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Abstract :

General expressions for the friction and diffusion coefficients for high energy proton-nucleus collisions are derived from an exact collisional description of the proton dynamics. Analytical expressions for both coefficients in nuclear matter are given. For low momentum transfers the Pauli blocking inhibites the friction and diffusion processes and this corresponds to the more realistic situations for proton incident energies $\sim$ 1 GeV. Results are compared with the ones derived within the context of Glauber Theory[1)] where Pauli blocking effects are not considered and the friction coefficient vanishes automatically.

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1. Introduction

A kinetic description of inelastic high energy proton nucleus collisions ($\sim$1 GeV incident energy) has been given [1)] within the context of Glauber Theory[2)]. The one nucleon inclusive cross section is shown to be related to the space integral of a function which depends on space and momentum variables of the nucleon to be detected. A Boltzmann-type equation for this function is derived as well as the diffusion coefficient for the proton. One of the open questions in ref [1)] concerns the connection between the above mentioned function and the Wigner function. Another important problem is due to the fact that energy conservation is not taken into account in Glauber's Theory. Therefore the friction coefficient is automatically zero. Also Pauli blocking effects are neglected.

It is the aim of the present paper to fill in these gaps. We derive a kinetic equation for the proton's Wigner function within the context of a projection formalism[3)] and work out the necessary approximations which lead to a Boltzmann-type equation. The usual momentum expansion of the Boltzmann equation leads directly to very simple expressions for the friction and diffusion coefficients. Analytical expressions for these coefficients are derived for nuclear matter in a model proposed by Bertsch and Scholten[4)]. In the limit of high momentum transfer ($q > 2\, p_F$) we show that the diffusion coefficient coincides with that of ref[1)]. Corrections to the kinetic coefficients due to Pauli blocking are explicitely considered both for low momentum transfers ($q < 2\, p_F$), where they are shown to be important, and for the opposite limit, where they vanish.

The question of overall energy conservation has been very often raised in the context of collisional descriptions especially in extended Hartree-Fock theories. The present formulation deals with two interacting subsystems (the proton and the nucleus). The proton's effective dynamics is approximated and therefore it is not at all obvious that the total energy is still conserved. We show explicitely that our approximations do not violate energy conservation.

In section 2 we derive the Boltzmann equation from the exact kinetic equation for the proton's effective dynamics and discuss the necessary approximations in the context of high energy collisions. General expressions for the kinetic coefficients are given. In section 3 we calculate friction and diffusion coefficients for nuclear matter. Summary and conclusions are given in section 4.

2. From the exact effective proton dynamics to the Boltzmann equation : Transport coefficients

Since we are considering high energy collisions we shall treat the incoming proton and the target as two independent subsystems of a many body system. We are thereby neglecting anti-symmetrization between the incident particle and the nucleus, which is a good approximation for such collisions. Futhermore if we call ρ the proton density matrix and R the nucleus density matrix, the exact coupled equations which govern their effective dynamics can be written as [3] : ($\hbar$ = 1)

$$i\dot{\hat{\rho}} = [H_p,\hat{\rho}] + [\langle H'\rangle_N,\hat{\rho}] - i\, tr_N\left\{\int_0^t dt'\, \Delta L_t\, G(t,t')\, \Delta L_{t'}\, \hat{R}(t')\hat{\rho}(t')\right\} \quad (2.1\ a)$$

$$i\dot{\hat{R}} = [H_N,\hat{R}] + [\langle H'\rangle_p,\hat{R}] - i\, tr_p\left\{\int_0^t dt'\, \Delta L_t\, G(t,t')\, \Delta L_{t'}\, \hat{R}(t')\hat{\rho}(t')\right\} \quad (2.1\ b)$$

In the above equations H_p and H_N are the free hamiltonians of the proton and the nucleus respectively. $\langle H'\rangle_p$ and $\langle H'\rangle_N$ are defined as

$$\langle H'\rangle_p = tr\,(H'\hat{\rho}) \quad (2.2\ a)$$

$$\langle H'\rangle_N = tr\,(H'\hat{R}) \quad (2.2\ b)$$

where H' is the interaction hamiltonian. The symbol ΔL_t stands for

$$\Delta L_t. = [H',.] - [\langle H'\rangle_p,.] - [\langle H'\rangle_N,.]$$

and G(t,t') is a many body Green's function which propagates the correlated state to its right from time t' to time t. Its formal definition is given in ref[3].

The next approximation consists in neglecting all averaged interaction terms (mean field contributions), assuming their effect on the effective dynamics to be small as compared to the other terms (e.g. kinetic energy). At this point we still have a highly non linear non markovian system of coupled equations. The next simplifying assumption amounts to considering only the lowest order correlation corrections in the collision term. In this case the equations decouple :

$$i\dot{\hat{\rho}} = [H_p,\hat{\rho}] - i\, tr_N\left\{\int_0^t dt'\, [H', G_0(t,t')\, [H',\hat{\rho}(t')\hat{R}_0]]\right\} \quad (2.3\ a)$$

$$i\dot{\hat{R}} = [H_N,\hat{R}] - i\, tr_p\left\{\int_0^t dt'\, [H', G_0(t,t')\, [H',\hat{\rho}_0\hat{R}(t')]]\right\} \quad (2.3\ b)$$

where $\hat{\rho}_0$ and $\hat{R}_0$ are the initial densities, and $G_0(t,t')$ propagates the correlated state to its right with the free dynamics of both subsystems.

These equations are not exact any longer. Despite of this fact the total energy is conserved (see Appendix). The equations are now decoupled and we can treat the effective dynamics of the proton separately. Eq. (2.3 a) can be rewritten as follows

$$\dot{\rho}_{mn} = -\, i(\varepsilon_m - \varepsilon_n)\rho_{mn} - \sum_{ij} \int_0^t dt' \;\; K_{mn,ij}\,(t - t')\;\rho_{ij}(t') \tag{2.4}$$

where

$$K_{mn,ij}(t-t') = \sum_{k\ell}$$

$$\langle p_m N_0 | H' | p_k N_\ell \rangle \langle p_k N_\ell | H' | p_i N_0 \rangle \,\delta_{nj} \exp\left[i\,(E_0 - E_\ell - \varepsilon_k + \varepsilon_n)\,(t-t')\right]$$

$$+ \langle p_k N_\ell | H' | p_n N_0 \rangle \langle p_j N_0 | H' | p_k N_\ell \rangle \,\delta_{im} \exp\left[i\,(E_\ell - E_0 + \varepsilon_k - \varepsilon_i)\,(t-t')\right]$$

$$- \langle p_j N_0 | H' | p_n N_\ell \rangle \langle p_k N_\ell | H' | p_i N_0 \rangle \;\; \delta_{kn} \exp\left[i\,(E_0 - E_\ell + \varepsilon_j - \varepsilon_m)\,(t-t')\right]$$

$$- \langle p_m N_\ell | H' | p_i N_0 \rangle \langle p_j N_0 | H' | p_k N_\ell \rangle \,\delta_{km} \exp\left[i\,(E_\ell - E_0 - \varepsilon_i + \varepsilon_n)\,(t-t')\right] \tag{2.5}$$

where $\langle p_i N_j | H' | p_k N_\ell \rangle$ stands for the coupling matrix element between unperturbed proton (p_i) and nuclear (N_j) states. The phases contain unperturbed energies of the proton (small letters) and nucleus (capital letters). The memory integral in eq. (2.4) represents an effect of quantum correlations. It is possible to treat the memory by means of a Laplace transform since we have now a linear equation. But in order to get to the Boltzmann equation we have to make the Markov approximation and assume that the time dependence of $\rho_{ij}(t')$ is much slower than that in the integral

Kernell $K_{mn,ij}(t-t')$. With this approximation, $\rho_{ij}(t') \approx \rho_{ij}(t)$ and

$$K_{mn,ij} \simeq \pi \sum_{k\ell} \Big\{ \delta\,(E_\ell - E_0 - \varepsilon_n + \varepsilon_k) \langle p_m N_0 | H' | p_k N_\ell \rangle \langle p_k N_\ell | H' | p_i N_0 \rangle \,\delta_{nj}$$

$$+ \delta(E_\ell - E_0 - \varepsilon_k + \varepsilon_n) \langle p_j N_0 | H' | p_k N_\ell \rangle \langle p_k N_\ell | H' | p_n N_0 \rangle \;\delta_{im}$$

$$- \delta(E_0 - E_\ell - \varepsilon_j + \varepsilon_n) \langle p_j N_0 | H' | p_n N_\ell \rangle \langle p_k N_\ell | H' | p_i N_0 \rangle \;\delta_{km}$$

$$- \delta(E_\ell - E_0 - \varepsilon_n + \varepsilon_i) \langle p_m N_\ell | H' | p_i N_0 \rangle > p_j N_0 | H' | p_k N_\ell \rangle \;\delta_{kn} \Big\} \tag{2.6}$$

In order to get to the Boltzmann equation, we write eq. (2.4) in Wigner representation, neglect the momentum spreading of the proton density and assume that the proton - nucleus interaction is of short range in coordinate space. We then get

$$\dot{\rho}_W(\underset{\sim}{p},\underset{\sim}{R},t) - \frac{p}{m}\frac{\partial}{\partial \underset{\sim}{R}}\;\rho_W(\underset{\sim}{p},\underset{\sim}{R},t) =$$

$$= \;\; -2\pi \sum_\ell \int d\underset{\sim}{p_1}\,\Omega(\underset{\sim}{p_1})\,\delta\Big(\hbar\omega_\ell - \frac{p^2}{2m} - \frac{p_1^2}{2m}\Big)\;|\langle \underset{\sim}{p}\, N_\ell | H' | \underset{\sim}{p_1} N_0 \rangle|^2 \;\; \rho_W(\underset{\sim}{p},\underset{\sim}{R},t)$$

$$+ 2\pi \sum_\ell \int d\underset{\sim}{p_1}\;\Omega(\underset{\sim}{p_1})\,\delta\Big(\hbar\omega_\ell - \frac{p^2}{2m} - \frac{p_1^2}{2m}\Big)\;|\langle \underset{\sim}{p_1} N_\ell | H' | \underset{\sim}{p}\, N_0 \rangle|^2 \;\; \rho_W(\underset{\sim}{p_1},\underset{\sim}{R},t) \tag{2.7}$$

where $\hbar\omega_\ell = E_\ell - E_0$ and $\Omega(\underset{\sim}{p})$ is the level density of the proton with momentum $\underset{\sim}{p}$. On the right hand side of eq. (2.7) one easily recognizes the loss and gain terms of a Boltzmann-type equation. This is the main result of this section.

If one considers that the proton's Wigner function is peaked around $\underset{\sim}{p_1} \approx \underset{\sim}{p}$, i.e., that the spreading in momenta is small, it is then reasonable to expand the second term on the r.h.s. of eq. (2.7) around p.

We get, up to second order :

$$\rho_W(\underset{\sim}{p},\underset{\sim}{R},t) - \frac{p}{m}\frac{\partial}{\partial \underset{\sim}{R}} \rho_W(\underset{\sim}{p},\underset{\sim}{R},t) = \underset{\sim}{\Gamma}\frac{\partial}{\partial \underset{\sim}{p}} \rho_W(\underset{\sim}{p},\underset{\sim}{R},t) + \sum_{ij} D_{ij} \frac{\partial^2}{\partial p_i \partial p_j} \rho_W(\underset{\sim}{p},\underset{\sim}{R},t) \tag{2.8}$$

with

$$\underset{\sim}{\Gamma} = -\frac{v}{V}\int d\,E_{\underset{\sim}{p_1}}\, d\Omega_{\underset{\sim}{p_1}} \frac{d^2\sigma}{dE_{\underset{\sim}{p_1}} d\Omega_{\underset{\sim}{p_1}}} (\underset{\sim}{p} - \underset{\sim}{p_1}) \tag{2.9 a}$$

$$D_{ij} = \frac{1}{2}\frac{v}{V}\int d\,E_{\underset{\sim}{p_1}}\, d\Omega_{\underset{\sim}{p_1}} \frac{d^2\sigma}{dE_{\underset{\sim}{p_1}} d\Omega_{\underset{\sim}{p_1}}} (\underset{\sim}{p} - \underset{\sim}{p_1})_i (\underset{\sim}{p} - \underset{\sim}{p_1})_j \tag{2.9 b}$$

where v is the proton's incoming velocity and V the normalized volume of the proton's wave function.

3. Calculation of the kinetic coefficients for Nuclear Matter

In this section we briefly introduce the approximations for the momentum transfers within the spirit of high energy collisions and present the model we use for $\frac{d^2\sigma}{d\Omega\, d\, E}$.

Finally we give analytical expressions for the transport coefficients in nuclear matter.

3.1. The Model

We express the longitudinal (along the beam axis) and transverse components of the transferred momentum ($\underset{\sim}{q} = \underset{\sim}{p} - \underset{\sim}{p}'$) in terms of the quantities $\delta p = |\underset{\sim}{p}| - |\underset{\sim}{p}'|$ and $\cos\theta = \frac{\underset{\sim}{p}\cdot\underset{\sim}{p}'}{|p|\,|p'|}$

Since we are interested in high energy collisions where the transferred momentum is small compared to the incident one, we consider these quantities to be small, so that we can write :

$$q = p - p'\cos\theta \simeq \delta p + \frac{p}{2}\theta^2 \tag{3.1 a}$$

$$q_x = -p'\cos\varphi \simeq -p\,\theta\cos\varphi \tag{3.1 b}$$

$$q_y = -p'\sin\varphi \simeq -p\,\theta\sin\varphi \tag{3.1 c}$$

φ being the corresponding azimuthal angle. On the basis of nucleon-nucleon elastic scattering it is easy to see that the magnitudes δp and $p\theta^2$ are of the same order.

Following ref [4] we express the double differential cross-section in the form

$$\frac{d^2\sigma}{d\Omega\, dE} = N_{eff} \left.\frac{d\sigma}{d\Omega}\right|_{NN} S(q,E) \tag{3.2}$$

The first factor N_{eff} is the effective number of target particles calculated geometrically as in Glauber Theory for one step reactions. It should also take into account the loss of flux in the one-step channel due to compound nucleus processes. The second factor $\frac{d\sigma}{d\Omega}|_{NN}$ is the nucleon-nucleon cross-section at the corresponding laboratory energy and angle. In the range of energies considered, it can be written as :

$$\frac{d\sigma}{d\Omega}|_{NN} = \sigma_0 \, e^{\frac{-q^2}{\Delta^2}} \approx \sigma_0 \, e^{\frac{-p^2\theta^2}{\Delta^2}} \qquad (3.3)$$

where σ_0 is a constant and $\Delta \approx 400$ MeV/c. The response function S (q,E) will be calculated in the Fermi gas model. It has different expressions, according to the magnitude of q (see ref[4),5)]). For $q \leqslant 2\,p_F$

$$S(q,E) = \frac{3\,m}{4p_F^3}\,\frac{1}{p\theta}\;2\,m\,E \equiv S^1$$

$$\text{if } 0 \leqslant 2\,m\,E \leqslant 2q\,p_F - q^2 \qquad (3.4a)$$

$$S(q,E) = \frac{3\,m}{4p_F^3}\,\frac{1}{p\theta}\left[p_F^2 - \left(\frac{p^2\theta^2}{2} - m\,E\right)^2/p^2\theta^2\right] \equiv S^2$$

$$\text{if } 2\,q\,p_F - q^2 \leqslant 2\,m\,E \leqslant 2q\,p_F + q^2 \qquad (3.4\,b)$$

for $q \geqslant 2\,p_F$

$$S\,(q,E) = S^2 \quad \text{if } q^2 - 2\,q\,p_F \leqslant 2\,m\,E \leqslant q^2 + 2\,q\,p_F \qquad (3.4c)$$

3.2. Results

Inserting eqs. (3.2), (3.3) and (3.4) into (2.9 a) and (2.9 b) we obtain to lowest in δp and $p\,\theta^2$:

$$\Gamma_{/\!/} = 2\,\pi\,v\,\rho_{eff}\,\sigma_0\left\{\frac{1}{2}\,p < \theta^3 > + \frac{3}{8}\,\frac{p^2}{p_F} < \theta^4 > - \frac{1}{32}\,\frac{p^4}{p_F^3} < \theta^6 > + p < \theta^3 >^c\right\} \qquad (3.5\,a)$$

$$\Gamma_x = \Gamma_y = 0 \qquad (3.5\,b)$$

$$D_{/\!/\,/\!/} = 0 \qquad (3.5\,c)$$

$$D_{xx} = D_{yy} = 2\,\pi\,v\,\rho_{eff}\,\sigma_0\left\{\frac{3}{16}\,\frac{p^3}{p_F} < \theta^4 > - \frac{1}{64}\,\frac{p^5}{p_F^3} < \theta^6 > + \frac{1}{4}\,p^2 < \theta^3 >^c\right\} \qquad (3.5\,d)$$

$$D_{x/\!/} = D_{y/\!/} = D_{xy} = 0 \qquad (3.5\,e)$$

where $\rho_{eff} = N_{eff}/V$ is the effective density and

$$< \theta^n > = \int_0^{2p_F/p} d\theta\;\theta^n\,e^{\frac{-p^2}{\Delta^2}\theta^2} \qquad (3.6\,a)$$

$$< \theta^n >^c = \int_{2p_F/p}^{\pi} d\theta\;\theta^n\,e^{\frac{-p^2}{\Delta^2}\theta^2} \qquad (3.6\,b)$$

The coefficients in eqs (3.5 b) and (3.5e) vanish due to axial symmetry while the longitudinal diffusion in (3.5 c) is of higher order in δp and $p\,\theta^2$.

In the low momentum transfer limit, eqs (3.5 a) and (3.5 d) reduce to (LMT) :

$$\Gamma^{LMT} = v\,\rho_{eff}\,\sigma^T_{NN}\,\frac{\Delta^2}{2p}\left\{1 + \frac{9\sqrt{\pi}}{8}\,\frac{\Delta}{2p_F} - \frac{15\sqrt{\pi}}{16}\,\frac{\Delta^3}{(2p_F)^3}\right\} \qquad (3.7\,a)$$

$$D^{LMT}_{xx} = D^{LMT}_{yy} = v\,\rho_{eff}\,\sigma^{T}_{NN}\,\Delta^2 \left\{ \frac{9\sqrt{\pi}}{32}\frac{\Delta}{2p_F} - \frac{15\sqrt{\pi}}{64}\frac{\Delta^3}{(2p_F)^3} \right\} \qquad (3.7\ b)$$

while in the high transfer regime they read (HMT) :

$$\Gamma^{HMT} = v\,\rho_{eff}\,\sigma^{T}_{NN}\,\frac{1}{p}\,\Delta^2 \qquad (3.8\ a)$$

$$D^{HMT}_{xx} = D^{HMT}_{yy} = \frac{1}{4}\,v\,\rho_{eff}\,\sigma^{T}_{NN}\,\Delta^2 \qquad (3.8\ b)$$

where

$$\sigma^{T}_{NN} = \int d\Omega \frac{d\sigma}{d\Omega}\Big|_{NN} \qquad (3.9)$$

Although the Pauli principle is taken into account in the present model it does not affect the kinetic coefficients in the high momentum transfer regime (eqs. (3.8 a) and (3.8 b)). In this case the diffusion coefficient coincides with the one obtained in Glauber Theory[1]). In the low momentum transfer regime the Pauli blocking is effective. For example if we compare the dominant contribution in Γ^{LMT} and Γ^{HMT} (eqs (3.7 a) and (3.8 a)) we observe that there is a reduction of the friction coefficient by a factor of two. Furthermore in the first case we find other correction terms which depend on the Fermi momentum and are absent in the HMT domain. Similar considerations also hold for the diffusion coefficient: it is also strongly affected by Pauli blocking.

In order to compare the relative importance of friction and diffusion processes, we estimate the ration between typical times associated with $\Gamma_{/\!/}$ and D_{xx}. We get :

$$\left.\frac{\tau_{\Gamma}}{\tau_{D_{xx}}}\right)^{LMT} \approx \frac{9\sqrt{\pi}}{16}\,\frac{\Delta}{2p_F} \qquad (3.10)$$

and

$$\left.\frac{\tau_{\Gamma_{/\!/}}}{\tau_{D_{xx}}}\right)^{HMT} = \frac{1}{4} \qquad (3.11)$$

We note that in both cases the momentum loss process requires shorter times, especially in the LMT regime where $\frac{\Delta}{2p_F} \ll 1$. In the HMT regime it is independent of the mean transferred momentum Δ.

4. Conclusions

Starting from the exact equations for the effective dynamics of a high energy proton interacting with a nucleus we derive a transport equation for the Wigner function of the proton density. We obtain simple general expressions for the kinetic coefficients and analytical expressions are given for nuclear matter. The available result for the diffusion coefficient calculated from Glauber Theory is reproduced in the high momentum transfer regime ($q > 2\,p_F$). In the low momentum transfer regime ($q < 2\,p_F$) the Pauli blocking inhibites the friction and diffusion processes strongly. Typical values of the momentum transfer in such collisions fall in this range.

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Appendix

In this appendix we show that the approximations which lead to eqs (2.3) do not violate the conservation of total energy. For that purpose we briefly recall the necessary steps in the derivation of the effective dynamics of subsystems ρ and R (see ref[3] for details). The total density of the many body sytem (in this case proton and nucleus) is written as [3]

$$\hat{F} = \hat{R}\hat{\rho} + \hat{F}' \qquad (A.1)$$

where $\hat{F}'$ corresponds to the correlated part of the density and $\hat{R}\hat{\rho}$ to the uncorrelated part. In the same approximation used to derive eq (2.3), $\hat{F}'$ can be written as [3] :

$$\hat{F}' = -i\int_0^t dt'\; G_0(t,t')\,[H', \hat{R}_0\hat{\rho} + \hat{R}\hat{\rho}_0] \qquad (A.2)$$

The total energy is given by (from eq (A.1))

$$E = tr\,(H\hat{F}) = tr\,(H\,\hat{R}\hat{\rho}) + tr\,(H\,\hat{F}') \qquad (A.3)$$

The first term on the r.h.s. of eq (A.3) corresponds to the uncorrelated part of the energy and the second term to the correlation energy. We will show in what follows that the energy lost from the uncorrelated part due to the interaction is stored in the correlated part so that the total energy remains constant. In order to see this we consider

$$\frac{dE}{dt} = tr\,\{H\,(\hat{R}_0\dot{\hat{\rho}} + \dot{\hat{R}}\hat{\rho}_0)\} + tr\,\{H\,\dot{\hat{F}}'\} \qquad (A.4)$$

The first term on the r.h.s can be evaluated using the equations of motion for $\hat{R}$ and $\hat{\rho}$ (the eqs (2.3)) and a straightforward calculation yields

$$tr\,\{H(\hat{R}_0\dot{\hat{\rho}} + \dot{\hat{R}}\hat{\rho}_0)\} = -\,tr\,\{H_p\,\hat{O}\,(\hat{R}_0\hat{\rho}) + H_N\hat{O}\,(\hat{R}\hat{\rho}_0)\} \qquad (A.5)$$

where

$$\hat{O}\,(\hat{R}\,\hat{\rho}) = \int_0^t dt'\,[H', G_0(t,t')\,[H',\hat{R}\hat{\rho}]\,] \qquad (A.6)$$

Inserting eq (A.2) in the second term of the r.h.s of eq (A.4) we get, after some algebra

$$tr\,(H\dot{\hat{F}}') = tr\,\{H_p\,\hat{O}\,(\hat{R}_0\hat{\rho}) + H_N\,\hat{O}\,(\hat{R}\,\hat{\rho}_0)\} \qquad (A.7)$$

and therefore $\frac{dE}{dt} = 0$

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