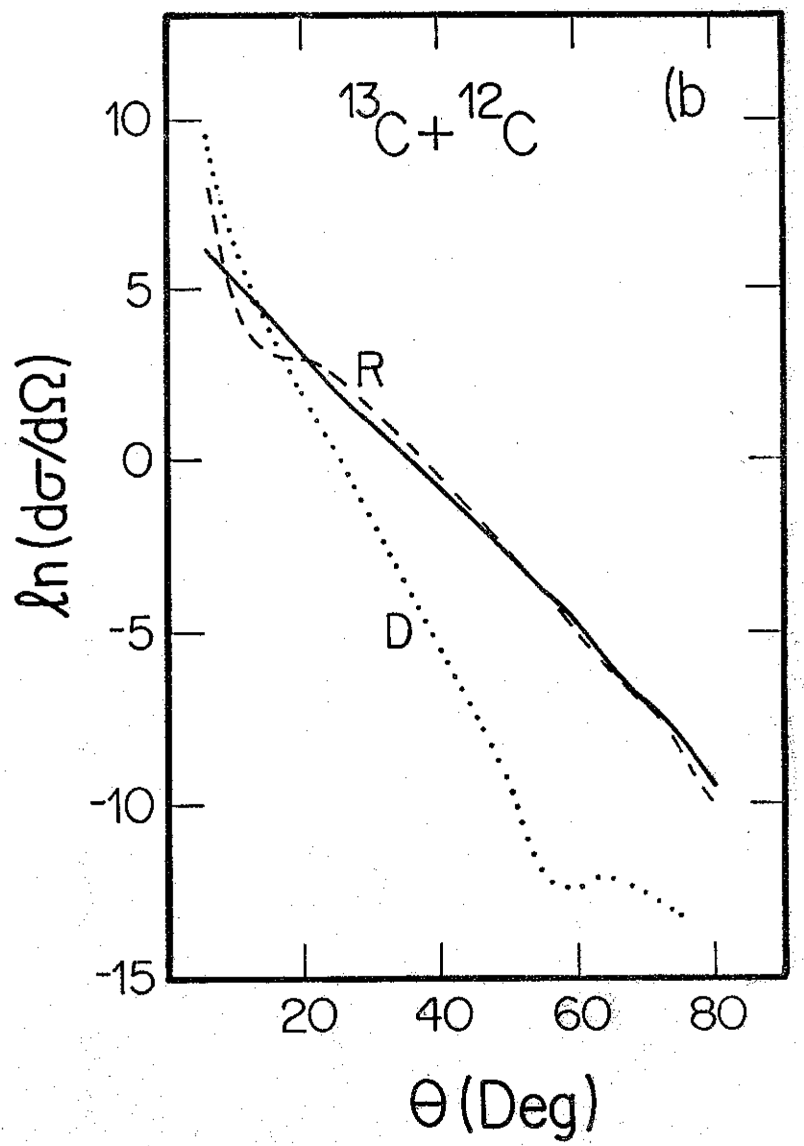
Figure 1 - a) $\ell n \frac{d\sigma}{d\vartheta}$ vs. ϑ calculated from the Near (dotted), Far (dashed) and total (full) amplitudes for $^{13}C + ^{12}C$ at $E_{Lab} = 260.00$ MeV (see text for details).

b) The diffractive (dotted) and refractive (dashed) components of the Far side cross section which is also shown as the full figure, for the same system.



$^{13}C+^{12}C$
(a
F
N
10
5
0
-5
-10
-15
20
40
60
80
$\ln(d\sigma/d\Omega)$
Θ (Deg)

Fig. 1



$^{13}C+^{12}C$
(b
R
D
10
5
0
-5
-10
-15
20
40
60
80
$\ln(d\sigma/d\Omega)$
Θ (Deg)

Fig. 1