UNIVERSIDADE DE SÃO PAULO

PUBLICAÇÕES

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

IFUSP/P-658

THE SIMPLEST GENTILIONIC SYSTEMS

M. Cattani and N.C. Fernandes
Instituto de Física, Universidade de São Paulo

Agosto/1987

THE SIMPLEST GENTILIONIC SYSTEMS

M. Cattani and N.C. Fernandes
Instituto de Física da Universidade de São Paulo
C.P. 20516, 01498 São Paulo, SP, Brasil

SUMMARY

Basic quantum mechanical properties of systems constituted by two and three gentileons are deduced in this paper. As an immediate and natural result of our theoretical analysis, it is shown how fundamental observed properties of composed hadrons can be rigorously predicted from first principles assuming quarks as spin 1/2 gentileons.

1. INTRODUCTION

In a recent paper(1) we have shown rigorously, according to the postulates of quantum mechanics and to the principles of indistinguishability, that three kinds of particles could exist in nature: bosons, fermions and gentileons. These results can be synthesized in terms of the following statement (Statistical Principle): "Bosons, fermions and gentileons are represented by horizontal, vertical and intermediate Young shapes, respectively".

Bosonic and fermionic systems are described by one-dimensional totally symmetric (ψ_S) and totally anti-symmetric (ψ_A) wavefunctions, respectively. For bosons and fermions the creation and annihilation operators obey bi-linear commutation relations.

Gentilionic systems are described by multi-dimensional (spinorial-like) wavefunctions (Y) with mixed symmetries. Since they are represented by intermediate Young shapes, only three or more gentileons can form a system of indistinguishable particles. This means that two identical gentileons are prohibited to constitute a system of indistinguishable particles. This implies that gentileons cannot appear freely. Indeed, if this were possible, two free identical gentileons could constitute a two-particle system in an occasional collision. For gentileons the creation and annihilation operators obey multi-linear matricial commutation relations. Finally, due to very peculiar geometric properties[1] of the intermediate states, there appear selection rules confining the gentileons and prohibiting the coalescence of gentilionic systems. Two systems like [ggg] and [gggg], for instance, cannot coalesce into a composite system of indistinguishable particles [ggggggg]. Only bound states [ggg] - [gggg] could be possible. The gentileon confinement appears as a consequence of the selection rule which prohibits the decomposition of a system [ggg...gg] into [ggg...g] and [g].

In our above quoted paper[1] only systems of identical gentileons have been considered. Let us now consider systems composed of two different kinds of gentileons, g and G. Taking into account the statistical principle we must expect that systems like [gG], [gggG], [gggGGG] and so on, are allowed. On the other hand, systems like [ggG], [ggGG], [gggGG] ... are prohibited because [gg] and [GG] are not allowed[1]. Of course, the coalescence of mixed systems is also forbidden, as can be easily verified. It is important to note that the commutation relations for the creation and annihilation operators for g and G in [gG] must be bi-linear since the state vector of the system is one-dimensional[1]. Thus, according to the special theory of relativity[2-4], they will be taken as commutative or anti-commutative for g(G) if the spin of g(G) is integer or half-integer, respectively.

The confinement and non-coalescence are intrinsic properties of gentileons as the total symmetrisation (anti-

symmetrisation) is intrinsic to bosons (fermions), not depending on their physical interpretation. Thus, they could be assimilated to individual real particles or to dynamical entities as quantum collective excitations. However, due to the selection rules imposed on the gentileons we think that they would be quite different from the usual particles and quantum collective states.

In section 2 we present a detailed study of the symmetry properties of the three gentileons state vector Y(123). We have emphasized the simplest non-trivial case of three particles aiming to apply the theory to the description of SU(3) models of strong interactions[5,6].

In section 3 we show that the $SU(3)_{color}$ representation can be naturally incorporated into the S_3 gentilionic symmetry.

In section 4 our theoretical results are applied to investigated some aspects of the hadronic physics. Assuming quarks as spin 1/2 gentileons we see that fundamental observed features of composed hadrons can be predicted from first principles.

In section 5 a modified quantum chromodynamics is suggested where, instead of fermions, gluons interact with gentileons. We verify that this approach and the usual QCD give identical predictions for hadronic properties when the Drell-Yan model is applied.

2. SYMMETRY PROPERTIES OF THE GENTILIONIC STATE VECTOR Y(123)

We present in this section a detailed study of the symmetry properties of the state vector $Y(123)$ of a system composed by three indistinguishable gentileons. This simplest three particles case (N=3) has been emphasized in order to apply the theory to the description of SU(3) models for strong interactions. Of course, it is possible to extend our results, concerning the structure of Y, for $N > 3$, at the expenses of unnecessary labour and non essential complications for our immediate purposes. Thus, according to our general results[1], the symmetry properties of $Y(123)$ is completely described in terms of the three quantum states α, β and γ. In analogy with the electromagnetic color theory these states will be named "primary colors". In terms of the colors α, β and γ, $Y(123) = Y(\alpha\beta\gamma)$ is written as[1]:

$$Y(\alpha\beta\gamma) = Y(123) = \frac{1}{\sqrt{4}} \begin{pmatrix} Y_1(123) \\ Y_2(123) \\ Y_3(123) \\ Y_4(123) \end{pmatrix} = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \qquad (2.1)$$

where,

$$Y_1(123) = (|\alpha\beta\gamma> + |\beta\alpha\gamma> - |\gamma\alpha\beta> - |\gamma\beta\alpha>)/\sqrt{4} \quad ,$$

$$Y_2(123) = (|\alpha\beta\gamma> + 2|\alpha\gamma\beta> - |\beta\alpha\gamma> + |\gamma\alpha\beta> - 2|\beta\gamma\alpha> - |\gamma\beta\alpha>)/\sqrt{12} \quad ,$$

$$Y_3(123) = (-|\alpha\beta\gamma> + 2|\alpha\gamma\beta> - |\beta\alpha\gamma> - |\gamma\alpha\beta> + 2|\beta\gamma\alpha> - |\gamma\beta\alpha> /\sqrt{12}$$

and $Y_4(123) = (|\alpha\beta\gamma> - |\beta\alpha\gamma> - |\gamma\alpha\beta> + |\gamma\beta\alpha>)/\sqrt{4}$. The state Y is decomposed into two parts, $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$, where $Y_+ = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ and $Y_- = \begin{pmatrix} Y_3 \\ Y_4 \end{pmatrix}$, corresponding to the duplication of the states implied by the reducibility of our representation in the intermediate gentilionic states[1,5]. We shall show, in what follows, that Y_+ and Y_- have a spinorial character[6], resulting a "bi-spinorial" character in Dirac's sense for $Y(123)$. The probability density function for $Y(123)$ is given by the permutation invariant function[1] $Y^{+}Y = |Y|^2 = (|Y_1|^2 + |Y_2|^2 + |Y_3|^2 + |Y_4|^2)/4$. The bi-spinorial character of $Y(123)$ is responsible for selection rules[1] predicting: (1) gentileon confinement and (2) non-coalescence of gentilionic systems. It is worthwhile to note that, in this context, our theory differs drastically from parastatistics[7-11] and fermionic theories. In the case of fermions, the three particles state function $\psi(123)$ would be given by $\psi_A(123) = (|\alpha\beta\gamma> - |\alpha\gamma\beta> - |\beta\alpha\gamma> + |\gamma\alpha\beta> + |\beta\gamma\alpha> - |\gamma\beta\alpha>)/\sqrt{6}$ and for para-particles $\psi(123)$ would be written as, $\psi_P(123) = a\,Y_1 + b\,Y_2 + c\,Y_3 + d\,Y_4$, where a, b, c and d are arbitrary constants. For these theories the wavefunction $\psi(123)$ is one-dimensional, from which the selection rules (1) and (2), above mentioned, cannot be deduced.

Our intention, in this section, is to show explicitely the spinorial character of $Y(123)$ and to establish fundamental properties of the gentilionic system that can be deduced from this spinorial character. In this way we must remember that, due to the six permutation operators of the symmetric group S_3, the Y(123) is transformed to[1]:

$$\begin{pmatrix} Y'_+ \\ Y'_- \end{pmatrix} = \begin{pmatrix} \eta_j & 0 \\ 0 & \eta_j \end{pmatrix} \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix} \quad (2.2)$$

where η_j $(j = 1, 2, \ldots, 6)$ are 2×2 matrices given by:

$$\eta_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \quad ; \quad \eta_2 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \quad ;$$

$$\eta_3 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix} \quad ; \quad \eta_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \quad (2.3)$$

$$\eta_5 = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \quad \text{and} \quad \eta_6 = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} .$$

From the point of view of group representation theory, Eq. (2.2) immediately suggests the reducibility of the intermediate representation. Due to the separation of Y into two components, Y_+ and Y_-, an interpretation of these objects is claimed.

Let us show that it is possible to interpret the transformations of Y_+ and Y_- in terms of rotations of an equilateral triangle in a particular Euclidean space E_3. That

is, we assume E_3 as a space where the color states are defined by three orthogonal coordinates (X,Y,Z). Due to this assumption, this space will be named "color space". It is also assumed that, in this color space, the colors α, β and γ occupy the vertices of an equilateral triangle taken in the (X,Z) plane, as seen in Fig. 1. The unit vectors along the X, Y and Z axes are indicated, as usually, by $\vec{i}$, $\vec{j}$ and $\vec{k}$. In Fig. 1, the unit vectors $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ are given by, $\vec{m}_4 = -\vec{k}$, $\vec{m}_5 = -(\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$ and $\vec{m}_6 = (\sqrt{3}/2)\vec{i} + (1/2)\vec{k}$, respectively.

(INSERT FIGURE 1)

We represent by Y(123) the state whose particles 1, 2 and 3 occupy the vertices α, β and γ, respectively. Thus, we see that the true permutations, (312) and (231), are obtained from (123) under rotations by angles $\vartheta = \pm 2\pi/3$ around the unit vector $\vec{j}$. As one can easily verify, the matrices η_2 and η_3, that correspond to these permutations are represented by:

$$\eta_2 = -I/2 + i(\sqrt{3}/2)\sigma_y = \exp[i\vec{j}.\vec{\sigma}(\vartheta/2)] \quad \text{and}$$
$$\eta_3 = -I/2 - i(\sqrt{3}/2)\sigma_y = \exp[i\vec{j}.\vec{\sigma}(\vartheta/2)] \quad (2.4)$$

where the σ_x, σ_y and σ_z are Pauli matrices.

Similarly, the transpositions (213), (132) and (321) are obtained under rotations by angles $\Phi = \pm\pi$ around the axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. The corresponding matrices are given by:

$$\eta_4 = \sigma_z = i\exp[i\vec{m}_4 \cdot \vec{\sigma}(\Phi/2)] \quad ,$$

$$\eta_5 = (\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_5 \cdot \vec{\sigma}(\Phi/2)] \quad \text{and} \qquad (2.5)$$

$$\eta_6 = -(\sqrt{3}/2)\sigma_x - (1/2)\sigma_z = i\exp[i\vec{m}_6 \cdot \vec{\sigma}(\Phi/2)] \quad .$$

According to our preceding paper[5], there is an algebraic invariant, $K^{[2,1]}_{(2,1)}$, with a zero eigenvalue, associated with the S_3 gentilionic states. In analogy with continuous groups, this invariant will be named "color Casimir"[5]. For permutations, that are represented by matrices with $\det = +1$, the invariant is given by $K_{rot} = \eta_1 + \eta_2 + \eta_3$. For transpositions, which matrices have $\det = -1$, it is defined by $K_{inv} = \eta_4 + \eta_5 + \eta_6$. Taking into account $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$ and Eqs. (2.5) we see that, $K_{inv} = \eta_4 + \eta_5 + \eta_6 = (\vec{m}_4 + \vec{m}_5 + \vec{m}_6) \cdot \vec{\sigma} = 0$. This means that the invariant K_{inv} can be represented geometrically, in the plane (X,Y) of the color space, by $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$, and that the equilateral triangle symmetry of the S_3 representation is an intrinsic property of $K_{inv} = 0$.

Eqs. (2.4) and (2.5) suggest a spinorial interpretation for Y_+ and Y_-. Here, starting from a general standpoint, we show the correctness of this contention. It is well known that the non-relativistic spinor can be introduced in several ways[12]. The interrelation of the various approaches is not obvious and can lead to misconceptions. In order to overcome the necessity of enumerating several approaches, let us stick on a geometrical image, recalling the very fundamental result on group isomorphism[13]: $S_3 \sim PSL_2(F_2)$, where $PSL_2(F_2)$ is the projective group associated with the special group SL_2 defined over a field F_2 with only two elements. Obviously, $PSL_2(F_2) \sim SL_2(F_2)/SL_2(F_2) \cap Z_2$, where the group in the denominator is the centre of SL_2 and corresponds to the central homotheties, since Z_2 is the intersection of the collineation group with SL_2.

If we consider the matrices (2.3) as representing transformations in a two-dimensional complex space characterized by homogeneous coordinates Y_1 and Y_2,

$$\begin{pmatrix} Y_1' \\ Y_2' \end{pmatrix} = \frac{1}{\rho}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \qquad (2.6)$$

where ρ is an arbitrary complex constant and the latin letters substitute the coefficients taken from (2.3), it is clear that (2.3) constitute a homographic (or projective) group.

Making use of definition (2.6) we can see from (2.3) that, apart from the identity η_1, the two matrices η_2

and η_3, which have $\det = +1$, are elliptic homographies with fixed points $\pm i$. If we translate these values for the variables of E_3, we see that η_2 and η_3 correspond to finite rotations around the $\vec{3}$ axis by an angle $\vartheta = \pm 2\pi/3$, agreeing thus with Eqs. (2.4). The remaining matrices η_4, η_5 and η_6 are elliptic involutions, with $\det = -1$. They correspond to space inversions in E_3, considered as rotations of $\pm\pi$ around the three axis $\vec{m}_4$, $\vec{m}_5$ and $\vec{m}_6$, respectively. These matrices completely define the axis of inversion and the angle $\pm\pi$, as is seen from Eqs. (2.5). It is an elementary task to establish the correspondence, via stereographic projection, between the transformations in the two spaces, $Y_+(Y_-)$ and E_3.

A topological image can help us to see the 4π invariance of Y_+ and Y_-. If we consider the rotation angle $\vartheta(\Phi)$ as the variable describing an Euclidean disc, the covering space associated to this disc is a Moebius strip[14]. Adjusting correctly the position of the triangles we can have a vivid picture of the rotation properties for each axis. This construction allow us to visualize the double covering of the transformation in E_3 and is a convincing demonstration of the spinorial link between E_3 and $Y_\pm$.

From this analysis we conclude that Y_+ and Y_- are spinors. As one can easily verify[15] by using the projective geometry, the four-dimensional state function $Y = \begin{pmatrix} Y_+ \\ Y_- \end{pmatrix}$ is a "bi-spinor" in Dirac's sense. Since E_3 is a "color space", Y_+ and Y_-, in analogy with the isospinor in the isospace, will be named "colorspinor".

We observe that the same transformation properties of Y_+ and Y_- can be obtained if, instead of the equilateral triangle shown in Fig. 1, we consider the triangle drawn in Fig. 2.

(INSERT FIGURE 2)

In the vertices of the equilateral triangle of the Fig. 2 we have the colors $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$. The unit vectors $\vec{m}_4^*$, $\vec{m}_5^*$ and $\vec{m}_6^*$ are given by, $\vec{m}_4^* = -\vec{m}_4$, $\vec{m}_5^* = -\vec{m}_5$ and $\vec{m}_6^* = -\vec{m}_6^*$. This means that, in this case, K_{inv} is represented geometrically by $\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0$. This two fold possibilities for depicting the triangle will be physically interpreted, in the next sections, in terms of the existence of colors and anti-colors.

3. THE S_3 SYMMETRY AND THE $SU(3)_{color}$ EIGENSTATES

In section 2, we have shown that it was possible to interpret the $Y(\alpha\beta\gamma)$ transformations in terms of rotations, in a color space E_3, of only two equilateral triangles with vertices occupied by three privileged color $\alpha(\bar{\alpha})$, $\beta(\bar{\beta})$ and $\gamma(\bar{\gamma})$. The Y must constitute symmetry adapted kets for S_3. In other words, their disposition in the plane of the triangle must agree with the imposition made by the color Casimir. According to Fig. 1, these colors are defined by, $\alpha = \vec{m}_5 = (-\sqrt{3}/2, 1/2)$, $\beta = \vec{m}_6 = (\sqrt{3}/2, 1/2)$ and $\gamma = \vec{m}_4 = (0,-1)$, and according to Fig. 2, $\overline{\alpha} = \vec{m}_5^* = -\vec{m}_5$, $\overline{\beta} = \vec{m}_6^* = -\vec{m}_6$ and $\overline{\gamma} = \vec{m}_4^* = -\vec{m}_4$. The equilateral triangle symmetry for S_3 plays a fundamental role in E_3, allowing us to obtain a very simple and beautiful geometrical interpretation for the invariant $K_{inv} = 0$. Indeed, since the S_3 symmetry, according to section 2, implies that $\vec{M} = \vec{m}_4 + \vec{m}_5 + \vec{m}_6 = 0$ $(\vec{M}^* = \vec{m}_4^* + \vec{m}_5^* + \vec{m}_6^* = 0)$, we conclude that the <u>total color quantity</u> of the system, pictured in E_3, is a constant of motion, which is null.

At this point we compare our color states α, β and γ with the $SU(3)_{color}$ eigenstates[16], blue, red and green. These color states are eigenstates of the color hypercharge $\tilde{Y}$ and of the color isospin $\tilde{I}_3$, both diagonal generators of the algebra of the $SU(3)_{color}$. The eigenstates blue (b), red (r) and green (g) are written as $|b\rangle = |-1/2, 1/3\rangle$, $|r\rangle = |1/2, 1/3\rangle$ and $|g\rangle = |0,-2/3\rangle$.

Taking into account that the SU(3) and S_3 fundamental symmetries are defined by equilateral triangles[16,17], it is quite apparent that the color states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ can be represented by eigenstates of $\tilde{I}_3$ and $\tilde{Y}$. Indeed, assuming that the axes X and Z (see Fig. 1) correspond to the axes $\tilde{I}_3$ and $\tilde{Y}$, respectively, and adopting the units along these axes as the side and the height of the triangle[17], we

verify that $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ would be given by, $|\alpha\rangle = |b\rangle = |-1/2, 1/3\rangle$, $|\beta\rangle = |r\rangle = |1/2, 1/3\rangle$ and $|\gamma\rangle = |g\rangle = |0,-2/3\rangle$. If we have considered the states $|\overline{\alpha}\rangle$, $|\overline{\beta}\rangle$ and $|\overline{\gamma}\rangle$, seen in Fig. 2, we should verify that these states would correspond to the anti-colors $|\overline{r}\rangle$, $|\overline{b}\rangle$ and $|\overline{g}\rangle$ of the $\overline{3}$ color representation.

Thus, if we assume that the states $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ correspond to $|b\rangle$, $|r\rangle$ and $|g\rangle$, respectively, each unit vector $\vec{m}_j$ ($j = 4, 5$ and 6) is represented, in the plane $(\tilde{I}_3, \tilde{Y})$ by the operator $\tilde{q} = \tilde{I}_3 + \tilde{Y}/2$. This means that the vector $\vec{M}$ will be represented by the operator $\tilde{M} = \tilde{q}_1 + \tilde{q}_2 + \tilde{q}_3$, where the indices 1, 2 and 3 refer to the three gentileons of the system. Thus, adopting the $SU(3)_{color}$ eigenvalues we see that $\tilde{M}$ will have a zero eigenvalue only when Y is given by $Y(brg)$. That is, the wavefunctions $Y(nnm)$, where $n,m = b$, r and g, with two particles occupying the same color state[1,5], are prohibited.

It is important to note that, in our previous paper[5], since the $SU(3)_{color}$ scheme was not adopted, we have assumed that two gentileons could occupy the same color state. This is a point that remains to be analysed: the existence of another kind of color representation, besides the $SU(3)_{color}$, which would be able to describe consistently the gentilionic approach.

4. THE GENTILIONIC HADRONS

Since gentileons are confined entities and their systems are non-coalescent it seems natural to think quarks as spin 1/2 gentileons(1,5). With this hypothesis it can be shown that(5) the baryons [qqq], that are formed by three indistinguishable gentileons in a color space, are represented by wave-functions $\Psi = \varphi.Y(\alpha\beta\gamma)$. The one-dimensional state vector $\varphi = (SU(6)\times O_3)_{symmetric}$ corresponds, according to the symmetric quark model of baryons(16), to a totally symmetric state. The four-dimensional state function $Y(\alpha\beta\gamma)$, that depends on three quantum color states $|\alpha>$, $|\beta>$ and $|\gamma>$, corresponds to the intermediate representation of the S_3 group. The colorspinor $Y(\alpha\beta\gamma)$ is written explicitely in section 2.

It will be assumed, in what follows, that the color states $|\alpha>$, $|\beta>$ and $|\gamma>$ are the $SU(3)_{color}$ eigenstates blue, red and green, respectively. Under these assumptions Ψ will be given by $\Psi = \varphi.Y(brg)$. If quarks are taken as fermions Ψ is given by $\Psi = \varphi.\psi_A(brg)$, where $\psi_A(brg)$ corresponds to the totally anti-symmetric fermionic color function. Thus, we see that in our theory the SU(3), flavor and color, continuous symmetry is maintained. Only the S_3 fermionic symmetry is substituted by the S_3 gentilionic symmetry, both discrete. We must also remark that, with the above results and following section 2 and 3, the $SU(3)_{flavor}\times SU(3)_{color}$ representation,

both 3 and $\bar{3}$, is naturally incorporated in our scheme. With this in mind and observing section 2 we see that in the gentilionic formalism one possibility is to define the individual quark charge as:

$$q = q_f + \tilde{q}_c = (I_3 + Y/2) + \lambda(\tilde{I}_3 + \tilde{Y}/2) \qquad (4.1)$$

where $q_f = I_3 + Y/2$ refers to flavour charge, $\tilde{q}_c = (\tilde{I}_3 + \tilde{Y}/2)$ refers to color charge and λ is an arbitrary constant that cannot be determined in the framework of the theory. With this definition, the total color baryon charge $\tilde{Q}$ is given by $\tilde{Q} = \lambda<\tilde{M}>$. Remembering that the expected value $<\tilde{M}>$ is a constant of motion equal to zero, that is, $<\tilde{M}>$ = constant = 0, for the states $Y(brg)$, as shown in section 3, we see that the generalized Gell-Mann-Nishijima relation is automatically satisfied(5) independently of the λ value. Putting $\lambda = -1$ we obtain integer quark charges, according to Han-Nambu, and if $\lambda = 0$ we have the fractional charges adopted by Gell-Mann(16). Note that the result $\tilde{Q}$ = constant = 0 can be interpreted as a selection rule for quark confinement in baryons.

In our approach(1,5) mesons are composed by a quark-antiquark pair $[q\bar{q}]$. According to the statistical principle, systems like [q], [qq], $]qq\bar{q}]$ and $[qq\bar{q}\bar{q}]$, for instance, are prohibited (it could exist only bound states $[q\bar{q}]-[q\bar{q}]$ of the mesons $[q\bar{q}]$). Since q and $\bar{q}$ are different particles in color space and both are spin 1/2 particles we can conclude, in agreement with section 1, that: (a) mesons are represented by one-dimensional state functions and (b) q and $\bar{q}$ in the system $[q\bar{q}]$ can be taken as fermions from the algebraic point of view. This permit us to conclude that mesons are represented by the same state vector in fermionic and gentilionic theories.

We are now in condition to make a summary of the fundamental properties that must be observed for composed hadrons if quarks are spin 1/2 gentileons: (1) quarks are confined, (2) baryons and mesons cannot coalesce, (3) baryonic number is conserved, (4) the hadron color charge is a constant of motion equal to zero and (5) only color singlet nadrons can exist.

The above mentioned hadronic properties have been predicted independently of the intrinsic nature of the gentileons; they could be particles, quantum collective excitations or something else. Consequently, no dynamical hypothesis, phenomenological or approximate arguments have been used to prove them. They have been deduced from first principles: from the statistical principle or by using the symmetries of the S_3 intermediate representation. Thus, if quarks are gentileons, even though we uphold the intrinsic geometrical nature of confinement, we cannot exclude the possibility that there may be hidden or explicit a confining mechanism in the dynamical laws. After all, we cannot reduce all the concepts, which enter into a dynamical law, to geometrical notions. The confining mechanism could be produced

by a very peculiar interaction between quarks, by an impermeable bag as proposed by the bag model or something else. At the moment these mechanisms are unknown. It is not our intention, in this paper, to study this problem.

In spite of our stimulating general results, there remains the crucial problem of determining the intrinsic nature of the quarks and their dynamical properties. According to the current theoretical ideas, quarks are fermionic elementary particles. The mathematical formulation of the fermionic model, the QCD, is a successful modern field theory since it is able to explain many properties of the hadrons. In next section, taking quarks as spin 1/2 gentileons, a quantum chromodynamics is proposed where, instead of fermions, gentileons interact with gluons. This formalism (QCDG) will be compared with the usual QCD. It will be shown that adopting the Drell-Yan model the QCD and the QCDG will give identical predictions for the hadronic properties.

5. A QUANTUM CHROMODYNAMICS FOR GENTILIONIC QUARKS

In this section a quantum chromodynamics using gentilionic quarks is proposed. This approach, indicated by QCDG, will be compared with the standard QCD. In this way, remembering that quarks have spin 1/2 and taking into account the SU(3) symmetry (color and flavor), the following Lagrangian density for gentilionic quarks interacting with gluons is suggested:

$$L = \sum_f \left[i\, q_a^+ \gamma^\mu \frac{\partial}{\partial x^\mu} q_a + g\, q_a^+ \gamma^\mu \left(\frac{\lambda_i}{2}\right)_{ab} A_\mu^i q_b - m_f q_a^+ q_a \right] +$$

$$- \frac{1}{4} \left(\frac{\partial A_\nu^i}{\partial x^\mu} - \frac{\partial A_\mu^i}{\partial x^\nu} + g\, f_{ijk} A_\mu^i A_\nu^k \right)^2 \qquad (5.1)$$

where the summation is over the flavors $f = u, d, s, c, \ldots$. The summation over repeated indices $a, b, \ldots,$ referring to color is understood. The $\lambda_i/2$ are the 3×3 matrix representation of the $SU(3)_{color}$ algebra generators, satisfying the commutation relations $[\lambda_i, \lambda_j] = i f_{ikj} \lambda_k/2$, where f_{ijk} are the SU(3) structure constants. The flavor symmetry is only broken by the lack of degeneracy in the quark masses. Finally, the quark free fields $q(x)$ are expanded in terms of positive and negative frequency solutions, $\varphi_{k+}(x)$ and $\varphi_{k-}(x)$, of Dirac's equation,

$$q(x) = \sum_k \left\{ a_{k+} \varphi_{k+}(x) + a_{k-}^+ \varphi_{k-}(x) \right\} .$$

It is important to remark that, with the above assumptions, both theories, QCD and QCDG, will have the same gluons and the same Lagrangian densities. However, the creation and annihilation quark operators obey different commutation

relations in these theories: in QCD they are bi-linear fermionic and in QCDG they are matricial gentilionic(1). This difference will be analysed in what follows.

In QCD, quarks being fermions, a_α and a_α^+, obey the well known bi-linear relations, independently of the hadronic system:

$$\left[a_\alpha^+, a_\beta\right]_+ = \delta_{\alpha\beta}$$

$$\left[a_\alpha^+, a_\beta^+\right]_+ = \left[a_\alpha, a_\beta\right]_+ = 0 \qquad (5.2)$$

Considering now the gentilionic hadrons, let us see first the mesons $[q\bar{q}]$. According to section 1, the commutation relations for $\bar{q}$ and q are determined only by their spins. Since these are equal to 1/2, q and $\bar{q}$ can be taken as fermions from the algebraic point of view. Consequently, for processes involving only mesons, the QCDG and the QCD would give exactly the same predictions.

For baryons the quantum field calculations, in the general case, would be more complicated since the creation and annihilation operators obey gentilionic matricial relations(1). However, a simplification is introduced when the color states are taken as the eigenstates of the $SU(3)_{color}$, blue, red and green. In this case $Y(\alpha\beta\gamma)$ must necessarily be composed by these three different colors, resulting for the baryon wavefunctions, $\psi = \varphi . Y(brg)$. In these conditions, the quarks in [qqq] that, according to φ and $Y(brg)$, have disponible an infinite number of quantum states, cannot assume the same color in the color space. In other words, two quarks in [qqq] cannot occupy the same quantum state. With this fermionic characteristic it is not difficult to verify(1) that the number of independent gentilionic commutation relations are reduced, remaining only a few ones:

$$\left[a_\alpha^+ , a_\beta\right]_+ = \delta_{\alpha\beta} , \quad a_i a_i = a_i^+ a_i^+ = 0 ,$$

$$a_i^+ a_j^+ a_k^+ = G\binom{\alpha\beta\gamma}{ijk} a_\alpha^+ a_\beta^+ a_\gamma^+ \quad \text{and} \tag{5.3}$$

$$a_i a_j a_k = G\binom{kji}{\gamma\beta\alpha} a_\alpha a_\beta a_\gamma ,$$

where the indices i, j and k can assume the values α, β and γ and G(...) are 4×4 matrices given elsewhere(1). From the trilinear relations we can obtain, for instance, the following transpositions, considered as bi-linear relation

$$a_\beta^+ a_\alpha^+ = G\binom{\alpha\beta\gamma}{\beta\alpha\gamma} a_\alpha^+ a_\beta^+ \quad \text{and} \tag{5.4}$$

$$a_\beta a_\alpha = G\binom{\gamma\alpha\beta}{\gamma\beta\alpha} a_\alpha a_\beta .$$

In spite of the great simplifications that have been introduced, it is evident from Eqs. (5.3) and (5.4) that

gentilionic quarks cannot rigorously be taken as fermions since only the first three relations, $\left[a_\alpha^+ , a_\beta\right]_+ = \delta_{\alpha\beta}$, $a_i a_i = a_i^+ a_i^+ = 0$ are bi-linear fermionic. However, the bi-linear relations, $a_\beta a_\alpha$ and $a_\beta^+ a_\alpha^+$, and the tri-linear relations need to be employed only if we intend to take into account properties which are common to pairs of particles or to three particle in the [qqq] system. Thus, if we assume, in a first approximation, that in the baryonic processes only one quark participates, and the remaining two are spectators (Drell-Yan model(16)), the bi-linear and the tri-linear feature of the commutation relations will be irrelevant in cross section calculations. Under these circumstances only the bi-linear fermionic commutation relations need to be used and, consequently, gentileons can be taken as fermions from the algebraic point of view.

Thus, we see from the above analysis that, when the Drell-Yan model is applied, the QCD and the QCDG will give identical predictions for the hadronic properties. In both approaches the following additional conditions are assumed: (a) quark confinement, (b) non-coalescence of hadrons, (c) baryon number conservation, (d) only color singlet hadrons exist and (e) the hadron color charge is a constant of motion equal to zero. In spite of this we must note that the fermionic and the gentilionic theories are not equivalent. Indeed, in the QCDG these fundamental additional conditions appear naturally, deduced rigorously from first principles, whereas in the QCD they are imposed "ad hoc".

In terms of cross section calculations, using the QCD and QCDG, probably it will be possible to decide if quarks are fermions or gentileons taking into account correlations between quarks in baryons. This, however, seems to be an extremely difficult task not only for gentileons but also for fermions[18].

ACKNOWLEDGEMENTS

The authors thank Dr. D. Spehler for helpful discussions on QCD and the Conselho Nacional de Pesquisas CNPq) for the financial support.

REFERENCES

1) M. Cattani and N.C. Fernandes, Nuovo Cimento A79, 107 (1984).

2) W. Pauli, Phys. Rev. 58, 716 (1940).

3) G. Lüders and B. Zumino, Phys. Rev. 110, 1450 (1958).

4) N. Burgoyne, Nuovo Cimento 8, 607 (1958).

5) M. Cattani and N.C. Fernandes, Nuovo Cimento B87, 70 (1985).

6) M. Cattani and N.C. Fernandes, Phys. Lett. A (1987). To be published.

7) J.B. Hartle and J.R. Taylor, Phys. Rev. 178, 2043 (1969).

8) J.B. Hartle, R.H. Stolt and J.R. Taylor, Phys. Rev. D2, 1759 (1970)

9) R.H. Stolt and J.R. Taylor, Nucl. Phys. B19, 1 (1970).

10) R.H. Stolt and J.R. Taylor, Nuovo Cimento A5, 185 (1971).

11) A.B. Govorkov, J. Phys. A13, 1679 (1980).

12) F.A.M. Frescura and B.J. Hiley, Am. J. Phys. 49, 152 (1981).

13) J. Dieudonné, La Géométrie des Groupes Classiques (Springer, Berlin, 1955).

14) Yu. Borisovich, N. Bliznyakov, Ya. Izrailevich and T. Fomenko, Introduction to Topology (Moscow, 1985).

15) M. Cattani and N.C. Fernandes, preprint IFUSP/P-588 (1986).

16) F.E. Close, An Introduction to Quarks and Partons (Academic Press, London, 1979).

17) E. Lifshitz and L. Pitayevski, Théorie Quantique Relativiste (Éditions MIR, Moscow, 1973).

18) G.T. Bodwin, preprint Argonne National Laboratory ANL-HEP-PR-84-64, September (1984).

FIGURE CAPTIONS

Fig. 1 - The equilateral triangle in the color space (X,Y,Z) with vertices occupied by the colors α, β and γ.

Fig. 2 - The equilateral triangle in the color space (X,Y,Z) vertices occupied by the colors α, β and γ.

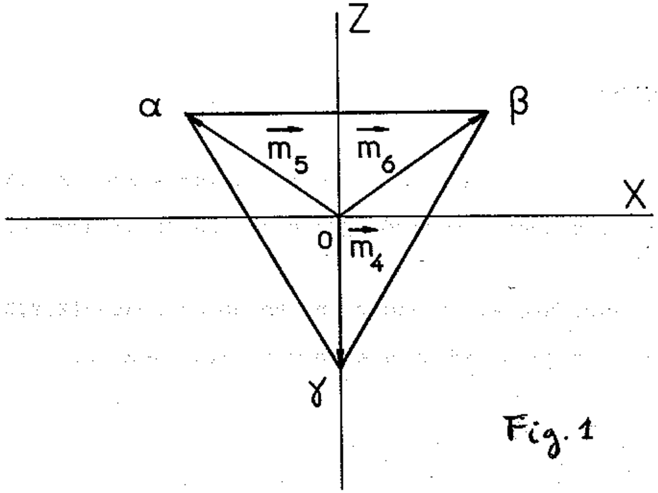

Fig. 1

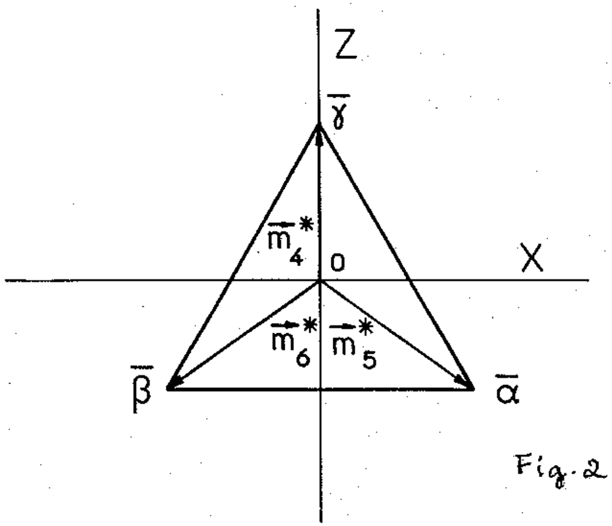

Fig. 2