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UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA  
CAIXA POSTAL 20516  
01498 - SÃO PAULO - SP  
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QUARK STRUCTURE OF THE NUCLEON AND QUANTUM  
HADRODYNAMICS

**T. Frederico, B.V. Carlson and R.A. Rêgo**

Centro Técnico Aeroespacial, Instituto de  
Estudos Avançados, 12000 São José dos Campos,  
SP, Brazil

**M.S. Hussein**

Instituto de Física, Universidade de São Paulo

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T. Frederico, B.V. Carlson and R.A. Rêgo

Centro Técnico Aeroespacial,  
Instituto de Estudos Avançados,  
12000 São José dos Campos, S.P., Brazil

and

M.S. Hussein\*\*

Instituto de Física, Universidade de São Paulo,  
C.P. 20516, 01498 São Paulo, S.P., Brazil

**ABSTRACT**

The effect of the quark structure of the nucleon on nuclear matter properties is investigated within a  $\sigma$ - $\omega$ - $q$  model. We have found that in order to reproduce the known properties of normal nuclear matter, the mass of the scalar meson must decrease with increasing nuclear density, in accord with the recent finding of Bernard, Meissner and Zahed.

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Relativistic description of nuclear structure and reactions, within Quantum Hadrodynamics (QHD) has been greatly developed during the last several years<sup>1)</sup>. In such a theory mesons appear explicitly together with the nucleons, with the Dirac equation used to describe the latter's motion. Both mesons and nucleons, however, are treated as structureless. Although several major advantages over the conventional non-relativistic theory with effective two-body interactions have been clearly demonstrated, it is expected, however, that certain nuclear observables may eventually call for the meson and nucleon substructure for their explanation, thus requiring a major revision of QHD.

A particularly important question which bears on the above statement is how the theory describes nuclear matter at high densities. This is an important issue both for astrophysics in connection with supernova explosions and nuclear heavy-ion reactions at relativistic energies.

The purpose of this Letter is to investigate the effect of quark substructure of the nucleon on the density dependence of the physical parameters entering QHD. The theoretical framework we adopt here is a  $\sigma$ - $\omega$  model Lagrangian for the quarks. The interaction Lagrangian density is taken to be

$$\mathcal{L}_I^{q(\sigma-\omega)} = \bar{\Psi}_q g_S^q \phi \Psi_q - \bar{\Psi}_q g_V^q \gamma^\mu \gamma_5 \Psi_q \quad (1)$$

where  $\psi_q$  is the quark field and  $g_s^q$  and  $g_v^q$  are the scalar meson and vector meson-quark coupling constants, respectively.

$\phi$  and  $V^\mu$  are scalar and vector meson fields respectively.

Eq. (1) should be contrasted with the QHD equation for the point nucleon

$$\mathcal{L}_I^{N(\sigma-\omega)} = \bar{\Psi}_N g_s^N \phi \Psi_N - \bar{\Psi}_N g_v^N V^\mu \gamma_\mu \Psi_N \quad (2)$$

By comparing 1 and 2, it becomes natural to make the identification

$$g_s^N \bar{\Psi}_N \phi \Psi_N = g_s^q \langle N | \sum_{i=1}^3 \bar{\Psi}_q \phi \Psi_q | N \rangle \quad (3)$$

$$g_v^N \bar{\Psi}_N V^\mu \gamma_\mu \Psi_N = g_v^q \langle N | \sum_{i=1}^3 \bar{\Psi}_q V^\mu \gamma_\mu \Psi_q | N \rangle \quad (4)$$

where the  $\langle N | | N \rangle$  matrix element are nucleon ground state expectation values. The model Lagrangian, Eq. (2) is very close to that employed by Celenza et al.<sup>3)</sup> in their many-body soliton dynamics. The nucleon emerges as a solitary wave. Confinement in this model is insured through the introduction of a fictitious scalar  $\chi$ -meson coupled to the quarks just like the  $\sigma$ . In our discussion below, we instead introduce an ad-hoc term,  $\bar{\Psi}_q \frac{1}{2} (1+\beta) V(r) \Psi_q$ , guaranteed to insure confinement as demonstrated by Ferreira and Zagury<sup>3)</sup>.

The quark field wave function  $\psi_q(r)$  is obtained from the following Dirac equation

$$[i\gamma - M_0 + g_s^q \phi - g_v^q \gamma_0 V^0 - \frac{1}{2} (1+\beta) U(r)] \psi_q = 0 \quad (5)$$

Because of its analytical simplicity we choose a harmonic oscillator form for the potential  $U(r)$ <sup>3)</sup>

$$U(r) = U_0 + \frac{1}{2} K r^2 \quad (6)$$

where  $U_0$ ,  $K$  and  $M_0$  are chosen to adjust the mass and a root mean square radius of 0.85 fm for a free nucleon.

The static solution for the fundamental level of the quark inside the nucleon is given by

$$\psi_q(r) = \frac{1}{A} \begin{pmatrix} f_0(r) \\ i r \frac{\sqrt{K/2}}{\sqrt{E'+M'_0}} f_0(r) \vec{n} \cdot \vec{\sigma} \end{pmatrix} \frac{\chi_m}{\sqrt{4\pi}} \quad (7)$$

where,

$$f_0(r) = \exp \left[ -\frac{r^2}{2} \sqrt{\frac{K(E'+M'_0)}{2}} \right],$$

and

$$A^2 = \frac{\sqrt{\pi}}{4} \frac{\left[ 1 + \frac{3}{2} \frac{\sqrt{K/2}}{(E'+M'_0)^{3/2}} \right]}{\left[ K/2 (E'+M'_0) \right]^{3/4}} \quad (8)$$

The effective energy and effective mass for the quark are defined by:

$$E' = E - g_V^q V^0, \quad M_0' = M_0 - g_s^q \phi \quad (9)$$

where  $E'$  is given by

$$E' = M_0' + 3 \sqrt{\frac{\kappa}{2(E'+M_0')}} \quad (10)$$

We construct the internal nuclear wave function as a symmetrized product of three independent quark wave functions.

We now calculate the effective nucleon meson couplings by equating the corresponding Lagrangean interaction terms<sup>4)</sup>. For the vector meson coupling we have, from Eq. (3):

$$g_V^N \bar{\Psi}_N \gamma_\mu V^\mu \Psi_N = g_V^q \langle N | \sum_{i=1}^3 \bar{\Psi}_q \gamma_\mu V^\mu \Psi_q | N \rangle \quad (11)$$

Evaluating this expression in the C.M. of the nucleon, we obtain:

$$g_V^N = 3 g_V^q \quad (12)$$

This simple result is due to the vector character of the coupling and our independent quark model for the nucleon.

Using the same approach, we obtain for the scalar

meson coupling

$$g_s^N = 3 g_s^q \frac{\left[ 1 - \frac{3}{2} \frac{\sqrt{\kappa/2}}{(E'+M_0')^{3/2}} \right]}{\left[ 1 + \frac{3}{2} \frac{\sqrt{\kappa/2}}{(E'+M_0')^{3/2}} \right]} + g_s^C \quad (13)$$

where we have introduced, what we may call the "colored" coupling constant,  $g_s^C$ , which takes into account the short range  $\sigma$ -meson coupling with all other nucleon constituents aside from the three valence quarks, which contribute the first term. Note that there is no need for  $g_V^C$  to be added to Eq. (12) since  $g_V^N$  is a constant.

We are now in a position to study the properties of the meson-nucleon system inside nuclear matter. Before presenting the results within our model we first cite those obtained by Walecka within his first simple QHD calculation<sup>5)</sup>. He finds for the energy density of the nuclear system the following

$$E = \frac{g_V^2}{2m_V^2} \rho^2 + \frac{M_S^2}{2(g_S)^2} (M - M^*)^2 + \frac{4}{(2\pi)^3} \int_0^{k_F} d^3k (k^2 + M^{*2})^{1/2} \quad (14)$$

where  $k_F$  is the Fermi momentum and  $M-M^*$  is the difference between the free and effective nucleon masses, given by  $g_S \phi$ . Minimizing  $E$  with respect to  $M^*$ , Walecka obtains the self consistency equation for  $M^*$ , namely

$$M^* = M - \frac{g_S^2}{m_S^2} \frac{M^*}{\pi^2} \left[ k_F E_F^* - M^{*2} \ln \left( \frac{k_F + E_F^*}{M^*} \right) \right] \quad (15)$$

where  $E_F^* = \sqrt{M^{*2} + k_F^2}$  (16)

It is a simple matter to repeat the above variational calculation given the nucleon-structure modified scalar coupling constant of Eq. (3), which clearly depends on the variational parameter  $M^*$  through the dependence of the quark effective mass  $M'_0$  Eq. (9) on the scalar field  $\phi = \frac{1}{g_S} (M-M^*)$ . As a result of this calculation, we obtain a new effective  $\sigma$ -nucleon coupling constant,  $\tilde{g}_S^N$ , given by

$$\frac{\tilde{g}_S^N}{m_S^2} = \frac{g_S^N}{m_S^2} \left[ 1 + \frac{27 \sqrt{\frac{k_F}{2}} \left( \frac{g_S}{m_S} \right)^2 4 M_N^*}{\left( 1 + \frac{3 \sqrt{k_F/2}}{2(E_F + M'_0)^{3/2}} \right)^3} \frac{k_F E_F^* - M_N^{*2} \ln \left( \frac{k_F + E_F^*}{M^*} \right)}{4\pi^2 (E_F + M'_0)^{5/2}} \right]^{-1} \quad (17)$$

and a new effective mass

$$M^* = M - \frac{\tilde{g}_S^N}{m_S^2} \frac{M^*}{\pi^2} \left[ k_F E_F^* - M^{*2} \ln \left( \frac{k_F + E_F^*}{M^*} \right) \right] \quad (18)$$

In figure 1 we plot the effective coupling constants,  $\tilde{g}_S^N/m_S$  and  $g_S^N/m_S$  vs the (nuclear density) $^{1/3} \equiv k_F$ , for two values of the "colored" coupling constants,  $g_S^C$ , which are given in the form of two values of the core radius,  $r_C = 0.85$  fm and  $r_C = 0.5$  fm. We observe in the figure that with the meson-quark coupling  $\frac{g_S^q}{m_S} = 3$ ,  $\tilde{g}_S/m_S$  decreases appreciably with  $k_F$ . At a nuclear matter density of  $k_F = 1.44$  fm $^{-1}$ , it attains a value of  $\sim 13$ , much smaller than the value of 17.4 obtained by Walecka in his nuclear matter calculation. Thus nuclear matter responds strongly to changing properties of the nucleon. This findings contradicts some of the assumptions made by Shakin and collaborators in their soliton description of the nucleon<sup>2)</sup>.

To restore nuclear matter to its "normal" behaviour, that is to raise the value of  $\tilde{g}_S/m_S$  to 17.4, which would guarantee the normal saturation properties, we are forced to change an important property of the  $\sigma$ -meson, namely its mass. Thus we have repeated the variational calculation described above with a density-dependent  $\sigma$ -meson mass,  $m_S(k_F)$ . We have assumed a linear dependence on the density,  $m_S(k_F)/m_S(0) = 1 - \lambda k_F^3$ . The self consistent calculation then gave rise to the dashed curve in figure 1, with  $\lambda = 0.027$ . The effective nucleon mass, for this value of  $\lambda$  is found to be  $0.56 M$ , quite close to the accepted value.

Another property of the nucleon which has attracted

a lot of attention in the last few years is the apparent increase of its effective size inside the nucleus. This is an important feature believed to underly the physics behind the EMC effect<sup>6)</sup>. We have calculated the rms radius of the nucleon  $\langle r_n^2 \rangle$  within our model, and have verified an appreciable increase with increasing nuclear density. At nuclear matter density there is about a 6% increase in  $\langle r_n^2 \rangle$  over the free space value for  $\lambda=0$  and a 10% increase with  $\lambda=0.027$ . Thus the reduction in the mass of the  $\sigma$ -meson at higher densities helps the nucleon attain a larger size, an important property required in the EMC effect.

The decrease of  $m_s$  with  $k_F$  obtained above, is very close to the behaviour, within the same range of  $k_F$  values, obtained by Bernard et al.<sup>7)</sup> within the Nambu-Jona-Lasinio model<sup>8)</sup>. Our finding certainly supplies more evidence in favor of a softer equation of state of nuclear matter than that obtained within the Walecka model, and contradicts the conclusion of Horowitz and Serot<sup>9)</sup> about an increasing  $m_s$  with increasing nuclear density.

In conclusion, we have studied the response of nuclear matter to changing properties of the nucleon, using a model meson-3q-nucleon coupled system both in free space and in the medium. We have found that in order to reproduce the known properties of normal nuclear matter the  $\sigma$ -meson mass must appreciably decrease with the nuclear density in accord with

the recent finding of Bernard et al.<sup>7)</sup>. It would be important to check how the  $q\bar{q}$  structure of the  $\sigma$ -meson affects the result. This will be investigated in a future work.

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FIGURE CAPTION

Fig. 1 - The effective  $\sigma$ -meson-nucleon coupling parameter  $\frac{g_S^N}{m_S}$  vs  $k_F$ . Lower full curve represent the case with a constant  $m_S$  ( $\lambda = 0$ ), whereas the upper full curve is obtained with  $m_S$  ( $\lambda = 0.027$ ) (see text). The long dashed curve represents  $\frac{g_S^N}{m_S}$  (coupling in free space). All these curves were obtained with nucleon core radius of 0.85 fm. The middle curves correspond to core radius of 0.5 fm; dashed-dotted, with  $m_S$  ( $\lambda = 0.027$ ) and small-dashed with  $m_S$  ( $\lambda = 0$ ).

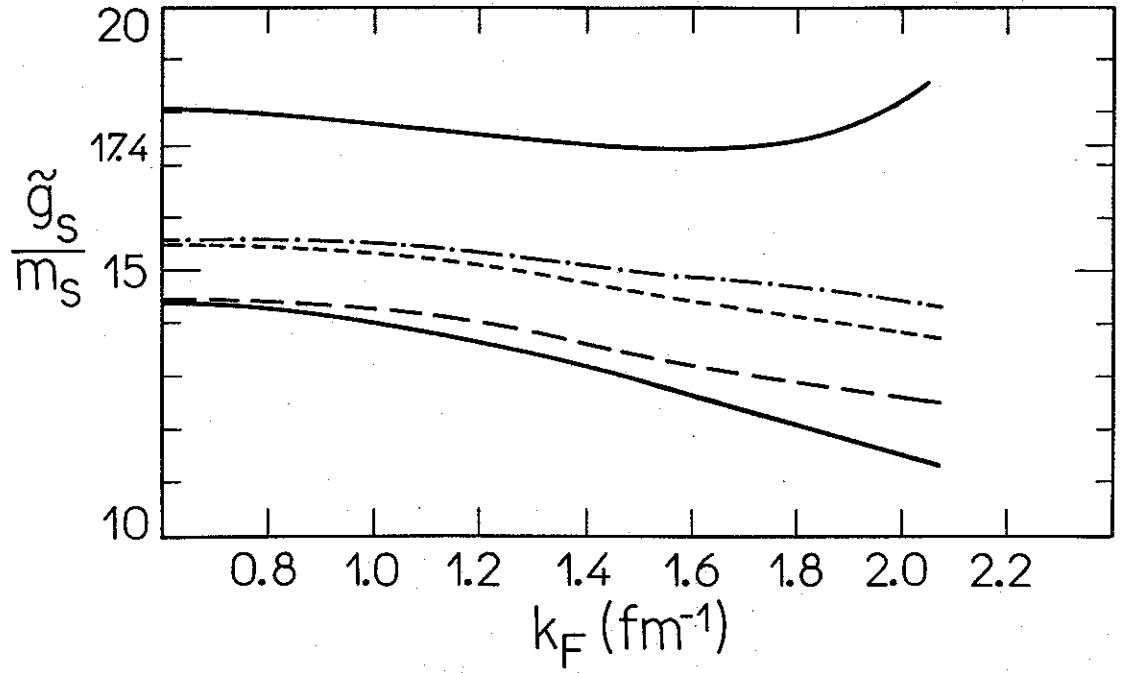


Fig. 1